Strength reduction factor (R factor) model and inelastic response spectra for forward-directivity ground motion

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ABSTRACT

The accuracy of six existing R-factor models for estimating inelastic response spectra is evaluated by using a suite of forward-directivity (FD) records and single-degree-of-freedom (SDOF) oscillators. Structural load-deformation mechanism is modelled using perfectly elastic-plastic, bilinear and stiffness degrading models. Two site conditions, rock site and soil site, are considered. A R-factor ratio (ratio of R factors from the existing R-factor Models to those from inelastic analyses) is used as a measure parameter. Results show that the Ordaz & Perez-Rocha model is suitable for use to estimate inelastic response spectra for FD ground motions. Based on the data set of FD records, a new R-factor model with considering the effect of site conditions is proposed. The accuracy of the new model has been checked, showing lower estimate error than the existing models.

KEYWORDS: R-factor, Near-fault ground motion, Response spectra

1. INTRODUCTION

Conventionally, strength reduction factor (R factor) is defined as the ratio of elastic strength to yield strength. The importance of estimating R factor originates in the need for directly deriving inelastic spectra. Brief steps for deriving an inelastic spectrum are: use an approximate R-factor model to estimate R factor, and then use the estimated R-factor to scale the elastic spectra. This is an approximate approach, but as expected, the error produced by the approach is tolerable in engineering practice.

In recent years, near-fault forward-directivity (FD) ground motion has attracted attention because it has caused an amount of structural damage in recent strong earthquake events. To obtain the inelastic spectra of FD ground motions, we should know which existing R-factor model is appropriate for use, as most existing R-factor models were developed mainly from far-field records. However, few papers have discussed the topic. For example, in the work of Fajfar (1999), he used the Vidic et al. R-factor model (1994), but he emphasized that his study excluded the effect of FD ground motions. The pulse R-factor model of Cuesta et al. (2001) might be the first one for FD ground motions, but the model is in an explicit form, so it is not convenient for end-users.

In this study, we have two objectives: 1) to verify which one of existing R-factor models is suitable for use in estimating R factor for FD ground motions, and 2) to develop a new R-factor model by using FD records and accounting for the effect of site conditions. For the purposes, two groups of FD records, recorded on soil and rock sites, are selected carefully from world-wide strong motion databases, six ductility factors from 2 to 8 are used as target ductility factors, the longest period used is 4s, and three hysteretic models, perfectly elastic-plastic, bilinear, and stiffness degrading models, are used to represent structural load-deformation mechanism. In this study, the Newmark and Hall (1973) (N&H), Nassar & Krawinkler (1991) (N&K), Miranda (1993), Ordaz & Perez-Rocha (1998) (O&P), Cuesta et al. (2003), and Berrill et al. (1981) R-factor models are assessed, and all analyses are performed based on SDOF systems.

2. NEAR-FAULT FORWARD-DIRECTIVITY GROUND MOTION RECORDS

In this study, FD records are selected from world-wide earthquake record databases, focusing on the 1994 Northridge earthquake, the 1995 Kobe earthquake, and the 1989 Loma Prieta earthquake. Two groups of FD records, recorded on rock and soil sites are selected by referring to the papers of Somerville et al. (1997) and Bray and Rodriguez-Marek (2004). We replotted velocity time histories of these records and deleted those without remarkable velocity pulses from the data set. Then, the two components for each record are rotated to fault-normal and fault-parallel directions, and only the fault-normal components are used to assess the existing

R-factor models. In this study, the rock and soil sites correspond to Classes A and B and Classes C and D in UBC code, respectively.

Table 1 lists all records and the ratio of PGVs of fault-parallel to fault-normal components. Most of the ratios of PGVs are less than 1.0 except the Anderson dam record from the 1984 Morgan Hill earthquake which is close to 1.0. The mean ratio of PGVs for the records on soil sites is 0.66 and rock sites is 0.65, indicating that PGV in the fault-normal component is much larger than that in the fault-parallel component.

3. EXISTING R-FACTOR AND STRUCTURAL MODELS USED IN THIS STUDY

In the past decades, various R-factor models have been developed by using SDOF oscillators. In this study, we divide existing R-factor models into two groups. One group is expressed by ductility factor, period, viscous damping, and ground motion parameters, hereafter called GA model, and the other is expressed by ductility factor, period, and viscous damping, hereafter called MV model, as listed in Table 2. The GA model includes the N&H model, the O&P model, and the Cuesta et al. model. The MV model includes the N&K model, the Miranda model, and the Berrill et al. model.

Energy dissipation due to structural deformation is conventionally represented using hysteretic models, the effect of which on the R factor has been discussed in the work of Vidic et al. (1994) and Lee et al. (1999). Lee et al. shows that the effect of hysteretic and viscous damping is significant, but the difference of R factors between 0% and 5% viscous damping or between 5% and 10% viscous damping is approximately 10 - 15%, suggesting that using 5% viscous damping to assess the R factor is reasonable.

It is accepted that bilinear model can represent an upper bound to the behaviour of intrinsically ductile, non-deteriorating systems, like steel members, with moderate axial load, and the stiffness degrading model can represent the characteristic behaviour of reinforced concrete members, frames, and walls under predominantly flexural stresses. In order to evaluate R-factor models for various structures, perfectly elastic-plastic, bilinear, and stiffness degrading models are used. Another reason for using the three hysteretic models is that most existing R-factor models were developed based on the three hysteretic models. Note that in these existing R-factor models, the perfectly elastic-plastic model was widely used. Therefore, the perfectly elastic-plastic model is considered as a reference in comparisons.

4. EVALUATION OF EXISTING R-FACTOR MODELS

Mean R-factor ratio, defined as the ratio of estimated to exact R factors for target ductility factors, is used to evaluate the accuracy of the existing R-factor models. To show uncertainty of the R-factor ratios, the standard deviation is calculated at each period. Results only for ductility factors of 2, 4, and 8 are presented, as shown in Figures 1 to 4.

Rock sites

For ductility factor μ=2 and the three hysteretic models (see Figures 1a to 1c), estimated R-factor errors from the six existing R-factor models are within 20 percent. For periods from 0.5s to 3.0s, the N&H model and O&P model give better R-factor estimates. Figures.1a to 1c also show that for μ=2 the effect of the hysteretic models on the estimated R factor is slight. For ductility factor μ=4 (see Figures 1d to 1f), estimated R-factor errors within 20 percent are from the N&H model, the O&P model, and the Berrill model. For periods beyond 0.75s and for perfectly elastic-plastic and bilinear models, the estimated errors from the Miranda model and the N&K model are also within 20 percent. The effect of the hysteretic models on the estimated R factor for μ=4 are more significant than that for ductility factor μ=2. Figures 1g to 1i show the results for ductility factor μ=8. The best estimate is from the O&P model, and then the Miranda model which produce the errors within 20 percent at most periods. For ductility factor μ=8, the effect of the hysteretic models is remarkable, and the smallest estimated error is from the bilinear model, followed by the perfectly elastic-plastic model and the stiffness degrading model.

Figure 2 shows the standard deviations of the R-factor ratios for rock sites. The standard deviations for ductility factor μ=2 are about 0.2, except at periods less than about 0.4s. The differences of the standard deviations between these existing R-factor models are slight. For ductility factor μ=4, the standard deviations are around 0.4. The minimum standard deviation is from the O&P model. Comparisons between these hysteretic models show that the maximum standard deviation is from the stiffness degrading model. Figures 2g to 2i show the results for ductility factor μ=8. The results are similar to those for μ=4. Compared with the cases for μ=2 and 4, larger standard deviations for μ=8 are observed.

Soil sites
In a similar manner to those for rock sites, comparisons for ductility factors μ=2, 4 and 8 are shown in Figure 3. For μ=2 (Figures 3a to 3c), estimated R-factor errors from these existing R-factor models are within 20 percent, except those from the Miranda model at some periods. For periods beyond 2.0s, the accuracy of all models is nearly the same and for periods from 0.5s to 2.0s, the O&P model, the N&H model and the Berrill model give better estimates. For μ=4 (see Figures 3d to 3f), only the O&P model and the N&H model give estimated R-factor errors within 20 percent. Figures 3d to 3f also show that these existing R-factor models are suitable in order for the bilinear, perfectly elastic-plastic, and stiffness degrading models. For μ=8 (see Figures 3g to 3i), none of these existing models gives estimated R-factor errors within 20 percent for periods from 0.3s to 1,5s, but at periods over 2.0s the O&P model, the N&H model, and the Berrill model give estimated errors within 20 percent. Clearly, the estimated R-factor errors for μ=8 are much larger than those for μ=4. Note that the largest peak error appears around 0.7s which is consistent with the transition period of the Berrill model.

Figure 4 shows the standard deviations of the R-factor ratios for soil sites. Similar to Figure 4, for μ=2 the standard deviations are about 0.2, and the difference between these standard deviations from different hysteretic models are slight; for μ=4, most standard deviations are less than 0.4 and the minimum standard deviation is from the Berrill model; for μ=8, the standard deviations are much larger than those for μ=2 and 4. Compared with Figure 4, the main differences are: 1) as the period increases, the standard deviation decreases, and 2) the differences of the standard deviations from the three hysteretic models are slight.

Comparisons between Figures 1 and 3 show that the accuracy of these existing R-factor models are higher for rock sites and for ductility factors less than 8. Normally the estimated R-factor errors from the GA models are smaller than those from the MV models. The probable reason for this is that the GA models are linked directly with ground motion parameters, rather than independent of ground motion parameters as in the MV models.

5. EVALUATION OF INELASTIC SPECTRA USING EXISTING R-FACTOR MODELS

The purpose of estimating R factor is to quickly obtain inelastic spectra for structural designs. It is, therefore, necessary to estimate the error which was introduced by using these approximate R-factor models. Herein the methodology taken by Cuesta et al (2003) was adopted, where the mean value and standard deviation of the difference between exact and approximate inelastic spectra for each of the three hysteretic models are assessed by taking into account all ground motions, ductility factors, and periods. The formulae for the assessment are shown in Eqns 5.1 and 5.2.

$$E_{ijk} = SA_{exact} - \frac{SA_{exact}(\mu = 1)}{R_{\mathrm{mod}el}} \tag{5.1a}$$

Mean value: $$E = \frac{1}{n_r n_d n_p} \sum_{i=1}^{n_r} \sum_{j=1}^{n_d} \sum_{k=1}^{n_p} E_{ijp} \tag{5.1b}$$

Standard deviation: $$\sigma(E) = \sqrt{\frac{1}{n_r n_d n_p} \sum_{i=1}^{n_r} \sum_{j=1}^{n_d} \sum_{p=1}^{n_p} (E_{ijk} - E)^2} \tag{5.2}$$

where, n_r=number of ground motions employed (=19 for rock site and 18 for soil site), n_d=number of ductility values (6, that is μ=2, 3, 4, 5, 6, 8), n_p=number of periods considered (=80, between 0.05 to 4.0s).

The mean values and standard deviations for the six existing R-factor models for rock sites are shown in Figure 5. For the GA models, the O&P model has smallest error, and for the MV models, the Berrill et al model has the smallest error. For the three hysteretic models, the stiffness degrading model has the largest error and the bilinear model has the minimum error, but the difference between the maximum and minimum errors is slight. The standard deviation (see Figure 5b) shows that the difference caused by the three hysteretic models is negligible. Similar to the mean value, the O&P model and Berrill et al model has smallest standard deviation in the GA model and MV model, respectively.

Figure 6 shows mean values and standard deviations of the difference between exact and approximate R factors for soil sites. The characteristics revealed from the analyses for rock sites are also displayed in this figure, but interestingly note that the mean values at soil sites are smaller than those at rock sites. This seems to be not consistent with the R-factor ratios shown in Figures 1 and 3, but carefully checking these figures we can find that at long periods (>2.0s) the errors of estimated R factor at soil sites are smaller than those at rock sites. This could be the reason why the mean values at soil sites are lower than those at rock sites.

6. RECOMMENDED R-FACTOR MODEL

As mentioned above, the O&P model is the best one suitable for estimating R factor for FD ground motions. However, in several cases its estimated errors are over 20 percent, particularly for soil sites. In addition, in the O&P model, PGD is used as a parameter. Actually PGD is very difficult to estimate because there are only few existing PGD attenuation relations for use. To overcome the drawbacks in the O&P model, we propose a new model, as shown in Eqn 6.1, based on pseudo-displacement spectrum which is easily obtained from estimated seismic hazard acceleration spectrum. Also, in Eqn.6.1 a spectral displacement at 3.0s period replaces PGD. The reason for this is that normally constant-displacement region starts from a period of 3.0s, such as in the New Zealand Loading Standards NZS1170.5:2004 (2004).

$$R(T,\mu) = 1 + B * \left(\frac{PSD(T)}{PSD(3.0s)} \right)^{A} \tag{6.1}$$

Two-stage regression analysis is taken in the study. In the first stage, regression is carried out with respect to period for a given ductility factor, where coefficients A and B are obtained. In the second stage, coefficients A and B as a function of ductility factor are obtained, as shown in Eqn.6.2.

$$A = \alpha * (\mu - 1)^{\beta} \qquad B = \xi * (\mu - 1)^{\varsigma} \tag{6.2}$$

For each of these cases (the combining of hysteretic models and two site conditions), coefficients, α, ß, ξ, and ς, are obtained using least-squares method. To show the effect of ductility factor, coefficients (A and B) versus ductility factor were plotted for the three hysteretic models and two site conditions. We found that the effect of hysteretic models on coefficients is slight, but the effect of site conditions is significant. For simplicity, we average the coefficients, α, ß, ξ, and ς, of the three hysteretic models for the two site conditions, respectively. The recommended R-factor models for rock and soil sites are shown in Eqs.6.3 and 6.4.

For rock sites: $$R(T,\mu) = 1 + 1.1024(\mu - 1)^{0.9756} * \left(\frac{PSD(T)}{PSD(3.0s)} \right)^{0.2904(\mu-1)^{0.232}} \tag{6.3}$$

For soil sites: $$R(T,\mu) = 1 + 1.2202(\mu - 1)^{0.9477} * \left(\frac{PSD(T)}{PSD(3.0s)} \right)^{0.2614(\mu-1)^{0.256}} \tag{6.4}$$

To show the accuracy of the new models (Eqns.6.3 and 6.4), the R-factor ratios of the recommended to the exact R-factors for μ=2, 3, 4, 5, 6, and 8 on rock and soil sites, respectively, are shown in Figures 7 and 8. The errors produced by the new models are lower than 20 percent, even for μ=8. Comparison between Figures 1 and 7 (for rock sites) shows that the new model improves the accuracy of estimating R factor, even compared with the O&P model. Figures 3 and 9 show the estimated errors from existing models and the new model, respectively, on soil sites. The results show that the new model greatly reduces the estimated error, even when compared with those from the O&P model, suggesting that the effect of site conditions is significant. Figures 7 and 8 also show that for the three hysteretic models, the minimum errors are from the bilinear model.

8. CONCLUSIONS

From the above analyses based on SDOF systems, which are excited by the FD ground motion records, some conclusions have been reached:

1. Normally, estimated R factors from the GA model are closer to the exact solution than the MV model for FD records. The reason for this may be that the GA model is linked directly with ground motion parameters, such as PGD, PGV, and PGA, whereas the MV model is independent of any ground motion parameters.
2. The better R-factor models for FD ground motions is, in order, the O&P model, the N&H model, and the Berrill model. For rock sites, the three models are suitable for ductility factors less than 8, except the O&P model which can be used for μ=8; for soil sites, the O&P model and the N&H model are suitable for use to ductility factors less than 8;
3. As the ductility factor increases, the estimated R-factor error and standard deviation also increase. For ductility factors μ=2, 4, and 8, the standard deviations are around 0.2, 0.4, and 0.6 for rock and soil sites, but the estimated errors at soil sites are larger than those at rock sites, particularly at short periods. The results show that these approximate R-factor models are more suitable for the structures built-up on rock sites than on soil sites.
4. Compared with the effect of various load-deformation behaviours on the estimated R factor, the effect of site conditions is more significant. Therefore, more focus should be on the effect of site conditions, rather than the effect of hysteretic models.
5. A new R-factor model for FD strong motions has been developed. The model is based on the mathematic expression of the O&P model, but using pseudo-displacement spectra instead of displacement spectra (although the difference between the two is not large, but the pseudo-displacement spectra can be obtained directly from seismic hazard response spectra or design spectra), and through replacing PGD by PSD(3.0s) avoiding the difficulty of obtaining PGD in structural designs. In addition, the new model emphasizes the effect of site conditions by using different models for rock and soil sites. The new model is suitable for ductility factors up to 8 and periods of up to 4.0s.

ACKNOWLEDGEMENTS

The authors wish to thank the review and critical comments of Prof. P. Moss, the University of Canterbury. This study is supported in part by the Foundation for Research and Science and Technology of New Zealand, Contract number C05X0208 and C05X0301.

REFERENCES

Nassar, A.A. and Krawinkler, H. (1991). Seismic demands for SDOD and MDOF systems, Report No.95, The John A. Blume Earthquake Engineering Center, Stanford University, California, USA.

Fajfar, P. (1999). Capacity spectrum method based on inelastic demand spectra, *Earthquake Eng. Struct. Dyn* 28, 979-993.

Vidic, T., Fajfar, P. and Fischinger, M. (1994). Consistent inelastic Design Spectra: Strength and Displacement, *Earthqauke Engng. Struct. Dyn* 23, 507-521.
Cuesta, I. and Aschheim M. (2001). Inelastic response spectra using conventional and pulse R-factors, *J. of Structural Engineering* 129 (9), ASCE, 1013-1020.
Cuesta, I., Aschheim, M.A. and Fajfar P. (2003). Simplified R-factor Relationships for Strong Ground Motions, *Earthquake Spectra* 19 (1), 25-45.
Newmark, N.M. and Hall W.J. (1973). Seismic design criteria for nuclear reactor facilities, Report No.46, Building Practice for Disaster Mitigation, National Bureau of Standards, U.S. Department of Commerce, 209-236.
Miranda, E. (1993). Site-dependent strength reduction factors, *J. of Structural Engineering* 119 (12), ASCE, 3503-3519.
Ordaz, M. and Perez-Rocha, L.E. (1998). Estimation of strength-reduction factors for elastoplastic systems: a new approach, *Earthquake Engineering and Structural Dynamics* 27, 889-901.
Berrill, J.B., Priestley, M.J.N. and Park, R. (1981). Future comments on seismic design loads for buildings, *Bulletin of the New Zealand National Society for Earthquake Engineering* 14 (1), 3-11.
Somerville, P.G., Smith, N.F., Graves, R.W. and Abrahamson, N.A.(1997). Modification of empirical strong ground motion attenuation relations to include the amplitude and duration effects of rupture directivity, *Seismilogical Research Letters* 68 (1), 199-222.
Bray, J.D. and Rodriguez-Marek, A.(2004). Characterization of forward-directivity ground motions in the near-fault region, *Soil Dynamics and Earthquake Engineering* 24, 815-828.
New Zealand Loading Standards, 2004. Structural Design Actions Part 5: Earthquake actions-New Zealand, NZS1170.5.
Lee, L., Han, S.W. and Oh, Y.H. (1999). Determination of ductility factor considering different hysteretic models, *Earthquake Engng. Struct. Dyn* 28, 957-977.

Table 1a Near-fault records on rock sites

Station Name	Erathquake	PGA_FN (m/s^2)	PGV_FN (m/s)	PGA_FP (m/s^2)	PGV_FP (m/s)	$V_{P/N}$
Temblor	1966 Parkfield	3.223	0.200	2.294	0.122	0.61
Pacoma Dam	1971 Sanfernando	12.740	1.221	7.705	0.430	0.35
Tabas	1978 Tabas, Iran	8.831	0.859	10.210	0.620	0.72
Anderson Dam	1984 Morgan Hill	4.363	0.279	2.698	0.291	1.04
Coyote Lake Dam	*	10.350	0.698	8.115	0.640	0.92
Gilroy Ary6	*	2.412	0.358	3.132	0.126	0.35
Gilroy Ary1	1989 LomaPrieta	4.198	0.386	4.352	0.287	0.74
Gavilan Coll	*	2.937	0.310	3.787	0.276	0.89
Lexington Dam	*	4.435	1.180	3.806	0.457	0.39
LGPC	*	6.623	1.075	4.407	0.585	0.54
Lucerne	1992 Landers	6.952	0.826	6.587	0.541	0.65
Jensen GBG	1994 Northridge	5.081	0.713	10.230	0.685	0.96
LA Dam	*	3.998	0.755	3.325	0.669	0.89
Pacoima Dam	*	13.490	1.075	14.370	0.500	0.46
KJM	1995 Kobe	8.373	0.959	5.832	0.521	0.54
Kobe U.	*	3.221	0.491	2.593	0.388	0.79
Arcelik	1999 Kocaeli	1.471	0.396	2.146	0.177	0.45
Gebze	*	2.395	0.503	1.346	0.297	0.59

Table 1b Near-fault records on soil sites

Station	Earhquake	PGA_FN (m/s^2)	PGV_FN (m/s)	PGA_FP (m/s^2)	PGV_FP (m/s)	$V_{P/N}$
ElCenAry3	1979 Imperial Valley	2.612	0.468	2.169	0.399	0.85
ElCebAry4	*	3.532	0.766	4.761	0.374	0.49
Brawley Airport	*	1.550	0.361	2.062	0.359	0.99
EC Meloland overpass	*	3.708	1.150	2.611	0.273	0.24
Desert Hot Spring	1986 North-Palm Spring	3.362	0.297	2.802	0.230	0.77
Palm Springs Airpoty	*	1.572	0.147	1.510	0.103	0.70
El Centro Imp. co. cent	1987 Super-stition Hills	3.241	0.521	4.042	0.353	0.68
Parachute test site	*	4.249	1.090	3.502	0.468	0.43
Rinaldi Receiver	1989 Loma Prieta	8.767	1.848	3.892	0.660	0.36
Gilroy Ary2	*	3.985	0.457	2.970	0.276	0.60
Gilroy Ary3	*	5.230	0.493	4.476	0.371	0.75
Erzikan	1992 Erzincan Turkey	5.055	0.840	4.861	0.643	0.77
NewHall W. PicoCyn Rd	1994 Northridge	4.041	1.140	3.556	0.512	0.45
Sylmar City Hospital	*	7.183	1.221	5.834	0.550	0.45
OSA	1995 Kobe	0.700	0.197	2.082	0.170	0.87
BoluDuz	1999 Duzce	8.068	0.621	7.138	0.565	0.91
DuzceKo	1999 Kocaeli	3.062	0.589	3.511	0.464	0.79

Table 2 GA model and MV model

Classification	Existing R-factor models		
GA model	Newmark & Hall	Ordaz &Perez-Rocha	Cuesta et al.
MV model	Nassar & Knawinkler	Miranda	Berrill et al.

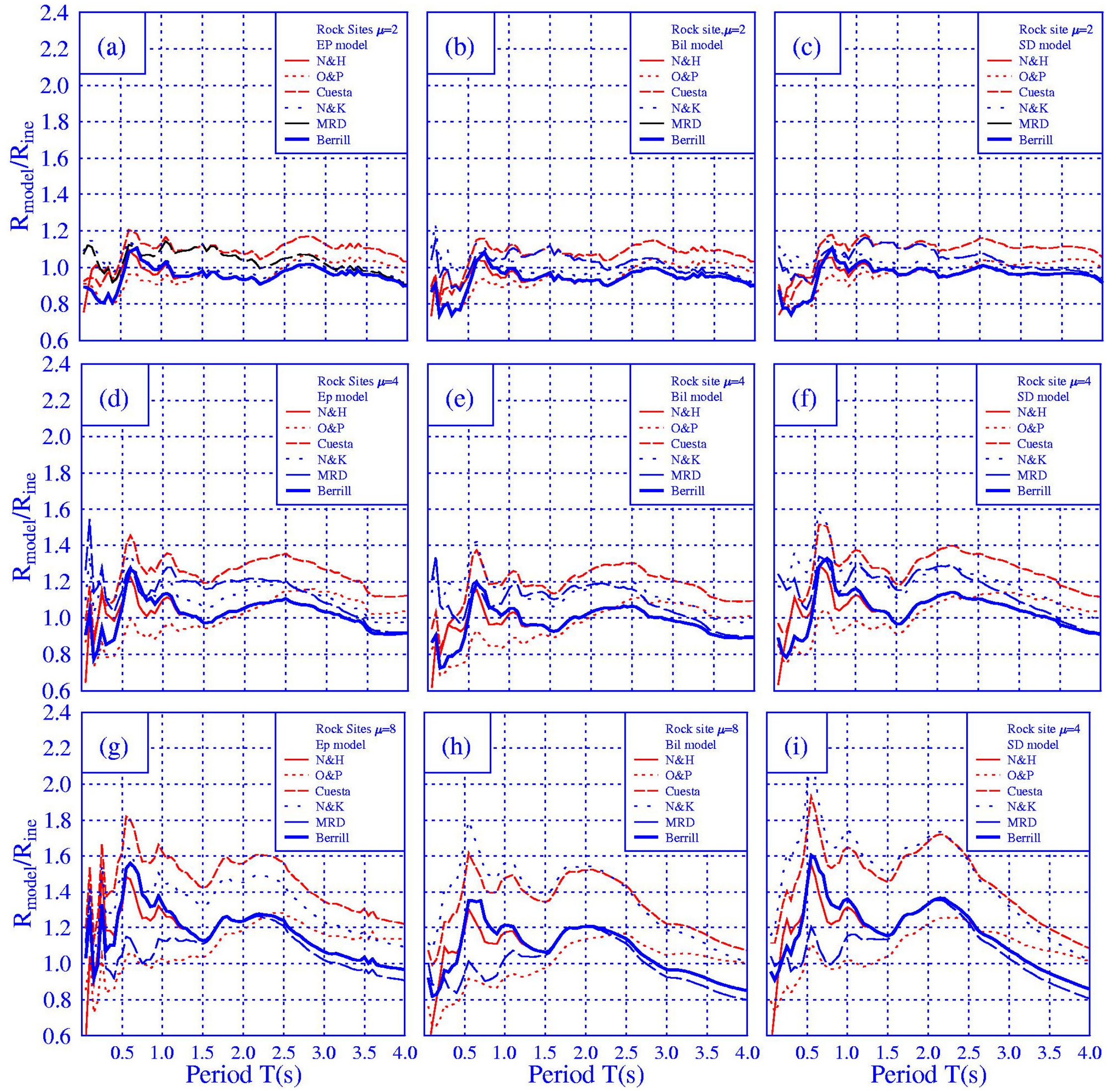

Figure 1 Comparisons of the R-factor ratios of the approximate to the exact R factors for μ=2, 4, and 8 on rock sites for the three hysteretic model (N&H: the Newmark & Hall model; Ordaz: the Ordaz and Perez-Rocha model; Cuesta: the Cuesta et al model; N&K: the Nassar & Krawinkler model; MRD: the Miranda model; Berrill: the Berrill et al model, and EP: perfectly elastic-plastic model; Bil: bilinear model; SD: stiffness degrading model).

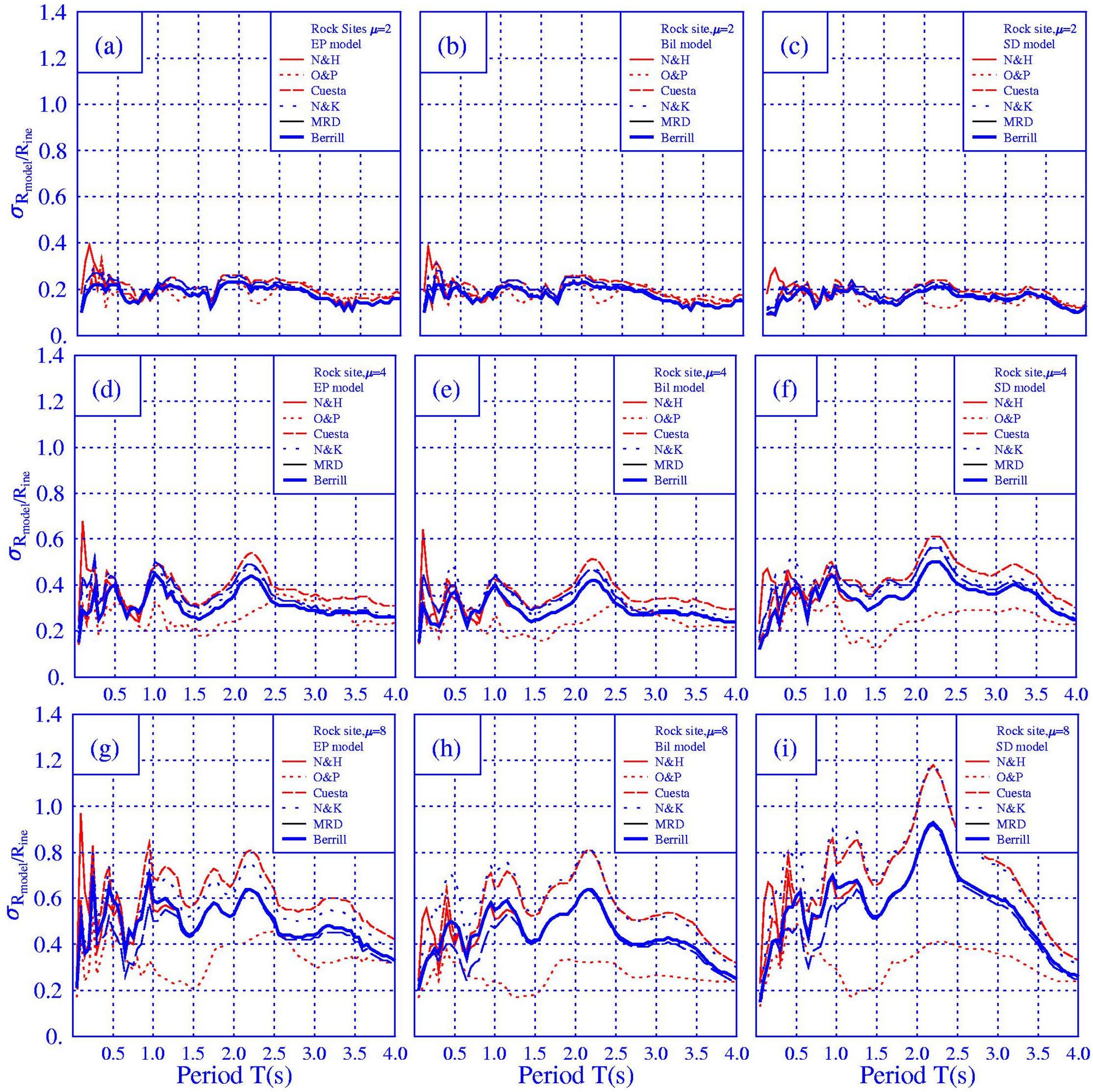

Figure 2 Comparisons of standard deviations of the R-factor ratios for μ=2, 4, and 8 on rock sites for the three hysteretic models (the meaning of these labels shows in the caption of Figure 1).

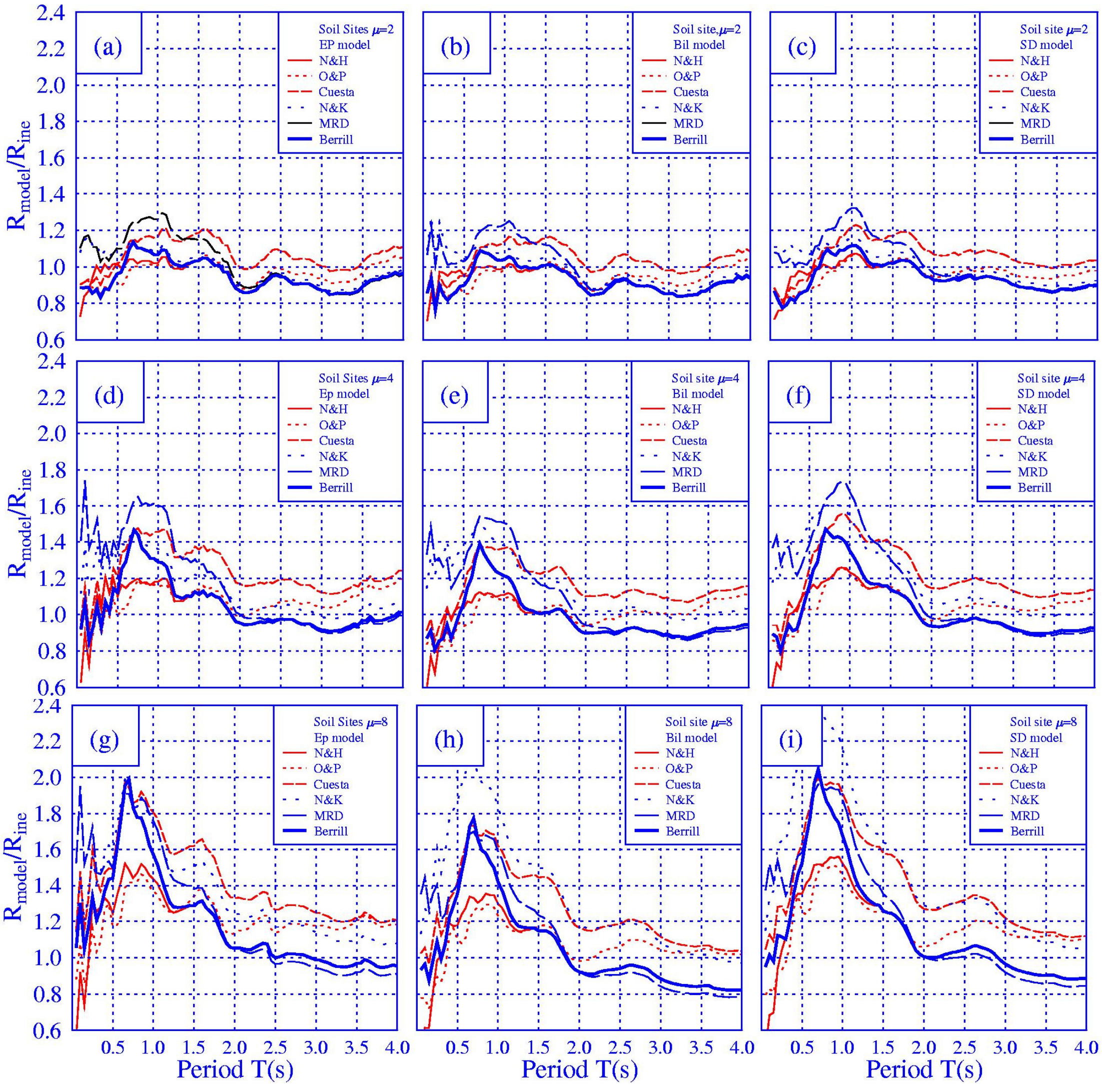

Figure 3 Comparisons the R-factor ratios of the approximate to the exact R factors for μ=2, 4, and 8 on soil sites for the three hysteretic models (the meaning of these labels shows in the caption of Figure 1)

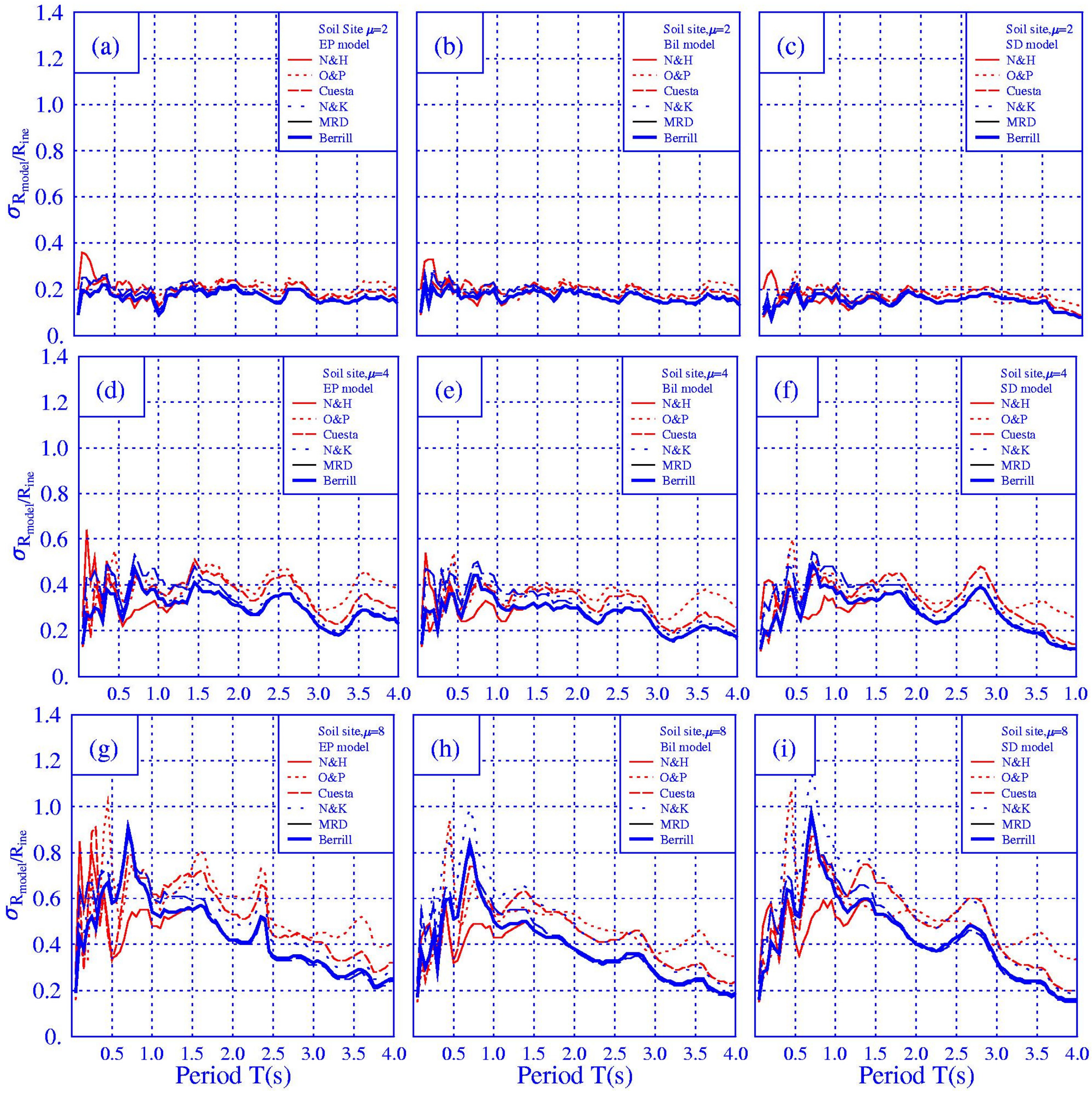

Figure 4 Comparisons of standard deviations of the R-factor ratios for μ=2, 4, and 8 on soil sites for the hysteretic models (the meaning of these labels shows in the caption of Figure 1).

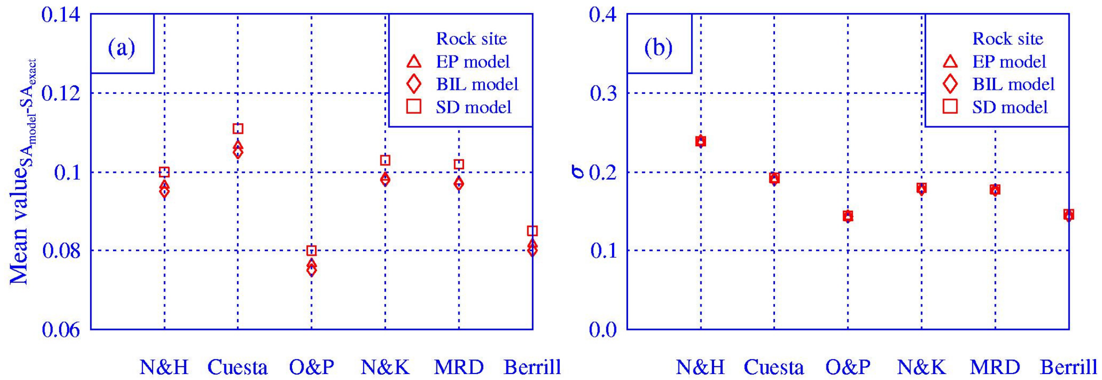

Figure 5 Comparisons of the mean values (a) and standard deviations (b) of the difference between the approximate and exact inelastic spectra for the existing 6 R-factor models on rock sites (EP: perfectly elastic-plastic model; Bil: bilinear model; SD: stiffness degrading model).

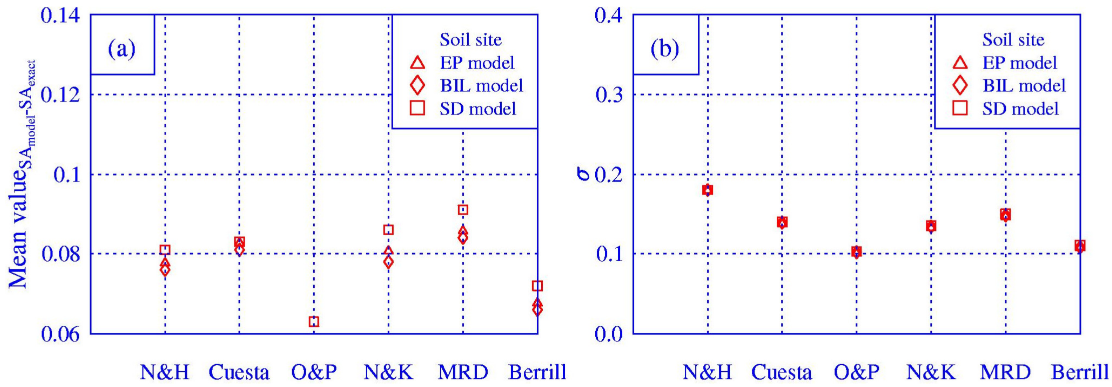

Figure 6 Comparisons of the mean values (a) and standard deviations (b) of the difference between the approximate and exact inelastic spectra for the existing 6 R-factor models on soil sites (EP: perfectly elastic-plastic model; Bil: bilinear model; SD: stiffness degrading model).

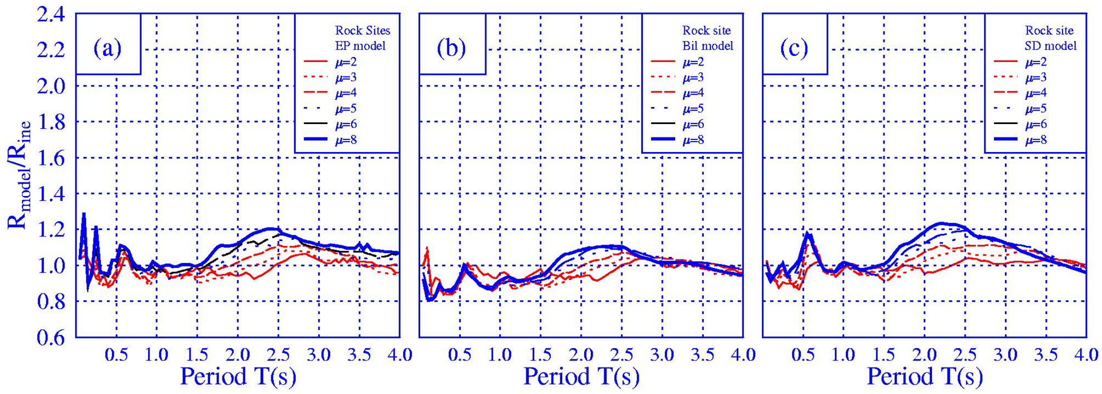

Figure 7 Comparisons of the R-factor ratios of the approximate to exact R-factors for μ=2, 3, 4, 5, 6, and 8 on rock sites for the three hysteretic models calculated from the new model (Eqn.6.3) (EP: perfectly elastic-plastic model; Bil: bilinear model; SD: stiffness degrading model).

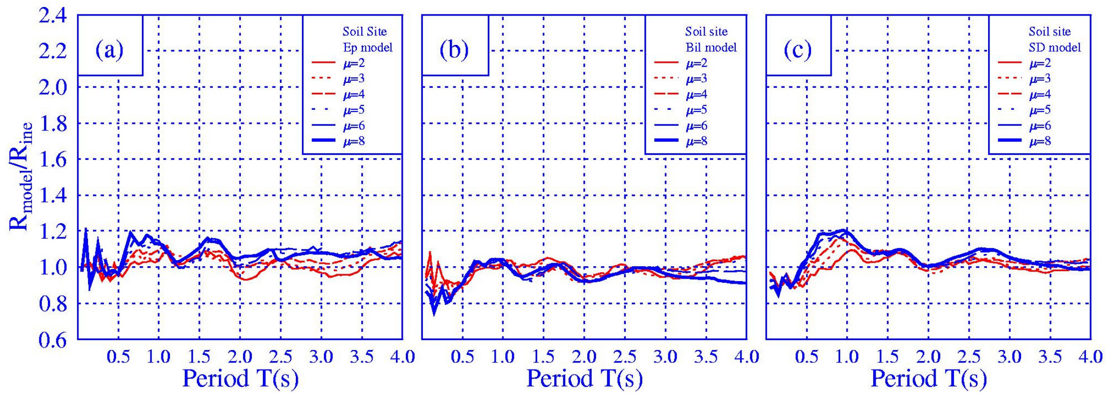

Figure 8 Comparisons of the R-factor ratios of the approximate to exact R-factors for μ=2, 3, 4, 5, 6, and 8 on soil sites for the three hysteretic models calculated from the new model (Eqn.6.4) (EP: perfectly elastic-plastic model; Bil: bilinear model; SD: stiffness degrading model).