INELASTIC RESPONSE SPECTRA FOR BIDIRECTIONAL GROUND MOTIONS

WANG Dong-sheng[1]， **LI Hong-nan**[2]， **WANG Guo-xin**[3]， **Fan Ying-Fang**[4]

[1]*Professor,Iinstitute of Road and Bridge Eng., Dalian Maritime University, Dalian, ,China*
[2]*Professor, School of Civil and Hydraul. Eng., Dalian Univ. of Technol., Dalian, China*
[3] *Associated Professor, School of Civil and Hydraul. Eng., Dalian Univ. of Technol., Dalian , China*
[4] *Associated Professor, Institute of Road and Bridge Eng., Dalian Maritime University, Dalian, China*
Email: dswang@newmail.dlmu.edu.cn, hnli@dlut.edu.cn, gxwang@dlut.edu.cn, fanyf@dlut.edu.cn

ABSTRACT : More attention has been paid to the inelastic design spectra with the development of performance based seismic design, because it can be used to estimate the maximum displacement demand of structures. The work is aimed at developing a bidirectional inelastic response spectrum by considering the multi-component earthquake excitation and coupled characteristics of structural inelastic response. The normalized equation of motion of the single-mass-system with two lateral freedoms along its perpendicular principal axes is formulated using strength reduction factors when subjected to two directional ground motions. Restore force characteristics of the two-degree-of-single-mass-system are determined by two-dimensional yield-surface plasticity theory. Then inelastic spectra for bidirectional ground motions is presented that is defined as the ratio of the maximum displacement responses in the same principal axes direction of the single-mass-system when subjected to two and one dimensional ground motions respectively. The pseudo-constant ductility inelastic spectra for bidirectional ground motions is given, which assumed ductility factor μ_x and μ_y keep constant only in one directional earthquake input is given. The effects of periods, force reduction factors or displacement ductility factors, and damping on spectra amplitude are discussed by statistic analysis of 3 groups of bidirectional recordings on hard, medium and soft site.

KEYWORDS: bidirectional ground motions, inelastic response spectra, strength reduction factors, ductility factors, soil conditions

1. INTRODUCTION

Elastic response spectrum analysis method has been widely accepted in seismic design code over the world since it was developed in the 1940s. Structures usually are designed suffered inelastic response during large earthquakes in considering of principle of economic rationality, then Newmark et al(1982)studied the seismic response of inelastic SDOF system and inelastic response spectra were presented. The general trend in developed inelastic response spectra is from their elastic counterparts, by means of a reduction factor R, named the force reduction factor. The factor R can be related to the system ductility μ according to the system periods T (R-μ-T relationship). After Northridge Earthquake in USA and Kobe Earthquake in Japan in the 1990s, performance based seismic design has been developed，and more attention has been paid to inelastic design spectra because it can be used to estimate the maximum displacement demand of structures. Especially the pushover analysis procedure is applied and responses of MDOF system are calculated by the equivalent inelastic SDOF system (Fajfar,1999, Chopra et al. 2001). The major research on inelastic earthquake spectra are confines on SDOF system and unidirectional ground motions(（Miranda et al. 1992 , Riddell et al. 2002,Zhai C. H. et al. 2004,Lu X.L. et al.2004, Chopra et al. 2004, Jarenprasert et al. 2006)，in fact the real structures under earthquake excitation will have multidimensional response. The work of the paper is aimed at developing a bidirectional inelastic response spectrum by considering the multi-component earthquake excitation and coupled characteristics of structural response, which is defined as the ratio of the maximum displacement responses in the same principal axes direction of a single-mass-system with two translational freedoms along its perpendicular principal axes when subjected to two and one dimensional ground motions respectively.

2 INELASTIC RESPONSE SPECTRA FOR BIDIRECTIONAL GROUND MOTIONS

The motion equation of the SDOF system when subjected to the ground motion $\ddot{x}_g(t)$ is

$$m_x\ddot{x}(t)+c_x\dot{x}(t)+f(x,t)=-m_x\ddot{x}_g(t) \qquad (1)$$

Where m_x, c_x and $f(x,t)$ =the mass, damping coefficient and restoring force of the system respectively, and the subscript x indicate earthquake input direction.

The yield displacement of the system is x_y, and the yield force is $f_{x,y}=k_x x_y$,where k_x=system stiffness. If non-dimensional displacement time history $\mu_x(t)=x(t)/x_y$ is defined, it is convenient to normalize (1) to (Miranda et al.1992)

$$\ddot{\mu}_x(t)+2\xi_x\omega_x\dot{\mu}_x(t)+\omega_x^2\frac{f(x,t)}{f_{x,y}}=-\frac{\omega_x^2}{\eta_x}\frac{\ddot{x}_g(t)}{\max(|\ddot{x}_g|)} \qquad (2)$$

where $\omega_x=\sqrt{k_x/m_x}$ is system natural circular frequency; $\xi_x=c_x/2\sqrt{m_x k_x}$ is system damping ratio ; $\eta_x=f_{x,y}/[m_x\cdot\max(|\ddot{x}_g|)]$ is system non-dimensional strength. The system ductility $\mu_x=\max(|x(t)|)/x_y$, defined by the ratio of the system maximum displacement response to its yield displacement, then $\mu_x=\max(|\mu_x(t)|)$.

The force response of the elastic SDOF system corresponding to the above inelastic one can be got by elastic response spectra with assumption of low damping ratio, it is

$$f_{x,e}=m_x\cdot\beta_x(\omega_x,\xi_x)\cdot\max(|\ddot{x}_g|) \qquad (3)$$

Where $\beta_x(\omega_x,\xi_x)$ is the amplification factor response spectra for ground motion $\ddot{x}_g(t)$.

The strength reduction factor R_x can be expressed as

$$R_x=f_{x,e}/f_{x,y} \qquad (4)$$

Substitution of (3) into (4) , the definition of η_x gives

$$\eta_x=\beta_x(\omega_x,\xi_x)/R_x \qquad (5)$$

Substitution of （5）into（2）leads to

$$\ddot{\mu}_x(t)+2\xi_x\omega_x\dot{\mu}_x(t)+\omega_x^2\frac{f(x,t)}{f_{x,y}}=-\frac{\omega_x^2\cdot R_x}{\beta_x(\omega_x,\xi_x)}\frac{\ddot{x}_g(t)}{\max(|\ddot{x}_g|)} \qquad (6)$$

(6) can be seemed as basic equation of R-μ-T(ω_x) for inelastic response spectra. Using (6) and characters of elastic spectra, the limit characters of inelastic spectra can be proved: （1）$\omega_x\to 0, R_x=\mu_x$；（2）$\omega_x\to\infty, R_x=1$。

(6) can be developed to the condition of bidirectional ground motion excitations. For a single mass system, has two translational freedoms along its perpendicular principal axes when subjected to bidirectional ground motions, equations of the motions are

$$\begin{cases} m_x\ddot{x}(t)+c_x\dot{x}(t)+f(x,t)=-m_x\ddot{x}_g(t) \\ m_y\ddot{y}(t)+c_y\dot{y}(t)+f(y,t)=-m_y\ddot{y}_g(t) \end{cases} \qquad (7)$$

Where subscript *x* and *y* represent its principal axes respectively. The assumptions of (7) are displacement responses are small and torsional effects are omitted.

(7) can be worked as the same as (1), and get

$$\begin{cases} \ddot{\mu}_x(t)+2\xi_x\omega_x\dot{\mu}_x(t)+\omega_x^2\dfrac{f(x,t)}{f_{x,y}}=-\dfrac{\omega_x^2\cdot R_x}{\beta_x(\omega_x,\xi_x)}\dfrac{\ddot{x}_g(t)}{\max(|\ddot{x}_g|)} \\ k\ddot{\mu}_y(t)+2k\xi_y\omega_y\dot{\mu}_y(t)+k\omega_y^2\dfrac{f(y,t)}{f_{y,y}}=-k\dfrac{\omega_y^2\cdot R_y}{\beta_y(\omega_y,\xi_y)}\dfrac{\ddot{y}_g(t)}{\max(|\ddot{y}_g|)} \end{cases} \qquad (8)$$

Where $k = \frac{m_y}{m_x} \cdot \frac{S_{dy}}{S_{dx}} \cdot \frac{R_x}{R_y}$，$S_{dx}$ and S_{dy} are displacement spectra of earthquake ground motions about x axis、 y axis respectively. Usually m_x=m_y is assumed.

The parameters k in (8) can not be divided because $f_{x,y} = \omega_x{}^2$, $f_{y,y} = k\omega_y{}^2$ are defined in the step-by-step integration of (7), the parameters k ensure the yield surface function similar to （9）, then the ductility factor responses of (7) and (8) are the same.

The two-dimensional yield-surface function is

$$F(f(x,t), f(y,t)) = \left|\left(\frac{f(x,t)}{f_{x,y}}\right)\right|^a + \left|\left(\frac{f(y,t)}{f_{y,y}}\right)\right|^b = 1.0 \qquad (9)$$

Where a=b=2，like a ellipsoid。

The motion equations of (8) satisfies: when $F(f(x,t), f(y,t)) < 1.0$, two translational freedoms are uncoupled, while $F(f(x,t), f(y,t)) = 1.0$, they are coupled, yielding and plastic flow are occurred. elastic-plastic restoring force model are used in the paper.

Supposed the response ductility factors are $\mu_x{}^b$和$\mu_y{}^b$ for equation (8)， and $\mu_x{}^s$ for SDOF system in equation (6), Bidirectional Ground Motion Spectra（BGMS）can be defined as

$$BGMS = \mu_x{}^b / \mu_x{}^s \qquad (10)$$

Effects on BGMS , in spite of earthquake ground motion, hysteretic characters, damping as the same as inelastic spectra for SDOF system， are system circular frequency ω_x 、ω_y, strength reduction factor R_x、 R_y and parameters k.

The system circular frequency ω_x 、ω_y are taken as horizontal ordinate(X-axis) and longitudinal coordinates(Y-axis) respectively. The strength reduction factors R_x、R_y are taken as parameters. k can be determined by strength reduction factors and earthquake excitations. If R_x、 R_y are defined as invariable the constant strength reduction factors BGMS are given(Wang D.S. et al. 2004); while μ_x or μ_y in (6) are selected as invariable the pseudo-constant ductility BGMS are got.

The constant strength reduction factors BGMS have some drawbacks that $\omega_x \to \infty$, R_x should be 1.0, this means the structures should be elastic. The pseudo-constant ductility BGMS are discussed in this paper.

Given μ_x in (6) then $R_x(\mu_x, \omega_x, \xi_x)$ are got by the theory of inelastic spectra of SDOF system. When ground motions act along y axis alone, $R_y(\mu_y, \omega_y, \xi_y)$ also are obtained by the same procedure.

Substitution $R_x(\mu_x, \omega_x, \xi_x)$ and $R_y(\mu_y, \omega_y, \xi_y)$ into (8)

$$\begin{cases} \ddot{\mu}_x(t) + 2\xi_x\omega_x\dot{\mu}_x(t) + \omega_x{}^2 \frac{f(x,t)}{f_{x,y}} = -\frac{\omega_x{}^2 R_x(\mu_x,\omega_x,\xi_x)}{\beta_x(\omega_x,\xi_x)} \frac{\ddot{x}_g(t)}{\max(|\ddot{x}_g|)} \\ k\ddot{\mu}_y(t) + 2k\xi_y\omega_y\dot{\mu}_y(t) + k\omega_y{}^2 \frac{f(y,t)}{f_{y,y}} = -k\frac{\omega_y{}^2 R_y(\mu_y,\omega_y,\xi_y)}{\beta_y(\omega_y,\xi_y)} \frac{\ddot{y}_g(t)}{\max(|\ddot{y}_g|)} \end{cases} \qquad (11)$$

Where parameters $k = \frac{m_y}{m_x} \cdot \frac{S_{dy}}{S_{dx}} \cdot \frac{R_x(\mu_x, \omega_x, \xi_x)}{R_y(\mu_y, \omega_y, \xi_y)}$.

$\mu_x{}^b$ calculated by step-by-step integration of (11) , $\mu_x{}^s$ according to the $R_x(\mu_x, \omega_x, \xi_x)$ in (6) , are substituted into (10), then the pseudo-constant ductility BGMS are presented.

If give $\ddot{y}_g(t)$=0, (8) turns out to (6), and the denominator, the numerator of BGMS are multiplied by the yield displacement x_y, BGMS became the ratio of the maximum displacement responses in the same principal axes direction of the single-mass-system when subjected to two and one dimensional ground motions respectively.

3 STATISTIC ANALYSIS OF PSEUDO-CONSTANT DUCTILITY BGMS

3 earthquake recording groups, 10 recordings are included in each group, classified by hard, medium and soft site conditions are selected, and the average BGMS are calculated. The rules for the earthquake recording election are that: (1) the earthquake magnitudes are greater then Ms6.0, so that structures may be damaged, (2)fault distances are 20-40km, near fault ground motions are not included, (3)the high pass frequencies are lower then 0.2Hz。the details of the earthquake records can be found in Wang D. S(2004).

In the statistical analysis, ductility factors μ_x and μ_y take the value of 2, 4, 6 and its combination, the damping ratios $\xi_x = \xi_y = 0.05$.

3.1 Effects Of Periods On BGMS

The Average pseudo-constant ductility BGMS in hard site is shown in fig.1. horizontal ordinate T_x and longitudinal coordinate T_y of the spectra are in the range of 0.25s～5.0s. For real structures the ratios of T_x/T_y=0.4～2.5, thus the BGMS inside these areas are concerned(areas between dot lines in fig.1).

BGMS are divided into two parts by T_x/T_y=1.0. In the part of T_x/T_y<1.0, BGMS=0.9～1.1, bidirectional ground excitation has little effects on structural response, while in the part of T_x/T_y>1.0, BGMS>1.1 and increase with T_x/T_y , the maximum value is about 1.5. Those shown that bidirectional ground motions usually amplify the structural responses in the long period direction, if the periods of the structure in two principal axes are much different, the negative effects of bidirectional ground motions are more distinct. Especially for the structures with longer period T_x in the ranges of 1s～3s.

BGMS have two characteristic periods, the first is at T_x＝0.8s～1.0s, independent on ductility factors , the second is at T_x＝2.0s～3.0s,varied with ductility factors.

3.2 Effects Of Ductility Factors On BGMS

BGMS in fig.1 are arranged by ductility factors. It is found that the ratios of μ_y/μ_x have significant influence on BGMS which increase with the ratio. This means that structural inelastic responses in shorter period direction are larger，the effects of bidirectional ground motion excitations are more serious.

3.3 Effects Of Damping On BGMS

$\xi_x = \xi_y = 0.02$ and $\xi_x = \xi_y = 0.10$ are used to study damping effects(fig.2). Results are that since BGMS are defined by the ratio of the maximum displacement responses, damping has little effects.

3.4 Effects Of Site Conditions On BGMS

μ_x＝4.0 and μ_y＝2.0,4.0,6.0 are used to investigated influences of soil conditions on BGMS. The damping ratios are $\xi_x = \xi_y = 0.05$.

BGMS on different sites are shown by fig.3 ～fig.4. On medium site BGMS are almost similar to that on hard site , the only difference is that the first characteristic period is decreased to 0.5s. BGMS on medium site are slightly greater than that on hard site when periods T_x >3s-4s.

BGMS on soft site are quite different from that on hard or medium site. In the rectangular area of T_x＝0.25s－0.5s and T_y＝0.25s－1.5s BGMS are more greater than other areas, which increase with the decreasing of T_x or T_y and increasing of μ_y, the maximum value reaches 2.0, while the BGMS are about 1.2 in other areas.

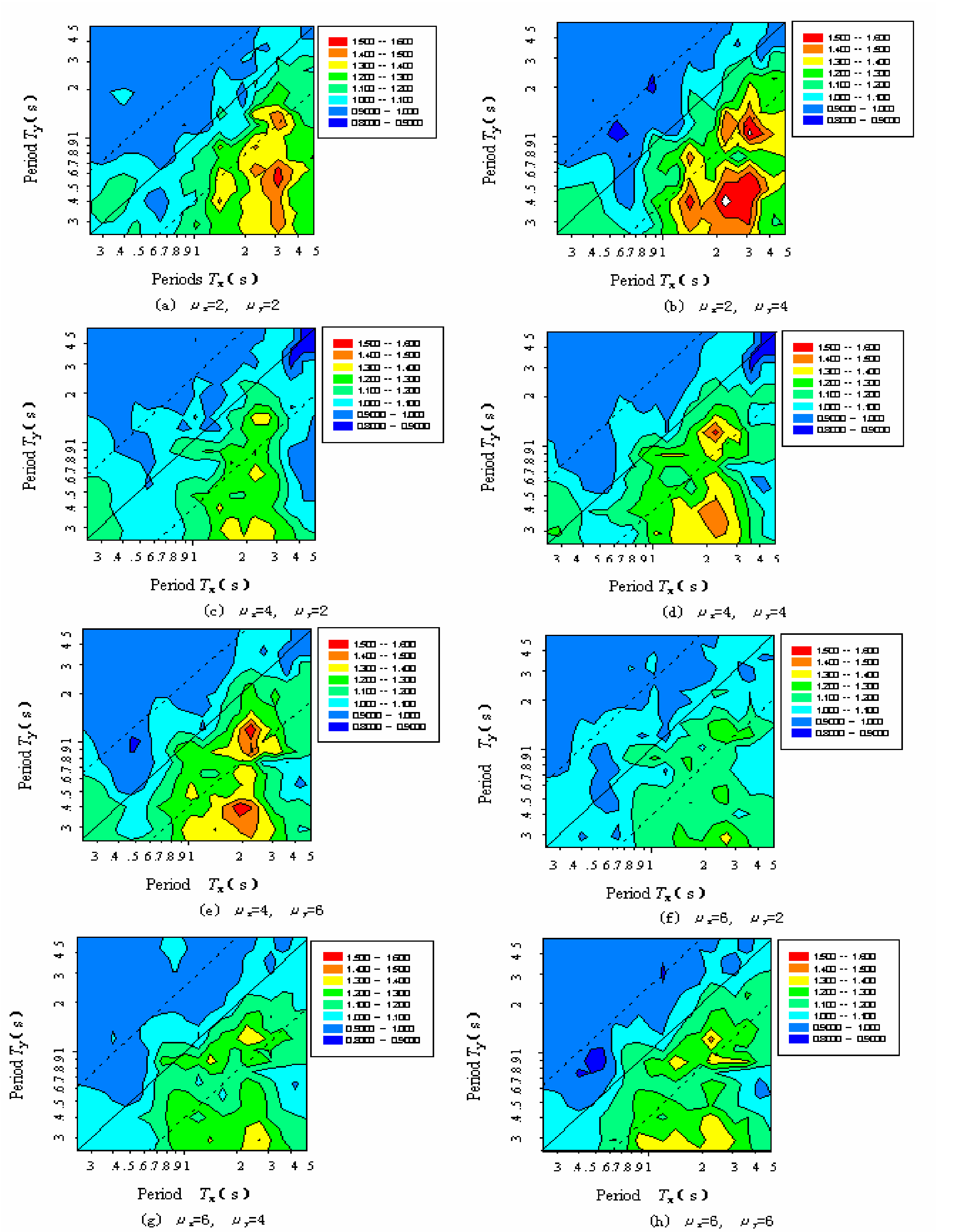
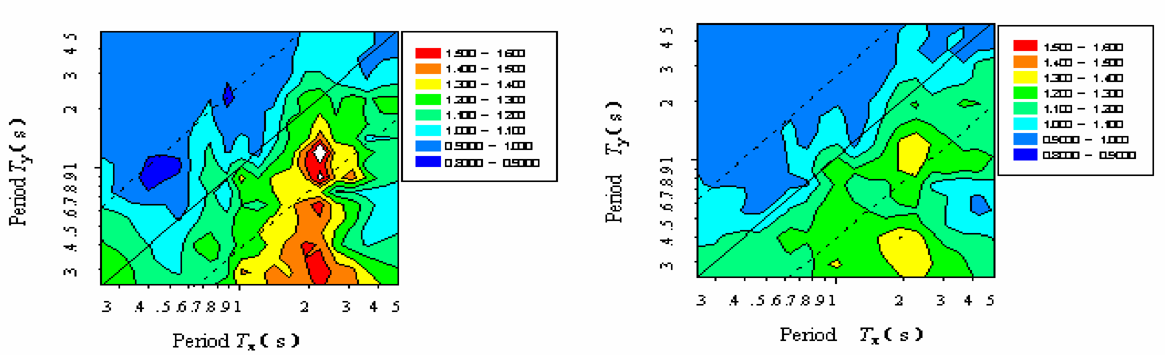
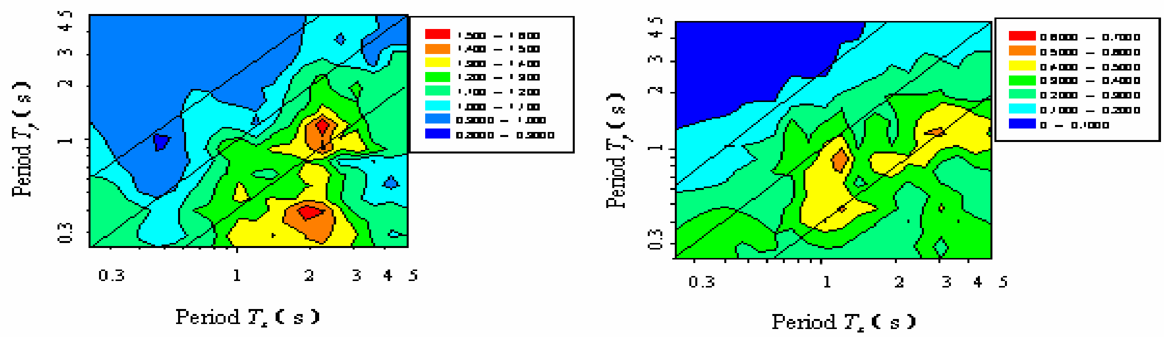


Fig.1 Average pseudo-constant ductility inelastic spectra for bi-directional ground motions on hard site

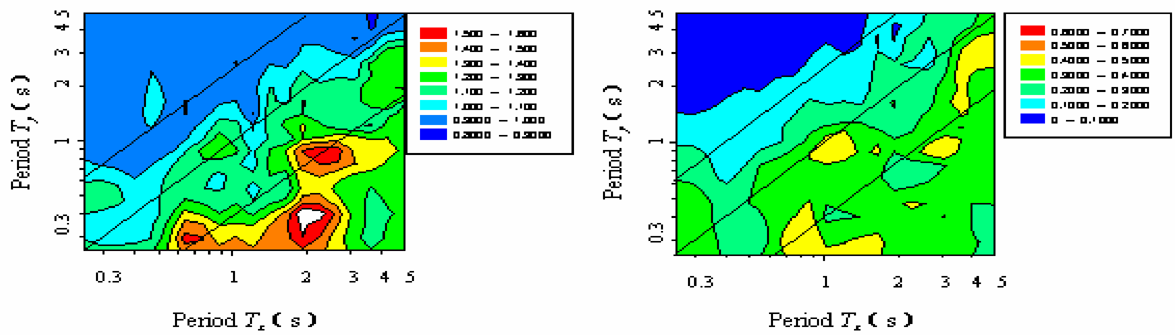


(a) μ_x=4, μ_y=6, ξ=0.02　　(b) μ_x=4, μ_y=6, ξ=0.10

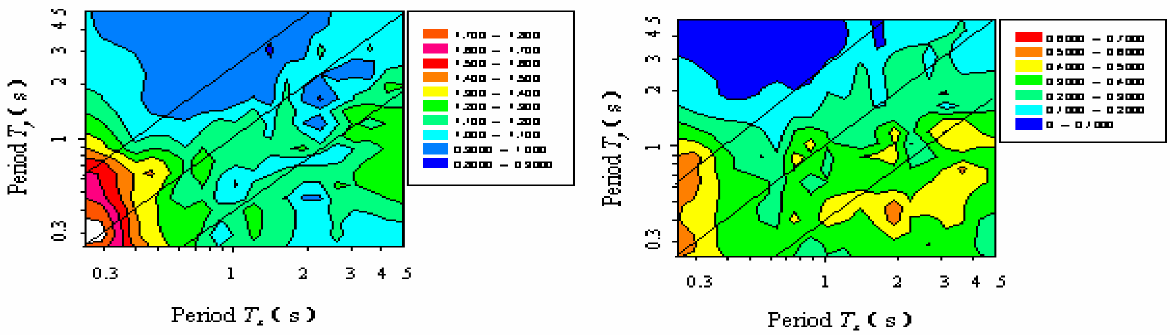
Fig. 2 Effect of damping on pseudo-constant ductility BGMS on hard site



(a) Hard site (left : BGMS, right :COV)

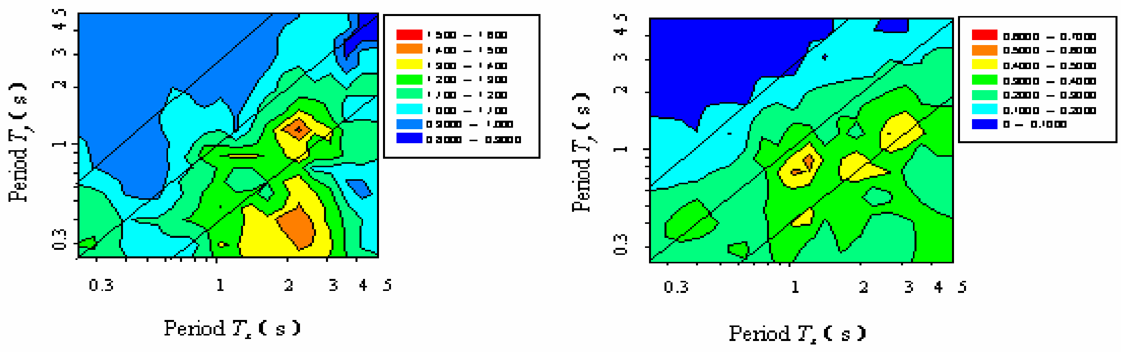


(b) medium site (left : BGMS, right :COV)

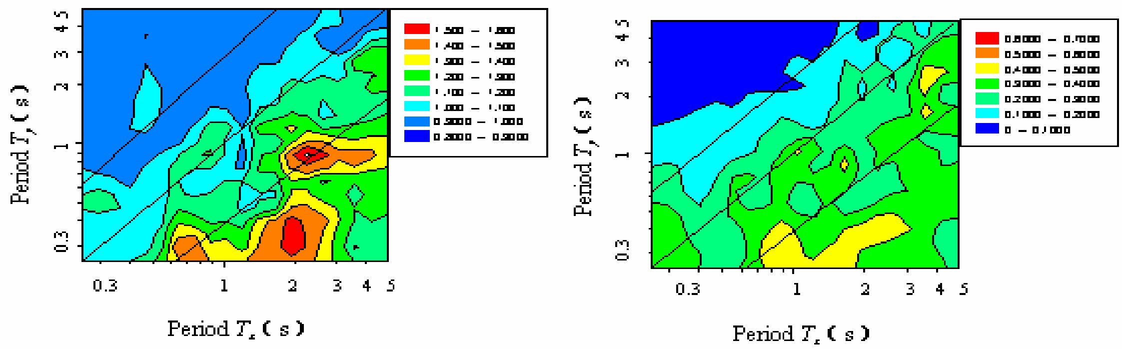


(c) Soft site (left : BGMS, right :COV)

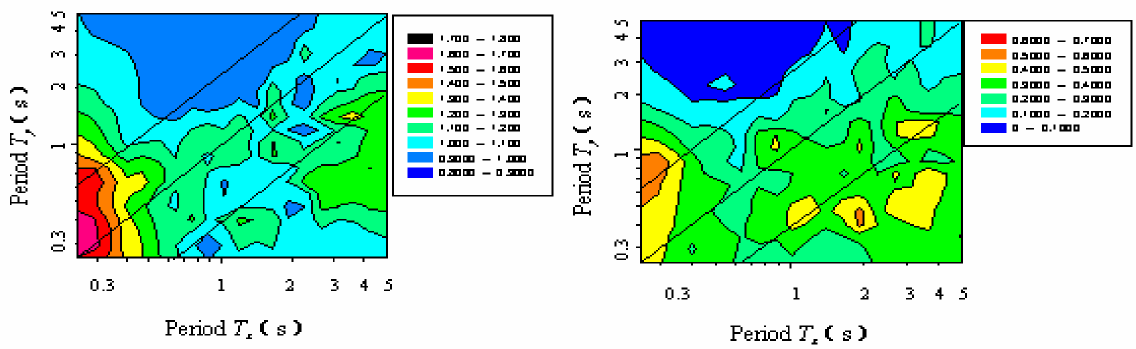
Fig.3 BGMS and its coefficient of variation (COV) on different sites（μ_x=4.0, μ_y=6.0）



(a) Hard site (left : BGMS, right :COV)



(b) medium site (left : BGMS, right :COV)



(c) Soft site (left : BGMS, right :COV)

Fig.4 BGMS and its coefficient of variation (COV) on different sites（μ_x=4.0, μ_y=4.0）

Coefficient of variations (COVs) of BGMS are compared. COVs are almost the same on different sites except for that COVs on soft site is got slightly greater in the rectangular area of T_x=0.25s－0.5s and T_y=0.25s－1.5s, which reach 60%.

Earthquake experiences have shown that structures on soft side usually suffer more damage than that on hard site. In Mexico City Earthquake in 1985, many high buildings with 6-15 stories are collapsed. The reasons are magnification effects of the soft site on long period portion of earthquake ground motions and the resonance vibration of structures.

Besides above reasons the authors supposes that bidirectional earthquake excitations may be another main causes, because the period of building structures is about 0.1N(N is structural story), so periods of 6-15 story buildings are 0.6s－1.5s, just in the ranges of T_x=0.25s－0.5s and T_y=0.25s－1.5s, the bidirectional ground motions will quite increase the structural ductility demands.

4. CONCLUSION

Inelastic spectra for bidirectional ground motions are presented and structural responses under bidirectional earthquake excitation are investigated. It is found that: 1) comparing with one-directional excitation, bidirectional earthquake excitations will mainly increase the maximum structural displacement response in the direction with longer period; 2) by decreasing the designed strength reduction factors or designed displacement ductility factors of structures in the direction with shorter period, the negative effects of bidirectional ground motions on structural response can be reduced; 3) since it is defined by the ratio of the maximum displacement responses, damping has little effect on the inelastic spectrum; 4) site conditions, periods and nonlinear response levels in two structural principal axes are the major effective factors on structural bidirectional response.

REFERENCES

Newmark N M , Hall W J(1982). Earthquake Spectra and Design. Berkeley: EER I, USA.

Fajfar P(1999). Capacity Spectrum Method Based on Inelastic Demand Spectra. *Earthquake Eng. Stru. Dyn.* 28(9): 979-993

Chopra A.K. , Goel R.K(2001). Direct Displacement-Based Design: Use of Inelastic vs. Elastic Design Spectra. *Earthquake Spectra*, 17(1):47-64

Chopra A. K., Chintanapakdee Ch(2004). Inelastic Deformation Ratios for Design and Evaluation of Structures: Single-Degree-of-Freedom Bilinear Systems. *Journal of Structural Engineering*, ASCE, 130(9) :1309-1319

Miranda E(1992). Evaluation of Site-Dependent Inelastic Seismic Design Spectra. *Journal of Structural Engineering*, ASCE, 119(6):1319-1337

Miranda E., Bertero V.V(1994). Evaluation of Strength Reduction Factors for Earthquake-resistant Design. *Earthquake Spectra*, 10(2):357-379

Miranda E., Ruiz-Garcia J(2002). Evaluation of Approximate Methods to Estimate Maximum Inelastic Displacement Demands. *Earthquake Eng. Stru. Dyn.*, 31(3): 539-560

Ruiz-Garca J., Miranda E(2006). Inelastic Displacement Ratios for Evaluation of Structures Built on Soft Soil Sites. *Earthquake Engng Struct. Dyn.*, 35:679–694

Riddell R., Garcia J.E., Garces E(2002). Inelastic Deformation Response of SDOF Systems Subjected to Earthquake. *Earthquake Eng. Stru. Dyn*. 31(3): 515-538

Zhai Chang-hai, Gong Mao-Sheng，Zhang Mao-Hua, et al(2004). On Seismic Resistance Spectra of Constant Ductility of Structures**.** *Earthquake Engineering and Engineering Vibration*，24（1）：22－29（in Chinese）

Lu Xi-lin, Zhou Ding-Song(2004). Ductility Demand Spectra and Inelastic Displacement Spectra Considering Soil Conditions and Design Characteristic Period**.** *Earthquake Engineering and Engineering Vibration*，24（1）：39－48（in Chinese）

Jarenprasert S., Bazán E., Bielak A(2006). Inelastic Spectrum-Based Approach for Seismic Design Spectra. *Journal of Structural Engineering*, ASCE, 132(8) :1284-1292

Hachem M. M(2003). BISPEC Program Description and User's Manual. Http://nisee.berkeley.edu/NISEE Engineering Software &Manuals/, USA

Wang Dong-sheng, Li Hong-nan, Wang Guo-xin(2005). Inelastic Response Spectra for Bi-directional Ground Motions. *Journal of Dalian University of Technology*, 45(2):248-254（in Chinese）

Wang Dong-sheng, Li Hong-nan, Wang Guo-xin(2004). Pseudo-Constant Ductility Inelastic Spectra For Bi-Directional Ground Motions. *Earthquake Engineering and Engineering Vibration*, 24(4):25-31（in Chinese）