SEISMIC SIMULATION OF PRESTRESSED CONCRETE STRUCTURES

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ABSTRACT :

Wall-type or shell-type prestressed concrete structures, such as prestressed concrete I-girders, box girders, nuclear containment vessels, offshore structures, shear walls, etc can be visualized as assemblies of membrane elements. Their behavior can be predicted if the behavior of the membrane elements is thoroughly understood. This paper reports the Softened Membrane Model for Prestressed Concrete (SMM-PC) developed using the Universal Panel Tester available at the University of Houston. The SMM-PC generalized the previously developed Softened Membrane Model (SMM) for reinforced concrete and can be used for prestressed as well as reinforced concrete. The new SMM-PC includes the following three new constitutive laws: (1) A constitutive law of concrete in tension that includes the decompression stage. (2) A new prestress factor W_p proposed for incorporation into the softening coefficient of the constitutive laws of concrete in compression. (3) A smeared (average) stress-strain relationships of prestressing strands embedded in concrete. In this paper the SMM-PC has also been extended to include cyclic behavior, thereby creating a Cyclic Softened Membrane Model for Prestressed Concrete (CSMM-PC). This has been accomplished by implementing the cyclic behavior of reinforced concrete previously developed at the University of Houston through the Cyclic Softened Membrane Model (CSMM). The SMM-PC including cyclic behavior is implemented into a non-linear finite element program based on the framework of OpenSees to predict the behavior of prestressed concrete structures under cyclic loading. Two new modules are developed in OpenSees to define the uniaxial cyclic constitutive laws of concrete under prestressing as well as those of prestressing tendons. The uniaxial cyclic constitutive laws of mild steel have been previously implemented into OpenSees at the University of Houston in a nonlinear finite element program called the Simulation of Concrete Structures (SCS). All these uniaxial constitutive laws are combined to develop two-dimensional PrestressConcretePlaneStress module, which can be used for developing finite element models of prestressed concrete structures. The accuracy of the new program is confirmed by comparing simulated responses with available test data.

KEYWORDS: prestressed concrete, constitutive laws, Cyclic Softened Membrane Model, OpenSees, PrestressConcretePlaneStress

1. INTRODUCTION

The past three decades have seen a rapid development of knowledge in shear of reinforced concrete structures. Various rational models have been proposed that are based on the smeared-crack concept and can satisfy Navier's three principles of mechanics of materials (i.e., stress equilibrium, strain compatibility and constitutive laws). These rational or mechanics-based models on the "smeared-crack level" (in contrast to the "discrete-crack level" or "local level") include the compression field theory (CFT) [Vecchio and Collins, 1981], the modified compression field theory (MCFT) [Vecchio and Collins, 1986], the rotating-angle softened truss model, (RA-STM) [Hsu, 1993; Belarbi and Hsu, 1995; Pang and Hsu 1995], the fixed-angle softened truss model, (FA-STM) [Pang and Hsu, 1996; Hsu and Zhang, 1996], the softened membrane model, (SMM) [Zhu, 2000; Hsu and Zhu, 2002], and the cyclic softened membrane model, (CSMM) [Mansour, 2001; Mansour and Hsu, 2005a, 2005b; Hsu and Mansour, 2005]. In recent years the SMM has also been extended to prestressed concrete (PC) to develop SMM-PC [Wang, 2006]. This can be used in conjunction with the CSMM to rationally predict the hysteresis loops for prestressed concrete structures.

In this paper, the cyclic behavior of the materials as obtained from CSMM is combined with SMM-PC to establish the Cyclic Softened Membrane Model for Prestressed Concrete (CSMM-PC). The paper will briefly discuss the analytical model of CSMM-PC by summarizing the equilibrium and compatibility equations, the concept of biaxial strains vs. uniaxial strains, the constitutive relationships of materials, the solution algorithm and the comparison of CSMM-PC predictions with the test results of a full-scale post-tensioned precast concrete bridge column subjected to reversed cyclic loads.

2. FUNDAMENTALS OF CYCLIC SOFTENED MEMBRANE MODEL FOR PRESTRESSED CONCRETE

Figure 1(a) shows a prestressed concrete element subjected to in-plane stresses. Two reference Cartesian coordinates are used in the CSMM for PC, as shown in Figure 1(e). The first reference Cartesian $\ell - t$ coordinate system represents the directions of the longitudinal and transverse reinforcements. The second reference Cartesian $1-2$ coordinate system represents the directions of the applied principal stresses.

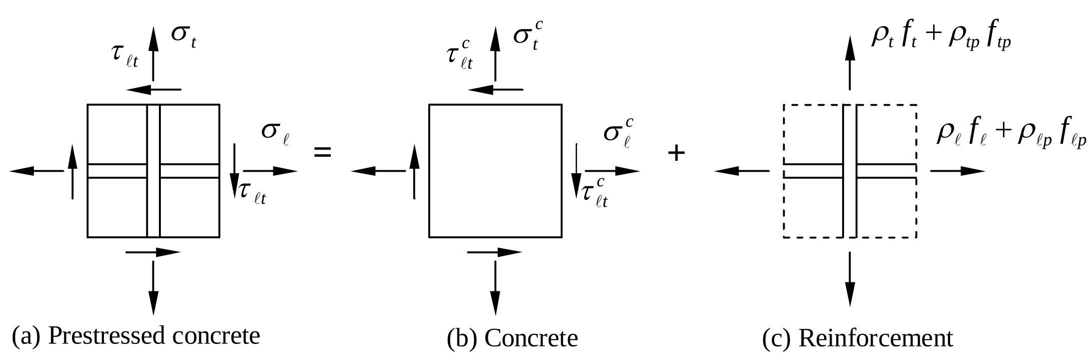
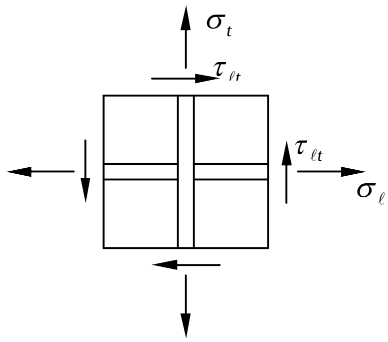
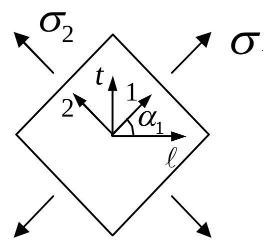


Figure 1 Coordinate System in a Prestressed Concrete Membrane Element



(d) Prestressed concrete element



(e) Principal coordinates 1-2 for applied stresses

Figure 1 Coordinate System in a Prestressed Concrete Membrane Element

2.1. Equilibrium and Compatibility Equations

The equilibrium equation, which relates the applied stresses (σ_ℓ, σ_t and $\tau_{\ell t}$) to the internal stresses of concrete (σ_1^c, σ_2^c and τ_{12}^c), mild steel (f_ℓ and f_t), and prestressing steel ($f_{\ell p}$ and f_{tp}) in a membrane element, is expressed in Eqn. 2.1. $[T]$ is the transformation matrix between the Cartesian $1-2$ coordinate system and the $\ell - t$ coordinate system. The compatibility equation, which represents the relationship between the strains (ε_ℓ, ε_t, and $\gamma_{\ell t}$) in the $\ell - t$ coordinates of the reinforcement and the strains ($\varepsilon_1, \varepsilon_2$, and γ_{21}) in the $1-2$ coordinates of the principal applied stress, is expressed in Eqn. 2.2.

$$\begin{Bmatrix} \sigma_\ell \\ \sigma_t \\ \tau_{\ell t} \end{Bmatrix} = [T] \begin{Bmatrix} \sigma_1^c \\ \sigma_2^c \\ \tau_{12}^c \end{Bmatrix} + \begin{Bmatrix} \rho_\ell f_\ell \\ \rho_t f_t \\ 0 \end{Bmatrix} + \begin{Bmatrix} \rho_{\ell p} f_{\ell p} \\ \rho_{tp} f_{tp} \\ 0 \end{Bmatrix} \tag{2.1}$$

$$\begin{Bmatrix} \varepsilon_\ell \\ \varepsilon_t \\ \frac{\gamma_{\ell t}}{2} \end{Bmatrix} = [T] \begin{Bmatrix} \varepsilon_2 \\ \varepsilon_1 \\ \frac{\gamma_{21}}{2} \end{Bmatrix} \tag{2.2}$$

2.2.Biaxial Strains vs. Uniaxial Strains

To solve the equilibrium and compatibility equations, the stress-strain relationships of concrete and reinforcements have to be provided. However, the set of strains ε_1, ε_2, ε_ℓ, and ε_t in (2.2) are biaxial strains, taking into account the Hsu/Zhu ratios (Zhu and Hsu 2002), while the constitutive laws relating the stresses to the biaxial strains are not unique and thus cannot directly be determined from experiments. Therefore, a “bridge” is required to relate the biaxial strains and the uniaxial strains. The relationships between the uniaxial strains ($\bar{\varepsilon}_1$, $\bar{\varepsilon}_2$, $\bar{\varepsilon}_\ell$, and $\bar{\varepsilon}_t$) and the biaxial strains (ε_1, ε_2, ε_ℓ, and ε_t) were established using the Hsu/Zhu ratios (Zhu, 2000; Hsu and Zhu, 2002).

2.3.Cyclic Uniaxial Constitutive Relationships of Concrete

The cyclic uniaxial constitutive relationships of cracked concrete in compression and tension are developed through panel tests (Mansour and Hsu, 2005a and 2005b). A damage coefficient D is incorporated in the compressive envelope which takes into account the effect of cyclic shear loading, where cyclic compression and tension occur in both principal directions. The damage coefficient due to compression is taken as a linear function of the maximum compression strain $\varepsilon_c^{'}$ as shown in Eqn. 2.3. $\varepsilon_c^{'}$ is the maximum compression strain normal to the compression direction under consideration and occurred in the previous loading cycles. ε_0 is the strain at maximum compressive stress. Compared to the tensile stress-strain relationship of non-prestressed concrete, the constitutive relationship of prestressed concrete in tension has a decompression stage (Wang, 2006). In compression stage, the stress-strain relationship in unloading is a straight line. The slope of the line is taken as the unloading modulus of the concrete in compression, which is the tangential slope of a parabolic compressive stress-strain relationship of concrete at the origin.

$$D = 1 - 0.4\frac{\varepsilon_c^{'}}{\varepsilon_o} \leq 1.0 \tag{2.3}$$

2.4. Cyclic Constitutive Relationships of Embedded Mild Steel Bars

The smeared (average) cyclic stress-strain relationships of mild steel rebars embedded in concrete, in tension, compression and during unloading and reloading were established thorugh panel tests at University of Houston (Mansour, 2005a and 2005b). The smeared tensile stress-strain relationships of mild steel bars embedded in concrete were found to have a yield stress lower than the actual yield stress of bare mild steel bars.

2.5. Cyclic Constitutive Relationships of Embedded Prestressing Tendons

The smeared (average) tensile stress-strain relationships of prestressing tendons embedded in concrete were established through tests on prestressed concrete panels at University of Houston (Wang, 2006). The cyclic behavior of mild steel can be extended to prestressing tendons. Hence during unloading and reloading stages, the stress strain relationship of prestressing tendons is the same as that of mild steel.

3. IMPLEMENTATION OF CSMM-PC

In a smeared crack model, cracked prestressed concrete is considered to remain a continuum. The material properties are characterized by the set of smeared (or average) stress-strain relationships for concrete, steel, and prestressing tendons described in the previous section. These average stress-strain relationships account for local bond slipping. An important advantage of using the averaging concept in tension stiffening [Hsu and Zhang, 1996] is that the tensile stress-strain relationship of concrete becomes mesh independent.

3.1. Nonlinear Analysis Algorithm

An iterative tangent-stiffness procedure under incremental load or displacement is developed to perform a nonlinear analysis for prestressed concrete structures. A flow chart of the procedure is described in Figure 2. Throughout the procedure, the material stiffness matrix $[D]$ is determined first, and the element stiffness matrix $[k]$ and the element resisting force increment vector Δf are calculated. Then the global stiffness matrix [K] and global resisting force increment vector ΔF are assembled. In each iteration, the material stiffness matrix $[D]$, the element stiffness matrix $[k]$ and the global stiffness matrix [K] are iteratively refined until convergence is achieved. It is noted that an additional iterative loop is defined to obtain the material stiffness matrix $[D]$ because the principal stress direction θ_1 is an unknown value before $[D]$ is found.

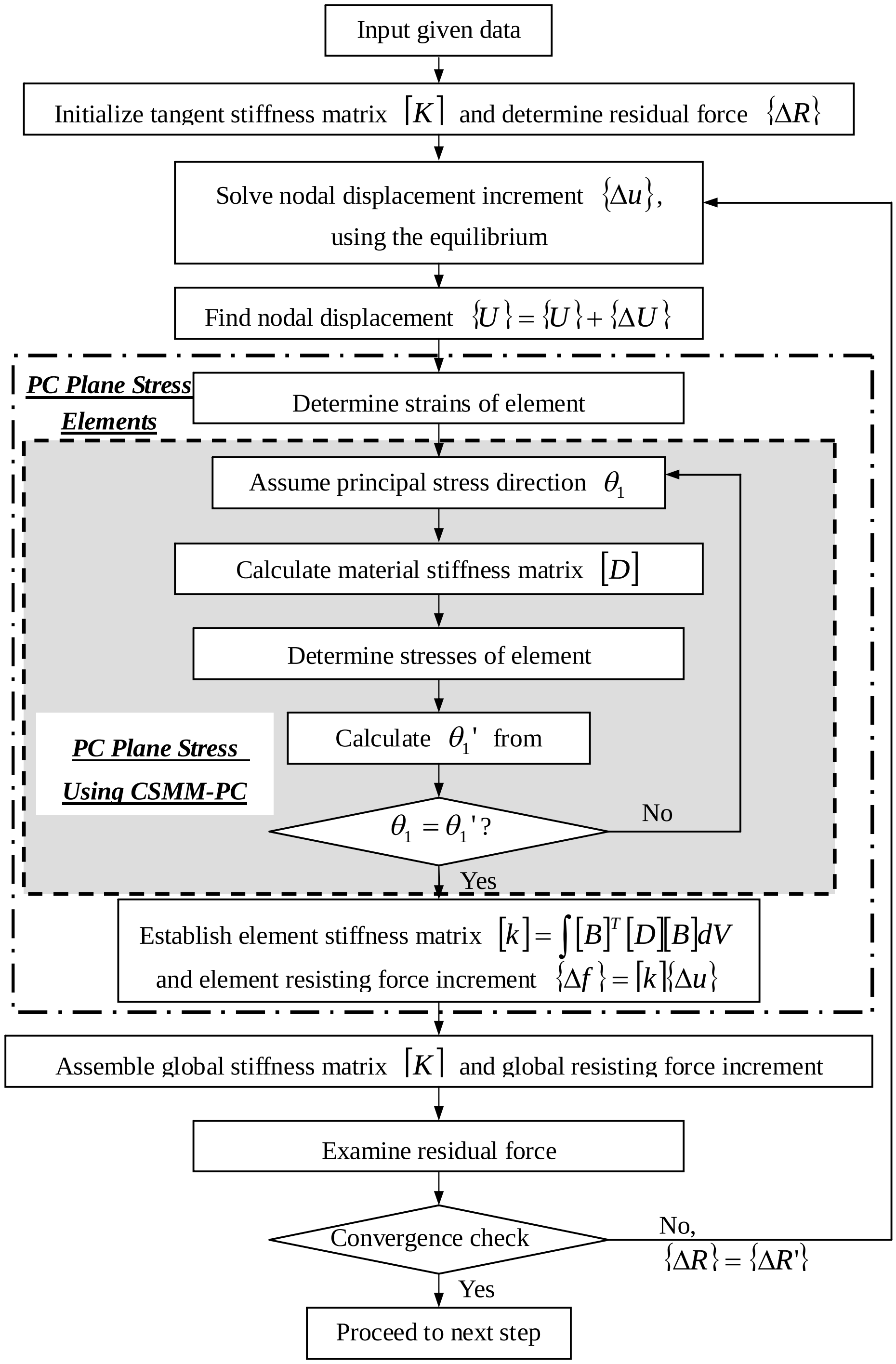


Figure 2. Nonlinear Analysis Algorithm

3.2.Implementation in OpenSees

OpenSees is an object-oriented framework for simulation applications in earthquake engineering using finite element methods. An object-oriented framework is a set of cooperating classes that can be used to generate software for a specific class of problem, such as finite element analysis. OpenSees is a communication mechanism for exchanging and building upon research accomplishments, and has the potential for a community code for earthquake engineering because it is open source. In order to implement the CSMM-PC into OpenSees, three new material modules, namely TendonL01, ConcreteL01 and PCPlaneStress are developed. TendonL01 and ConcreteL01 are the uniaxial material modules, in which the uniaxial constitutive relationships of prestressing tendons and concrete specified in the CSMM-PC are defined respectively. The uniaxial constitutive relationships of mild steel are already defined in the module SteelZ01 [Zhong, 2005]. The PCPlaneStress is implemented with the quadrilateral element to represent the four node prestressed concrete membrane elements. The uniaxial materials of SteelZ01, TendonL01 and concreteL01 are related with material PCPlaneStress to determine the material stiffness matrix of prestresseed concrete membrane in PCPlaneStress. Using the OpenSees as the finite element framework, a nonlinear finite element program titled Simulation of Concrete Structures (SCS) was thus developed for the simulation of prestressed concrete structures subjected to monotonic and reversed cyclic loading. In Section 4, SCS has been validated using the test results of a post-tensioned segmental bridge column subjected to reversed cyclic loads at the State university of New York (SUNY), Buffalo (Ou, 2002).

4. VALIDATION

A 5.7 m high post-tensioned segmental hollow bridge column tested under reversed cyclic loads has been used to validate SCS. The test setup of the specimen is given in Figure 3. The specimen had four prestressing tendons. Each tendon comprised of two seven-wire strands made of steel equivalent to ASTM A416 Grade 270. Each strand had a nominal diameter of 15.24 mm. The total design prestressing force in the specimen was 1042 kN.

4.1. Analytical Model

The specimen was modeled using the finite element mesh illustrated in Figure 3. The two flange sides of the bridge piers perpendicular to the bending direction, which are mainly under compression and tension under bending, are modeled as NonlinearBeamColumn elements with fiber sections. The NonlinearBeamColumn elements used herein were previously developed in the OpenSees framework. The two web sides of the bridge piers parallel to the bending direction, which resist the shear force, are defined by PCPlaneStress Quadrilateral elements. The prestressing tendons in the center of the specimens were modeled separately using NonlinearBeamColumn elements consisting only of fibers of TendonL01 material. The cap beam on the top of the column is defined as a rigid body in the finite element model. The boundary condition and load pattern in the finite element model were defined according to the test condition, as shown in Figure 3. The prestressing force was applied as vertical nodal loads acting at the top and bottom of the column.

4.2. Comparison of Experimental and Analytical Results

The measured and calculated load-drift relationships of the bridge column are shown in Fig. 4. Compared with the experimental results, the analysis could well predict the load versus drift characteristics of the specimen including precracking stiffness, postcracking stiffness, yielding of specimen, ultimate strength, and energy dissipation. The strength degradation in the post peak region was also well predicted in the analyses both in the positive as well as the negative directions. The nearly flattop envelopes of specimen, which is a typical behavior of the flexure mechanism, was also predicted by the analyses.

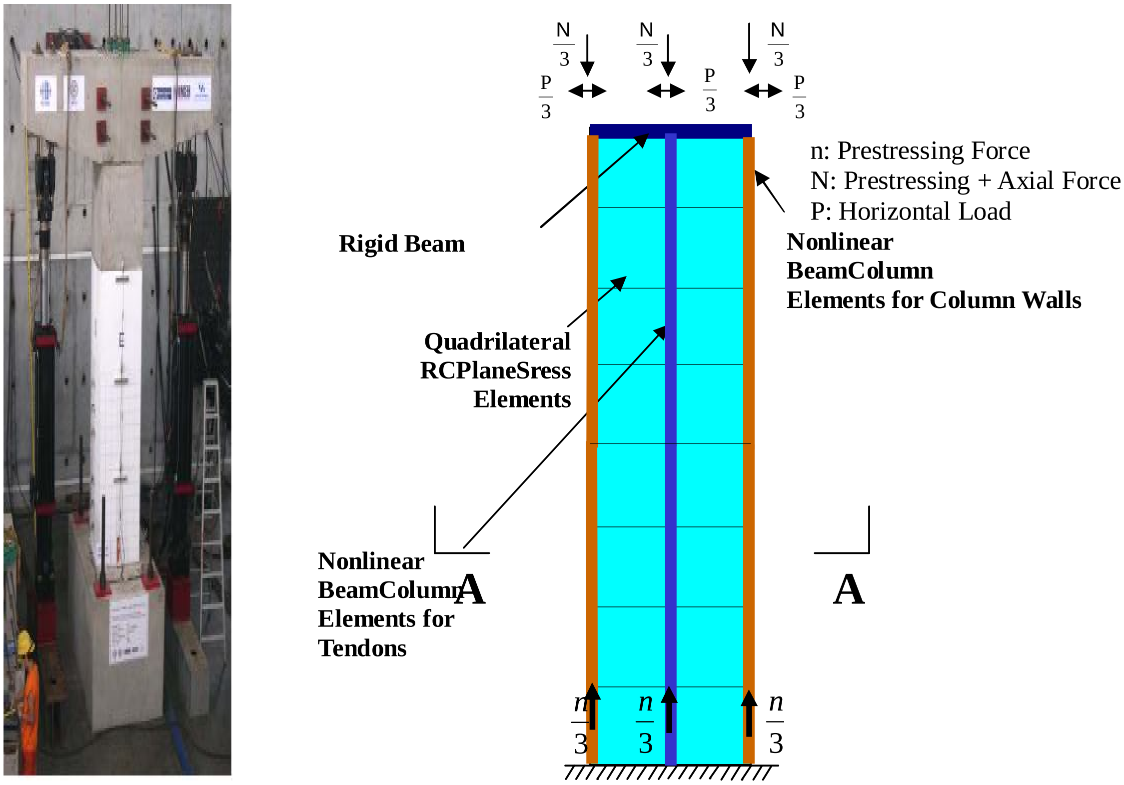


Figure 3 Setup and Finite Element Mesh of Column Specimen

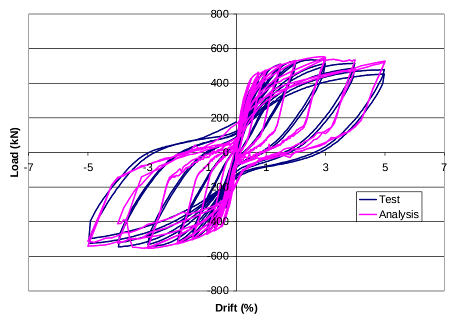


Figure 4 Experimental and Analytical Load Drift Diagram of Bridge Column

5. CONCLUSIONS

The CSMM-PC was implemented into a finite element program based on OpenSees platform to predict the structural behavior of prestressed concrete wall-related structures subjected to reversed cyclic loads. The developed finite element program was validated by tests of a 5.7 m high post-tensioned segmental column under horizontal reversed cyclic load. The predicted results agree well with the experimental data.

REFERENCES

Belarbi, A., and Hsu, T. T. C. (1995). Constitutive Laws of Softened Concrete in Biaxial Tension-Compression, *Structural Journal of the American Concrete Institute*, **92:5**, 562-573.

Fenves, G. L., (2005). Annual workshop on Open System for Earthquake Engineering Simulation. Pacific Earthquake Engineering Research Center, UC Berkeley.

Hsu, T. T. C. (1993) Unified Theory of Reinforced Concrete, Boca Raton: CRC Press Inc.

Hsu, T. T. C., and Zhang, L. X. (1996). Tension Stiffening in Reinforced Concrete Membrane Elements, *Structural Journal of the American Concrete Institute*, **93:1**, 108-115.

Hsu, T. T. C., and Zhu, R. R. H. (2002). Softened Membrane Model for Reinforced Concrete Elements in Shear, *Structural Journal of the American Concrete Institute*, **99:4**, 460-469.

Hsu, T. T. C., and Mansour, M. Y. (2005). Stiffness, Ductility, and Energy Dissipation of RC Elements under Cyclic Shear, *Earthquake Spectra, EERI*, **21:4**, Nov., 1093-1112.

Mansour, M. (2001). Behavior of Reinforced Concrete Membrane Elements Under Cyclic Shear: Experiments to Theory, Ph. D. Dissertation, Department of Civil and Environmental Engineering, University of Houston, Houston, TX.

Mansour, M. and Hsu, T. T. C. (2005a). Behavior of Reinforced Concrete Elements under Cyclic Shear:Part 1 – Experiments, *Journal of Structural Engineering, ASCE*, **131:1**, 44-53.

Mansour, M. and Hsu, T. T. C. (2005b). Behavior of Reinforced Concrete Elements under Cyclic Shear: Part 2 - Theoretical Model, *Journal of Structural Engineering*, ASCE, **131:1,** 54-65.

Ou, Y.C (2007). Precast Segmental Post-Tensioned Concrete Bridge Columns for Seismic Regions, Ph.D. Dissertation, Department of Civil, Structural, and Environmental Engineering, State University of New York, Buffalo, NY.

Pang, X. B., and Hsu, T. T. C. (1995). Behavior of Reinforced Concrete Membrane Elements in Shear, *Structural Journal of the American Concrete Institute*, **92: 6**, 665-679.

Pang, X. B., and Hsu, T. T. C. (1996). Fixed-Angle Softened-Truss Model for Reinforced Concrete, *Structural Journal of the American Concrete Institute*, **93:2**, 197-207.

Vecchio, F. and Collins, M. P. (1981). Stress-strain characteristic of reinforced concrete in pure shear, IABSE Colloquium, Advanced Mechanics of Reinforced Concrete, Delft, Final Report, International Association of Bridge and Structural Engineering, Zurich, Switzerland, 221-225.

Vecchio, F. J., and Collins, M. P. (1986). The modified compression field theory for reinforced concrete elements subjected to shear, *ACI Journal*, **83:2,** 219-231.

Wang, J. (2006). Constitutive Relationships of Prestressed Concrete Membrane Elements, Ph.D. Dissertation, Department of Civil and Environmental Engineering, University of Houston, Houston, TX.

Zhong, J. (2005). Model-Based Simulation of Reinforced Concrete Plane Stress Structures, Ph.D. Dissertation, Department of Civil and Environmental Engineering, University of Houston, Houston, TX.

Zhu. R. H., (2000). Softened Membrane Model for Reinforced Concrete Elements Considering Poisson Effect, Ph. D. Dissertation, Dept. of Civil and Environmental Engineering, University of Houston, Houston, TX.

Zhu, R. R. H., and Hsu, T. T. C. (2002). Poisson Effect of Reinforced Concrete Membrane Elements, *Structural Journal of the American Concrete Institute*, **99:5,** 631-640.