

# NATURAL PERIODS OF COUPLED SHEAR WALLS VIA DIFFERENTIAL TRANSFORMATION

Y. H. Chai<sup>1</sup> and Yanfei Chen<sup>2</sup>

<sup>1</sup>Professor, Dept. of Civil & Environmental Engineering, University of California Davis, USA <sup>2</sup>PhD candidate, School of Civil Engineering, Dalian University of Technology, China Email: <u>yhchai@ucdavis.edu</u>, <u>chenyfvip@163.com</u>

## **ABSTRACT:**

In multistory buildings, coupled shear walls are frequently used as the main lateral load resisting system, the seismic design of which requires some knowledge of their period of vibration. Although the vibrational characteristics of coupled shear walls have been extensively studied since the 1970's, most of the studies were based on approximations resulting in varying degree of accuracies and complexities. Methods used in these studies range from the Rayleigh's quotient to the Dunkerley's formula, to the Galerkin's method of weighted residuals, and to the solution of the Sturm-Louiville type differential equation. Motivated by conflicting results in the literature, particularly inaccuracies in a recent study, this paper re-examines the vibrational characteristics of coupled shear walls. The governing equation, established on the basis of replacing the coupling beams by equivalent laminae, is solved using the technique of differential transformation, which has been shown to provide rather accurate solution for eigenvalue problems. Solutions are shown to compare well with experimental results of model coupled shear walls published in the literature.

KEYWORDS: Coupled shear walls, differential transformation, fundamental period

## **1. INTRODUCTION**

Current construction of multistory buildings often rely on strategically arranged shear walls to form the lateral force resisting system of the building. Architectural as well as service constraints, however, frequently perforate these shear walls with windows, doors or access openings for corridors and elevators. Since shear walls are connected structurally by slabs or beams at each floor level, the connecting elements can induce significant axial force in the walls, which in turn increase the lateral stiffness of the building. Consequently coupled shear walls are expected to respond differently from flexural walls. For design purposes, coupled shear walls may be classified according to the degree of coupling, depending on the size of the opening, depth and spacing of the connecting members etc (Chaallal et al. 1996).

Requirements for symmetry and regularity of the floor plan in high-rise apartment buildings often result in uniform shear walls with a single vertical row of openings. Although such structural systems are readily amenable to finite element analysis, a commonly accepted technique assumes that the beams connecting the walls can be replaced by an equivalent system of independently acting laminae with properly assigned flexural properties. The laminae are assumed to be infinitely stiff in the axial direction so that individual shear walls undergo the same amount of lateral deflection upon lateral loading. The idealization of the coupling members by the equivalent system of laminae allows the lateral deflection of the shear wall to be described fully by a differential equation, similar to the idealization of flexural members by the Euler-Bernoulli beam (Smith and Coull 1991).

The effectiveness of coupled shear walls in resisting seismic forces in multistory buildings has long been recognized (Paulay and Priestley 1992). As these forces tend to be period-dependent, considerable attentions have been paid to the determination of the natural period of these shear walls since the 1970's (Tso and Chan 1971; Tso and Biswas 1972; Jennings and Skattum 1973; Coull and Mukherjee 1973; Mukherjee and Coull 1973; Rutenberg 1975; Tso and Rutenberg 1977). In nearly all of these studies, however, only approximate



formulations with varying degree of complexities and accuracies are available. For example, (Tso and Biswas 1972) used the Rayleigh's quotient to determine the eigenvalues where the mode shapes of vibrating flexural cantilevers were used as the approximating eigenfunctions. In a separate study, (Rutenberg 1975) used the Dunkerley's formula to approximate the natural frequencies of coupled shear walls, where the overall deflection of the wall is decomposed into two modes, one involving pure flexural deformation and the other involving flexural-shear deformation. The approximation was noted to result in fairly accurate natural frequencies of coupled shear walls. Other contributions on the subject include (Mukherjee and Coull 1973) and (Coull and Mukherjee 1973) where the Galerkin's method of weighted residuals was used to approximate the natural frequencies of the coupled shear walls. Although the Galerkin's method is considered numerically efficient, accuracy of results, particularly for the higher modes, was later questioned by (Tso and Rutenberg 1974). More recently, another attempt was made by (Wang and Wang 2005) to provide an approximate solution for the natural frequencies of coupled shear walls. Their approach was based on the assumption of a uniform distribution of lateral force, which results in a fourth-order Sturm-Louiville type differential equation. However, such assumption should be brought into question since the lateral force associated with the inertia of the building does not remain constant with height during free vibration. Intrigued by these conflicting results, the natural periods of coupled shear walls are re-examined in the hope of answering some of these questions. Solutions obtained using differential transformation, which is a relatively new technique capable of dealing with eigenvalue problems efficiently, are compared with experimental data.

#### 2. GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

The model considered in this paper consists of two planar shear walls, connected by coupling floors or beams, as shown Fig. 1(a). The shear walls are assumed to be subjected to a distributed lateral load w per unit height. In modeling the lateral response, the discrete connecting floor slabs or beams are replaced by an equivalent medium of laminae, assumed to deform anti-symmetrically with a point of contraflexure at mid-span of the laminae. The coordinate system is chosen such that the vertical coordinate  $\overline{x}$  is measured from the base of the wall. The task at hand is to determine the in-plane lateral vibrational period of the coupled shear wall.

Basic equations governing the vibration of coupled shear walls may be adapted from that developed under static condition. With time t introduced as the second variable, the governing equation can be written in the form of a partial differential equation (Smith and Coull 1991):

$$\frac{\partial^4 \overline{y}}{\partial \overline{x}^4} - p^2 r^2 \frac{\partial^2 \overline{y}}{\partial \overline{x}^2} = \frac{1}{EI_o} \left( \frac{\partial^2 M_o}{\partial \overline{x}^2} - r^2 (p^2 - 1) M_o \right)$$
(2.1)

where  $\overline{y}$  is the lateral deflection of the wall, which is a function of the vertical coordinate and time, E is the elastic modulus of the wall, assumed to be the same for both walls, coupling beams or slabs, and  $M_o$  is the overturning moment due to the lateral load, as depicted in Fig. 1(b), and is given by:

$$M_o = \int_{\tau=\bar{x}}^{H} (\tau - \bar{x}) w(\tau) d\tau$$
(2.2)

The term  $w(\tau)$  in Eqn. 2.2 corresponds to the lateral load, with  $\tau$  being a dummy variable, and H is the total height of the shear wall. The two parameters, p and r, in Eqn. 2.1 are defined as:

$$p^{2} \equiv 1 + \frac{A_{o}I_{o}}{A_{1}A_{2}L^{2}}$$
(2.3)







(b) Overturning moment due to lateral load

+ dτ

h

 $M_{o} = \frac{Overturning}{Moment}$ 

Fig. 1 - Coupled shear wall subjected to lateral load

and

$$r^2 = \frac{12I_{be}L^2}{b^3hI_a}$$
(2.4)

where  $A_1$  and  $A_2$  are the cross-sectional areas, and  $I_1$  and  $I_2$  are the moments of inertia, of walls 1 and 2, respectively. The term  $A_o$  represents the sum of the wall areas i.e.  $A_o = A_1 + A_2$ , whereas the term  $I_o$  represents the sum of the moments of inertia i.e.  $I_o = I_1 + I_2$ . The term L is the distance between the centriods of the two walls, b is the length of the coupling beam, and h is the story height, which is assumed to be constant over the height of the building. The term  $I_{be}$  in Eqn. 2.4 represents the effective moment of inertia of the coupling beam, including shear effects, and is given by:

$$I_{be} = \frac{I_b}{1+\alpha} \tag{2.5}$$

where the parameter  $\alpha$  characterizes the reduction in stiffness of the coupling beam as a result of shear deformation, and is given by:

$$\alpha = \frac{12 f E I_b}{G A_b b^2} \tag{2.6}$$

where  $I_b$  is the moment of inertia of the coupling beam, G is the shear modulus, and  $A_b$  is the cross-sectional area of the coupling beam, and f is the form factor, which may be taken as 1.2 for rectangular sections.

For free vibration analysis of coupled shear walls addressed in this paper, the inertial effects are considered assuming a constant distributed mass, denoted by  $\rho$ , along the height of the building. By virtue of the D'Alembert's principle, the lateral load associated with the distributed mass may be written as:

$$w(\overline{x},t) = -\rho \frac{\partial^2 \overline{y}}{\partial t^2} \quad \text{where} \quad \overline{y} = \overline{y}(\overline{x},t) \tag{2.7}$$

Thus the overturning moment  $M_o$  acting on the wall during free vibration can be written in integral form from Eqn. 2.2, as:

The 14<sup>th</sup> World Conference on Earthquake Engineering October 12-17, 2008, Beijing, China



$$M_{o} = -\rho \int_{\tau = \overline{x}}^{H} (\tau - \overline{x}) \frac{\partial^{2} \overline{y}}{\partial t^{2}} d\tau$$
(2.8)

By substituting the overturning moment into Eqn. 2.1 and taking twice the partial derivative with respect to  $\bar{x}$ , we have:

$$\frac{\partial^{6}\overline{y}}{\partial\overline{x}^{6}} - p^{2}r^{2}\frac{\partial^{4}\overline{y}}{\partial\overline{x}^{4}} = \frac{\rho}{EI_{o}} \left(r^{2}(p^{2}-1)\frac{\partial^{2}\overline{y}}{\partial t^{2}} - \frac{\partial^{4}\overline{y}}{\partial\overline{x}^{2}\partial t^{2}}\right)$$
(2.9)

Using the method of separation of variables, lateral deflection  $\overline{y}(\overline{x},t)$  of the wall may be expressed as the product of a position function  $\overline{\phi}(\overline{x})$  and a time function  $\eta(t)$  i.e.

$$\overline{y}(\overline{x},t) = \overline{\phi}(\overline{x})\eta(t) \tag{2.10}$$

By defining the following dimensionless quantities:

$$\phi \equiv \frac{\overline{\phi}}{H}, \quad x \equiv \frac{\overline{x}}{H}, \quad \beta \equiv \frac{12I_{be}L^2H^2}{b^3hI_o}, \quad \xi \equiv \frac{A_oI_o}{A_1A_2L^2}, \quad \lambda \equiv \frac{\rho\omega^2H^4}{EI_o}$$
(2.11)

where  $\omega$  is the circular frequency of the coupled shear wall, and using a spectral representation (Doyle 1989), Eqn. 2.9 can be written as:

$$\frac{d^6\phi}{dx^6} - \beta(1+\xi)\frac{d^4\phi}{dx^4} - \lambda\frac{d^2\phi}{dx^2} + \beta\xi\lambda\phi = 0$$
(2.12)

The above equation describes the free vibration of fixed-base coupled shear walls in non-dimensional form and can be solved in conjunction with six appropriate boundary conditions. Three of the boundary conditions may be prescribed at the base of the wall while the remaining three may be prescribed at the top of the wall. More specifically, the following conditions are available at the base of the wall:

$$\phi\big|_{x=0} = 0 \tag{2.13}$$

$$\frac{\partial \overline{y}}{\partial \overline{x}}\Big|_{\overline{x}=0} = 0 \implies \frac{d\phi}{dx}\Big|_{x=0} = 0$$
(2.14)

and

$$\frac{d^{3}\phi}{dx^{3}}\Big|_{x=0} + \lambda \int_{x=0}^{1} \phi(x) dx = 0$$
(2.15)

Similarly, at the top of the wall, the following conditions are available:

$$N\big|_{\overline{x}=H} = 0 \implies \frac{d^2\phi}{dx^2}\Big|_{x=1} = 0$$
(2.16)

$$\left. \frac{d^4 \phi}{dx^4} \right|_{x=1} - \lambda \phi \Big|_{x=1} = 0 \tag{2.17}$$

and

$$\left. \frac{d^5 \phi}{dx^5} \right|_{x=1} - \beta (1+\xi) \frac{d^3 \phi}{dx^3} \right|_{x=1} - \lambda \frac{d\phi}{dx} \right|_{x=1} = 0$$
(2.18)

For a given geometry of the coupled shear wall and material properties, Eqn. 2.12 may be solved together with Eqns. 2.13-2.18 as an eigenvalue problem. The resulting solution for the parameter  $\lambda$  is related to the circular frequency  $\omega$  or period T of the coupled shear wall by:



$$T = \frac{2\pi}{\omega} = 2\pi H^2 \sqrt{\frac{\rho}{\lambda E I_o}}$$
(2.19)

#### 3. DIFFERENTIAL TRANSFORMATION AND SOLUTION PROCEDURE

A convenient closed form solution for Eqn. 2.12 is generally difficult except for the simple case of coupled shear walls with equal properties. The homogeneous equation, however, can be solved numerically using the method of differential transformation, which can provide rather accurate solution for the eigenvalue problem. As basic concepts in differential transformation are readily available in the literature (e.g. Abdel-Halim Hassan 2002; Chen and Ho 1996), only salient features are highlighted here for the application to free vibration of coupled shear walls. By definition, the differential transformation of a function  $\phi(x)$  can be written as:

$$\Phi(k) = \frac{1}{k!} \left[ \frac{d^k \phi}{dx^k} \right]_{x=a} \quad \text{where} \quad 0 \le x \le 1$$
(3.1)

where  $\Phi(k)$  is sometimes referred to as the T-function at x = a. In many applications (e.g. Chen and Ho 1996; Bert Zeng 2004; Abdel-Halim Hassan 2002; Chai and Wang 2006; Chen and Ho 1999; Jang et al. 2003; Ho and Chen1998), the  $k^{th}$  derivative is evaluated at the origin by setting a = 0. The evaluation of derivatives at a = 0, however, is not a necessary condition but rather a convenient choice for implementation. Although not explicitly shown here, the value of a influences the convergence of the solution.

In the application of differential transformation to free vibration of coupled shear walls, the function  $\phi(x)$  may be regarded as the mode shape of the shear wall. The original function  $\phi(x)$  can then be recovered by a differential inverse transformation, which is given by:

$$\phi(x) = \sum_{k=0}^{\infty} (x-a)^k \Phi(k)$$
(3.2)

The combination of Eqns. 3.1 and 3.2 gives the Taylor series expansion of the function  $\phi(x)$  about x = a i.e.

$$\phi(x) = \sum_{k=0}^{\infty} \frac{(x-a)^k}{k!} \left[ \frac{d^k \phi}{dx^k} \right]_{x=a}$$
(3.3)

Thus, the differential transformation technique essentially converts a differential equation into an algebraic equation, which can be solved together with the boundary conditions of the problem.

By applying the differential transformation to the governing equation Eqn. 2.12, one obtains the following recursive equation:

$$\Phi(k+6) = \frac{\beta(1+\xi)(k+1)(k+2)(k+3)(k+4)\Phi(k+4) + \lambda(k+1)(k+2)\Phi(k+2) - \beta\xi\lambda\Phi(k)}{(k+1)(k+2)(k+3)(k+4)(k+5)(k+6)}$$
(3.4)

For the six boundary conditions associated with the problem, the inverse transformation of Eqn. 3.2 shall be applied. At the bottom of the wall, we have:

$$\phi|_{x=0} = 0 = \sum_{k=0}^{\infty} (-a)^k \Phi(k) = 0$$
 (3.5)

The 14<sup>th</sup> World Conference on Earthquake Engineering October 12-17, 2008, Beijing, China



$$\left. \frac{d\phi}{dx} \right|_{x=0} = 0 \implies \sum_{k=0}^{\infty} k \left( -a \right)^{k-1} \Phi(k) = 0 \tag{3.6}$$

$$\frac{d^{3}\phi}{dx^{3}}\Big|_{x=0} + \lambda \int_{x=0}^{l} \phi(x)dx = 0 \quad => \sum_{k=0}^{\infty} k(k-1)(k-2)(-a)^{k-3}\Phi(k) + \lambda \sum_{k=0}^{\infty} \frac{(l-a)^{k+1}}{k+1}\Phi(k) - \lambda \sum_{k=0}^{\infty} \frac{(-a)^{k+1}}{k+1}\Phi(k) = 0 \quad (3.7)$$

At the top of the wall, we have:

$$\frac{d^2\phi}{dx^2}\Big|_{x=1} = 0 \implies \sum_{k=0}^{\infty} k(k-1)(1-a)^{k-2}\Phi(k) = 0$$
(3.8)

$$\frac{d^{4}\phi}{dx^{4}}\Big|_{x=1} -\lambda\phi\Big|_{x=1} = 0 \implies \sum_{k=0}^{\infty} k(k-1)(k-2)(k-3)(1-a)^{k-4}\Phi(k) -\lambda\sum_{k=0}^{\infty} (1-a)^{k}\Phi(k) = 0$$
(3.9)

and

$$\frac{d^{5}\phi}{dx^{5}}\Big|_{x=1} -\beta(1+\xi)\frac{d^{3}\phi}{dx^{3}}\Big|_{x=1} -\lambda\frac{d\phi}{dx}\Big|_{x=1} = 0 \Longrightarrow$$

$$\sum_{k=0}^{\infty}k(k-1)(k-2)(k-3)(k-4)(1-a)^{k-5}\Phi(k) -\beta(1+\xi)\sum_{k=0}^{\infty}k(k-1)(k-2)(1-a)^{k-3}\Phi(k) -\lambda\sum_{k=0}^{\infty}k(1-a)^{k-1}\Phi(k) = 0 \quad (3.10)$$

The set of summation equations from Eqns. 3.5-3.10, together with the recursive equation of Eq. 3.4, completely describe the free vibration of the coupled shear wall. Although the representation of the wall deflected shape by T-functions requires a summation of an infinite number of terms, only a finite number of terms can be realized in actual applications. The solutions process may be facilitated by defining  $\Phi(0) = c_0$ ,  $\Phi(1) = c_1$ ,  $\Phi(2) = c_2$ ,  $\Phi(3) = c_3$ ,  $\Phi(4) = c_4$ , and  $\Phi(5) = c_5$ , and by using the recursive relation in Eqn. 3.4 to generate up to any *n* number of terms. The substitution of these terms into the boundary conditions, namely Eqns. 3.5 to 3.10, gives six simultaneous equations, which can be arranged into a matrix equation of the form:

$$[A]_{6\times6} \{C\}_{6\times1} = \{0\}_{6\times1}$$
(20)

where  $\{C\} = \langle c_o, c_1, c_2, c_3, c_4, c_5 \rangle^T$ . For non-trivial solutions, the determinant of the matrix [A] must be equal to zero, which furnishes a polynomial of n<sup>th</sup> order for the eigenvalue  $\lambda$ . The resulting characteristic polynomial can be solved numerically to give the natural period of the coupled shear wall.

#### 4. EXAMPLE - COMPARISON WITH MODEL COUPLED SHEAR WALLS

Two model coupled shear walls constructed of Plexiglas sheets were tested by Tso and Chan (1971), primarily to investigate the influence of coupling beams on the vibrational characteristic of coupled shear walls. One model corresponds to a twenty-story coupled shear wall with equal piers (denoted as model 1), while the second model corresponds to a fifteen-story coupled shear wall with unequal piers (denoted as model 2). Properties of the model coupled shear walls are listed in Table 4.1. The coupled shear walls were instrumented with strain gages at the base and were subjected to sinusoidal base excitation on a shaking table. Reported fundamental periods for each model are compared with the periods predicted by the differential transformation method.

Fig. 2(a) and (b) shows the fundamental period of the coupled shear wall as a function of the depth of connecting beam for models 1 and 2 respectively. The circle symbol represents the measured fundamental period while the solid line represents the period predicted by the differential transformation method using the equations developed in this paper. It can be seen from the figures that the fundamental period of the coupled shear wall decreases with increased depth of the coupling beam as expected. Although the predicted period is smaller than that measured, the predicted period is nonetheless accurate enough for design purposes as the error in prediction is generally less than 5%. The prediction is equally good for coupled shear walls of equal

# The 14<sup>th</sup> World Conference on Earthquake Engineering October 12-17, 2008, Beijing, China



and unequal pier width. Although not explicitly shown, mode shapes as well as periods for higher modes of vibration can be determined using the equations and technique proposed in this paper.

Table 4.1 - Properties of model coupled shear walls		
Parameter	Model 1	Model 2
Number of stories	20	15
Length of coupling beam (mm)	25.4	35.56
Width of right pier (mm)	76.2	35.56
Width of left pier (mm)	76.2	106.68
Story height (mm)	45.72	60.96
Young's modulus (MPa)	4.17e3	4.17e3
Shear modulus (MPa)	1.4e3	1.4e3
Mass density $(kg/m^3)$	1.2e3	1.2e3



Fig. 2 - Fundamental periods of model coupled shear walls - data from Chan and Tso (1971)

### 4. CONCLUSIONS

Coupled shear walls have long been recognized as being highly effective in resisting lateral loads in multistory buildings. Since seismic forces adopted for design are typically period-dependent, an accurate characterization of the dynamic properties of coupled shear walls is important, and as such, considerable attentions have been paid in the past to determine the natural period of coupled shear walls. Motivated by conflicting results in the literature, particularly inaccuracies in a recent study, this paper re-examines the dynamic characteristics of coupled shear walls and proposes a solution strategy for determining their periods of vibration. The governing equation, established on the basis of replacing the coupling beams by an equivalent medium of laminae, is derived together with the necessary boundary conditions. It is shown that the governing equation for free vibration of coupled shear walls is a sixth-order homogeneous differential equation, which can be solved as an eigenvalue problem using the technique of differential transformation.

The accuracy of the approach proposed in this paper is demonstrated using reported data of two model coupled shear walls with equal and unequal pier widths. The predicted period agrees well with the measured period of the model coupled shear walls for a wide range of coupling beam depth. The largest error in the prediction is about 5%, indicating that the methodology is sufficiently accuracy for practical design of coupled shear walls. The proposed method also permits the mode shapes and periods of higher modes to be determined.



#### REFERENCES

Abdel-Halim Hassan, I.H. (2002). On solving some eigenvalue problems by using a differential transformation. *Applied Mathematics and Computation*, **127**, 1-22.

Bert, C.W. and Zeng, H. (2004). Analysis of axial vibration of compound rods by differential transformation method. *Journal of Sound and Vibration*, **275:(3-5)**, 641-647.

Chaallal, O., Gauthier, D., and Malenfant, P. (1996). Classification methodology for coupled shear walls. *Journal of Structural Engineering, ASCE*, **122:12**, 1453-1458.

Chai, Y. H. and Wang, C. M. (2006). An application of differential transformation to stability analysis of heavy columns. *Int. Journal of Structural Stability and Dynamics*, **6:3**, 317-332.

Chen, C. K. and Ho, S. H. (1996). Application of differential transformation to eigenvalue problems. *Applied Mathematics and Computation*, **79**, 173-188.

Chen, C. K. and Ho, S. H. (1999). Transverse vibration of a rotating twisted Timoshenko beams under axial loading using differential transform. *Int. Journal of Mechanical Sciences*, **41**, 1339-1356.

Coull, A. and Mukherjee, P. R. (1973). Approximate analysis of natural vibrations of coupled shear walls. *Earthquake Engineering and Structural Dynamics*, **2**, 171-183.

Doyle, J.F. (1989). Wave Propagation in Structures. Springer-Verlag, 175, 5th Ave., New York.

Ho, S. H. and Chen, C. H. (1998). Analysis of general elastically end restrained non-uniform beams using differential transform. *Applied Mathematical Modeling*, **22**, 219-234.

Jang, M. J., Wang, J. S., and Liu, Y. C. (2003). Applying differential transformation method to parameter identification problems. *Applied Mathematics and Computation*, **139**, 491-502.

Jennings, P. C. and Skattum, K. S. (1973). Dynamic properties of planar, coupled shear walls. *Earthquake Engineering and Structural Dynamics*, **1**, 387-405.

Mukherjee, P. R. and Coull, A. (1973). Free vibrations of coupled shear walls. *Earthquake* Engineering and Structural Dynamics, 1, 377-386.

Paulay, T. and Priestley, M. J. N. (1992). Seismic Design of Reinforced Concrete and Masonry Buildings. Wiley Interscience, New York.

Rutenberg, A. (1975). Approximate natural frequencies for coupled shear walls. *Earthquake Engineering and Structural Dynamics*, **4**, 95-100.

Smith, B. S. and Coull, A. (1991). Tall Building Structures: Analysis and Design. Wiley Interscience Publication, New York.

Tso, W. K. and Biswas, J. K. (1972). An approximate seismic analysis of coupled shear walls. *Building Science*, **7**, 249-256.

Tso, Wai K. and Chan, Ho-Bong (1971). Dynamic analysis of plane coupled shear walls. *Journal of Engineering Mechanics Division, ASCE*, **97:1**, 33-47.

Tso, W. K. and Rutenberg, A. (1977). Seismic spectral response analysis of coupled shear walls. *Journal of Structural Division, ASCE*, **103:ST1**, 181-196.

Tso, W. K. and Rutenberg, A. (1974). Discussion on paper by A. Coull and P.R. Mukherjee - Approximate analysis of natural vibrations of coupled shear walls. *Earthquake Engineering and Structural Dynamics*, **3**, 105-107.

Wang, Q. and Wang, L. Y. (2005). Estimating periods of vibration of buildings with coupled shear walls. *Journal of Structural Engineering, ASCE*, **131:2**, 1931-1935.