

AN ENERGY BASED METHOD FOR SEISMIC EVALUATION OF STRUCTURES

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ABSTRACT :

This paper presents a seismic evaluation procedure based on an energy concept that has been recently developed and successfully used for design purposes called Performance-Based Plastic Design (PBPD). The underlying theory and the framework for carrying out the analysis are first presented. The skeleton force-displacement (capacity) curve of the structure is converted into energy capacity plot which is superimposed over the corresponding energy demand plot for the given hazard level to determine the expected peak response. The method is applied to a number of example single-degree-of-freedom (SDOF) and multi-degree-of-freedom (MDOF) structural systems with excellent results. The results are compared with those obtained from nonlinear dynamic analyses as well as those from methods proposed by other investigators including the Modal Pushover Analysis Method and the FEMA Displacement Coefficient Method. For SDOF systems, the results indicate that the proposed method provides response values that are identical to those obtained from a well-established procedure using inelastic design spectrum. For MDOF systems also, the proposed method provides response values that are reliable when compared to the results from non-linear dynamic analysis and other well-established nonlinear static procedures.

KEYWORDS: Energy Method, Energy Balance Concept, Nonlinear Static Analysis, Seismic Evaluation, Performance-Based Plastic Design

1 INTRODUCTION

Nonlinear Static Procedure (NSP) has been widely accepted as a useful tool for performance-based seismic design and evaluation of structures (FEMA 2006). In the short time that has elapsed since its introduction to the engineering community, the method has been a subject of extensive research and several new analysis approaches have been proposed. In most cases, the behavior of the structure is characterized by the capacity curve which is represented by a plot of the base shear versus the roof displacement. The capacity curve is used to establish an equivalent single-degree-of-freedom (SDOF) system. The target displacement can then be predicted using one of the methods such as nonlinear dynamic analysis, capacity spectrum approach, the modification coefficient approach, or the direct use of inelastic constant ductility spectra. Once determined, the target displacement can then be projected back to the roof displacement from which the story drifts and component forces can be obtained for comparison with available capacities. The absorbed energy (or work done by the external forces) has also been suggested as an alternative index to establish the capacity curve (Hernández-Montes et al. 2004; Mezzi et al. 2006).

The objective of this study is to present a seismic evaluation procedure based on an energy concept that has been recently developed and successfully used for design purposes called Performance-Based Plastic Design (PBPD) method. The underlying theory and the framework for carrying out the analysis are first presented. The analysis procedure is then applied to SDOF and multi-degree-of-freedom (MDOF) structures to estimate the displacement demands. The results obtained from the proposed procedure are then evaluated and compared with the results from non-linear dynamic procedure (NDP) as well as those from other well established NSPs.

2. ENERGY BALANCE CONCEPT IN PERFORMANCE-BASED PLASTIC DESIGN

Recently, an innovative performance-based design procedure which directly accounts for structural inelastic behavior has been developed (Leelataviwat et al. 1999; Lee and Goel 2001; Dasgupta et al. 2004; Chao and Goel 2006a; Chao and Goel 2006b). The design base shear for a selected hazard level and yield mechanism is calculated by equating the work needed to push the structure monotonically up to the target drift to that required by an equivalent elastic-plastic single degree of freedom system to achieve the same state. The method has been called Performance-Based Plastic Design (PBPD).

At the heart of PBPD methodology is the energy balance concept. This concept has been used by early researchers, in a design context, to obtain the amount of plastic energy to be “absorbed” by the structure (Housner 1956) as well as to derive constant ductility inelastic design spectra (Newmark and Hall 1982). The energy balance concept is based on the assumption that the energy computed from the monotonic load-deformation response of the inelastic system and the one computed from the corresponding elastic system are the same. For a SDOF system with elastic-plastic load-deformation characteristic, the energy balance equation is given by:

$$E = \frac{1}{2}MS_v^2 = \frac{1}{2}V_y D_y + V_y(D_m - D_y) \quad (1)$$

where M is the mass of the system, S_v is the pseudo velocity, V_y is the yield strength, D_y is the yield displacement, D_m is the maximum inelastic displacement.

As recognized by Newmark, the above energy balance relationship is valid only for systems with period in the acceleration sensitive regions of the spectrum. In the PBPD the energy balance equation is modified so that it can be used for all periods. Lee and Goel (2001) introduced the energy factor, γ , and modified the energy balance equation as follows:

$$\gamma E = \gamma \frac{1}{2}MS_v^2 = \frac{1}{2}V_y D_y + V_y(D_m - D_y) \quad (2)$$

in which γ is the energy factor and is defined as the ratio of the energy absorbed by the inelastic system to that of the equivalent elastic system. The energy factor can be computed by rearranging Equation 2:

$$\gamma = \frac{\frac{1}{2}V_y D_y + V_y(D_m - D_y)}{\frac{1}{2}MS_v^2} = \frac{\frac{1}{2}V_y D_y + V_y(D_m - D_y)}{\frac{1}{2}V_y D_y R_y^2} = \frac{2\mu - 1}{R_y^2} \quad (3)$$

where μ is the displacement ductility factor (D_m/D_y), and R_y is the yield strength reduction factor. The yield strength reduction factor, R_y , is defined as the ratio of the strength required for the system to remain elastic to the yield strength of the system. The modified energy balance concept is illustrated in Figure 1a.

In order to compute the energy factor in Equation 3, a relationship between R_y - μ - T is needed. Using R_y - μ - T equations such as the one developed by Newmark and Hall (1982), the factor γ can be determined for a given ductility level as shown in Figure 1b. For seismic design purposes, a target ductility level can be selected and, Equation 3 can then be used to obtain the required yield strength of the system

The energy balance equation can be extended to MDOF systems by using the concept of equivalent simple oscillator. The equivalent simple oscillator of a MDOF system in the n^{th} mode is defined as a SDOF system with the same period and carries a mass equal to:

$$M_n^* = \frac{(\phi_n^T \cdot \mathbf{m} \cdot \mathbf{1})^2}{\phi_n^T \cdot \mathbf{m} \cdot \phi_n} \quad (4)$$

where M_n^* is generally called the effective modal mass for the n^{th} mode of the MDOF system, $\mathbf{m}$ is the mass matrix, ϕ_n is the n^{th} mode shape, and $\mathbf{1}$ is a unit vector. The sum of all effective modal masses for the MDOF system is equal to the total mass of the MDOF system. Further, for this SDOF system carrying a mass equal to M_n^* , the SDOF becomes equivalent to the n^{th} mode of the MDOF system in terms of base shear.

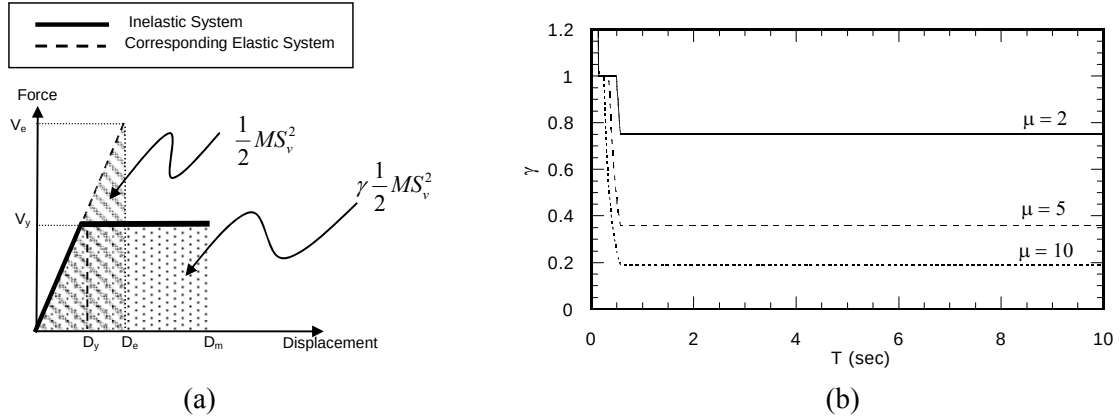


Figure 1. (a) Modified Energy Balance Concept (Lee and Goel, 2001); (b) Energy Factor as a Function of Period and Ductility Ratio based on Newmark and Hall's R_y - μ - T Equations.

By assuming that the coupling between the modes which arises due to yielding of the system can be neglected (Chopra and Goel 2002) and that the mode shapes remain constant even after yielding, it can be shown (Leelataviwat et al. 2007) that the equivalent simple oscillator can be used to calculate the energy demand of the n^{th} mode of the MDOF system resulting in the energy balance equation:

$$\gamma \frac{1}{2} M_n^* S_v^2 = \frac{1}{2} V_{bny} u_{ny}^* + V_{bny} (u_{nm}^* - u_{ny}^*) \quad (5)$$

where V_{bny} is the yield base shear of the MDOF in the n^{th} mode and u_{ny}^* and u_{nm}^* are the yield and maximum displacements at the center of the applied forces, respectively. The left hand side of Equation 5 is the energy demand and the right hand side of the equation is the work done by the applied forces or the absorbed energy. The energy balance concept for MDOF systems and the concept of equivalent simple oscillator are illustrated in Figure 2.

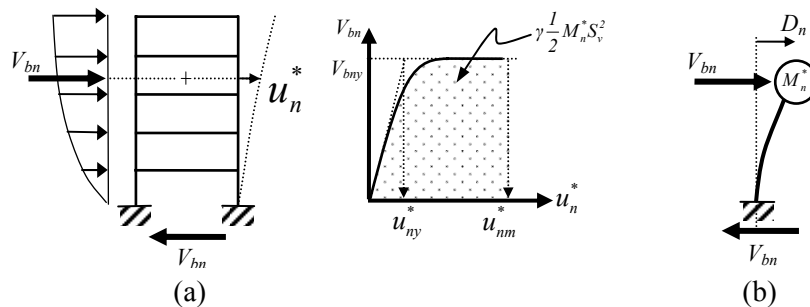


Figure 2. (a) Energy balance concept for MDOF system in the n^{th} mode; (b) Equivalent simple oscillator

3. SEISMIC EVALUATION BASED ON ENERGY BALANCE CONCEPT

The above energy balance concept can be applied in seismic evaluation of structures by evaluating the terms on both sides of Equation 5. The energy capacity plot for the system is first obtained. The conventional push-over curve (generally represented by the base shear versus roof displacement plot) can be generated and then converted into the energy capacity curve. The absorbed energy can be determined from the total absorbed energy due to individual lateral forces or from the sum of the areas under the lateral load-deflection curves at all

floor levels. This energy capacity curve represents the right-hand side of Equation 5.

For the demand side, an energy demand curve can be generated using the term on the left-hand side of Equation 5. As can be seen, the energy demand is a function of the energy factor, γ , which in turn is a function of ductility factor, μ , and the yield strength reduction factor, R_y . In dealing with MDOF systems, the concept of displacement ductility may be somewhat difficult to define. Hence the energy factor and the yield strength reduction factor can be more appropriately written in terms of energy ductility, μ_E , which is defined as the ratio of the maximum absorbed energy to the absorbed energy at yield and is related to the displacement ductility by:

$$\mu_E = \frac{V_y D_m - \frac{1}{2} V_y D_y}{\frac{1}{2} V_y D_y} = 2\mu - 1 \quad (6)$$

The energy factor in terms of the energy ductility is given by substituting Equation 6 into Equation 3:

$$\gamma_E = \frac{\mu_E}{R_y^2} \quad (7)$$

where γ_E is the energy factor in terms of energy ductility. The yield strength reduction factor, R_y , can also be cast in terms of energy ductility. The energy ductility can be approximately taken as the ratio of the total absorbed energy to the absorbed energy when system yielding occurs. This can be determined based on the ratio of the total absorbed energy to the absorbed energy at the yield point obtained from a bi-linear approximation of the pushover curve of the system.

The intersecting point of the energy demand and capacity curves determines the expected level of the response of the system. The response parameters of interest, corresponding to the displacement level obtained from the intersection of the energy demand and capacity curves can then be extracted. The proposed energy balance concept can be applied in the first or multiple mode analysis. However, combining independent modal responses is a complex problem. Future research is needed to explore the applicability of the energy-based multiple mode analysis. The concept of energy-based demand-capacity diagram is illustrated in Figure 3. It should be noted that the roof displacement used in the push-over curve serves only as a tool for visualization purposes. The estimate of the response does not depend on the choice of the location of the reference displacement. It is also noteworthy that there is no need to convert the structure into an equivalent SDOF system even though the concept of equivalent simple oscillator plays an important role in the underlying process. The response can be computed directly from the MDOF model.

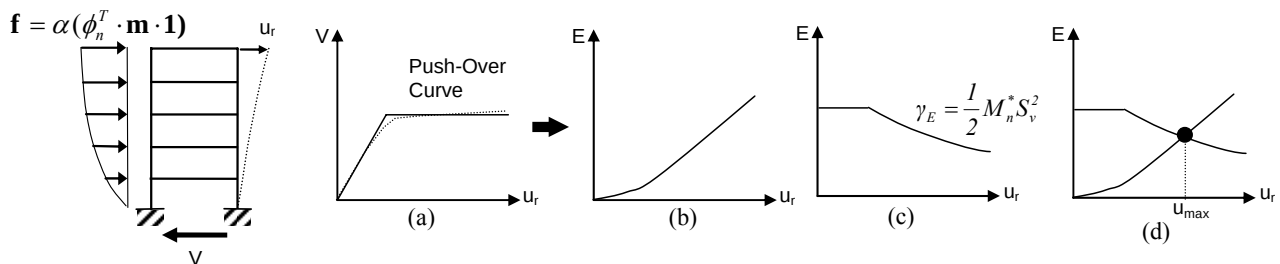


Figure 3. Proposed Energy-Based Evaluation Method: (a) Push-Over Curve, (b) Energy-Displacement Capacity Diagram, (c) Energy Demand Diagram, and (d) Determination of Displacement Demand.

4. SDOF SYSTEM EXAMPLES

In order to validate the proposed energy-based evaluation method, the procedure was implemented to determine maximum inelastic displacements of six elastic-plastic SDOF systems. These systems were first used in another study by Chopra and Goel (1999). In their study, Chopra and Goel used a capacity-demand diagram method that

was based on Newmark and Hall's inelastic design spectrum to obtain the maximum displacement of those systems. The procedure used by Chopra and Goel was an improved analysis procedure over the conventional Capacity Spectrum Method to determine peak deformation of inelastic systems. The properties of the elastic-plastic systems are summarized in Table 1. The peak displacements as determined by Chopra and Goel along with the peak displacements obtained by the proposed method discussed earlier are also shown in Table 1. For brevity, the resulting energy demand-capacity diagram of only one of the systems (System 6) is shown here in Figure 4.

Table 1. Properties of Six Elastoplastic Systems Used to Evaluate the Proposed Method.

System	T (s)	V_e/m (g)	V_y/m (g)	D_y (cm)	R_y (V_e/V_y)	Peak Response			
						Chopra & Goel		This Study	
						μ	D_m (cm)	μ_E	D_m (cm)
1	0.5	2.71	0.60	3.72	4.51	5.99	22.29	11.01	22.27
2	0.5	2.71	0.90	5.58	3.01	3.99	22.29	7.00	22.31
3	0.5	2.71	1.56	9.70	1.73	2.00	19.39	3.02	19.42
4	1	1.80	0.30	7.44	6.00	6.00	44.64	11.02	44.67
5	1	1.80	0.45	11.16	4.00	4.00	44.64	7.01	44.68
6	1	1.80	0.90	22.32	2.00	2.00	44.64	3.01	44.67

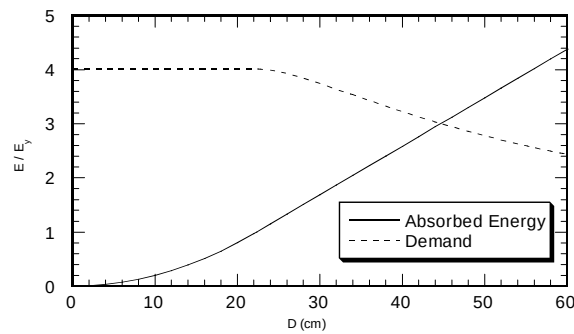


Figure 4. Energy Capacity-Demand Diagram of Example System 6.

As can be seen, the results obtained by the two methods are essentially identical. This is because the same inelastic response spectrum was used in both methods. Hence, for SDOF systems, it can be concluded that the proposed method gives identical results to those obtained from a well-established procedure based on inelastic design spectrum.

5. MDOF SYSTEM EXAMPLES

The procedure presented herein was applied to determine the response of example three-story and nine-story steel frames shown in Figure 5. These structures have been used in various studies and detailed information about these structures can be found elsewhere (Hernández-Montes et al. 2004, Goel and Chopra 2001).

For this study, the buildings were modeled using the centerline dimensions without considering the panel zone deformations. All members were assumed to have elastic-plastic behavior with 2% strain hardening. Damping of 5% critical was assigned for the first and second modes. Gravity load and P-delta effects were not included in the analysis for the three-story frame. For the nine-story frame, the P-delta effect was considered using dummy column representing the gravity columns in the building. The analyses were carried out using SAP2000NL (Computer and Structures 2000).

The example frames were analyzed to determine the story displacement and drift demands for the 1940 NS El Centro ground motion scaled in such a way that the roof displacements were equal to 1% and 2% in nonlinear dynamic analysis. This results in ground motion multipliers (GMMs) of 0.88 (1% drift) and 2.19 (2% drift) for

the three-story frame and GMMs of 1.15 (1% drift) and 3.08 (1% drift) for the nine-story frame.

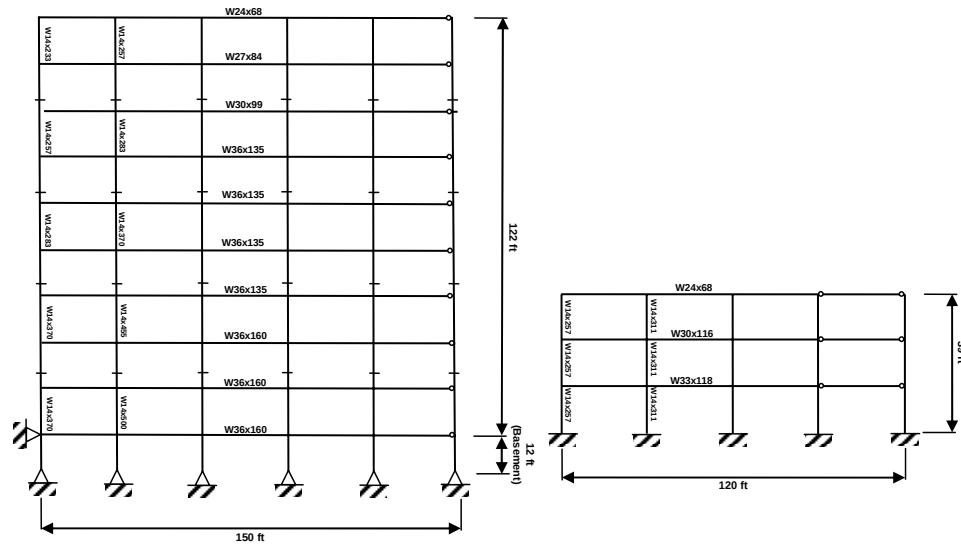


Figure 5. Nine-story and three-story example frames

For each frame, the capacity curve was first obtained by calculating the work done by the applied forces in the pushover analysis. Lateral forces were applied in proportion to the first mode shape. The absorbed energy was calculated by numerically integrating the lateral load-deflection curves at the floor levels. The demand curve was obtained based on the effective modal mass of the first mode and the pseudo-velocity value determined from the 5% damped response spectrum of the El Centro ground motion with corresponding ground motion multiplier. The energy factor, γ_E , was determined based on Newmark and Hall's R_y - μ - T relationship. The energy ductility used to evaluate the energy factor, γ_E , was based on the ratio of the total absorbed energy to the absorbed energy at system yield point determined from bi-linear approximation of the pushover curve.

The energy capacity and demand curves normalized by the absorbed energy at yield of the two frames are shown in Figure 6. Using the intersection points of the demand and capacity curves as the maximum displacement values, the story displacement and drift demands could be determined.

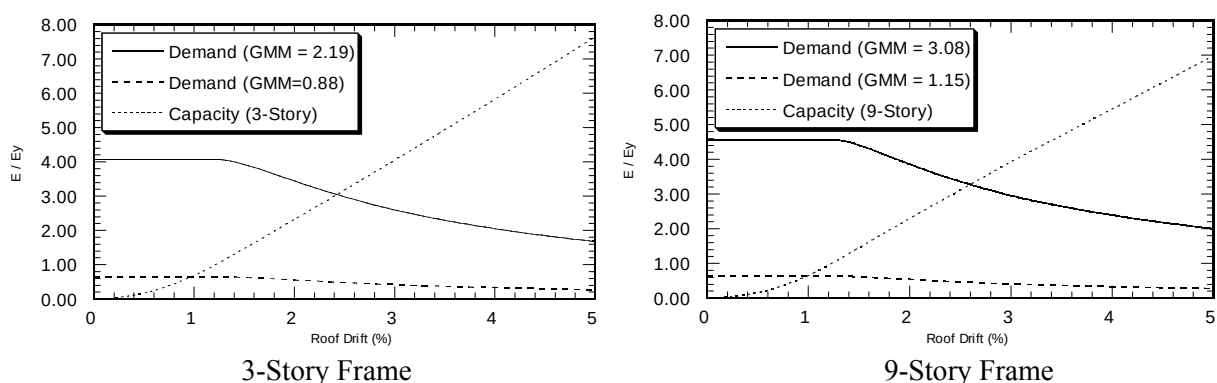


Figure 6. Energy demand and capacity curves of the three- and nine-story example frames

The accuracy of the method was assessed by comparing the results with those from the NDP and the results from other well-established NSPs including the modal pushover analysis (MPA) by Goel and Chopra (2001) and the displacement coefficient method according to FEMA (2000). For the MPA method, the first three modes were employed and, for each mode, modal pushover, in conjunction with dynamic analysis, was performed to determine the modal displacement demands. The displacements obtained from modal response of the first three modes were then combined. For the FEMA method, pushover analysis was performed for each

frame using the target displacement specified by FEMA.

Figure 7 shows floor displacement and story drift demand estimates from different analysis procedures for the three- and nine-story example frames. It can be seen that, for the 1% roof drift level (GMMs of 0.88 and 1.15), where the response remained essentially in the elastic range, all of the NSPs provided very close estimates of the response for both frames. For the 2% roof drift level (GMMs of 2.19 and 3.08) where inelastic response became significant, the floor displacement and story drift estimates were less accurate but were within acceptable limit considering the approximate nature of NSPs. As expected, the story drift estimates were less accurate than the floor displacement estimates for most the cases especially for the nine-story frame. For the case of the nine-story frame, the MPA provided closer drift results in the upper stories where the influence of the higher modes may be significant.

Overall, the proposed energy method provides results that are comparable to those from well-established procedures. It should be noted that in this example, the proposed energy method was carried out using Newmark and Hall's R_y - μ - T Relationship which was developed to represent response for a wide range of ground motions. The accuracy of the proposed energy method could be improved by using a more specific R_y - μ - T relationship developed for the ground motion under consideration.

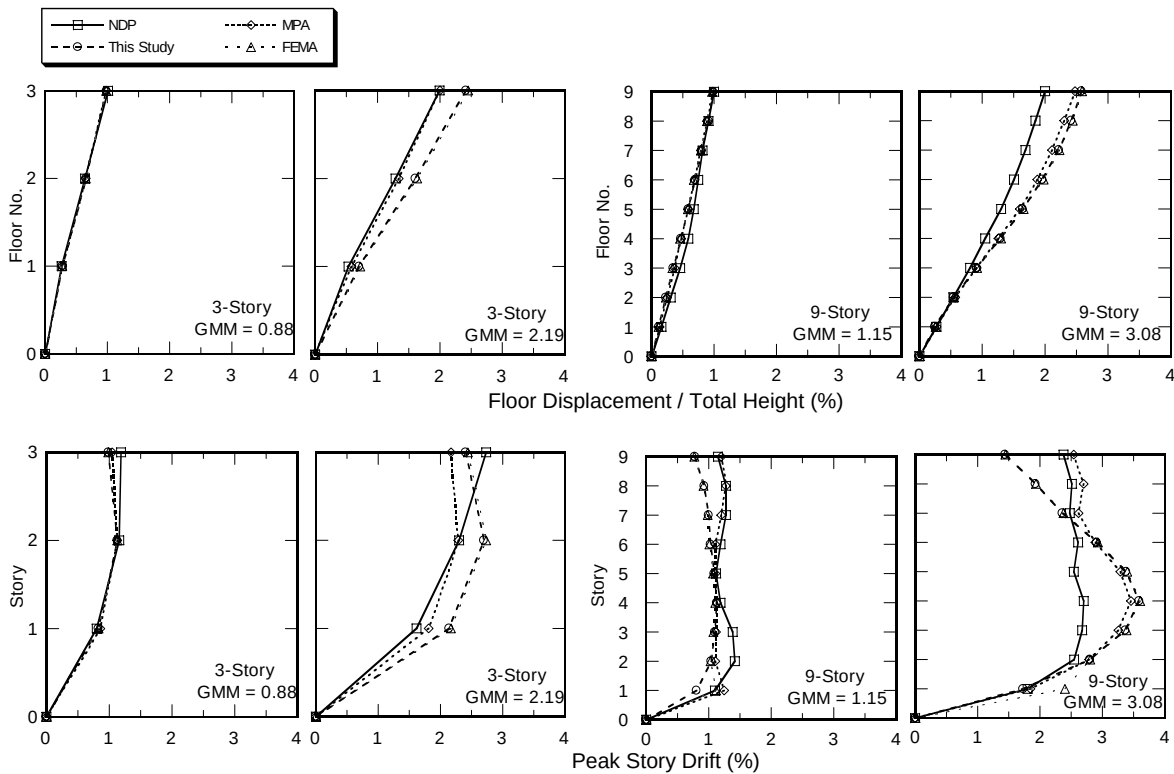


Figure 7. Floor displacements and story drifts from different analysis procedures

7. SUMMARY AND CONCLUSIONS

In this study, an application of the energy balance concept in seismic evaluation of structures is presented. The main findings of this study are:

1. Energy balance concept can be extended to MDOF systems and can be used in seismic evaluation of structures. The seismic demand curve is first established for varying displacement levels. The capacity curve of the structure is then presented in terms of the energy absorbed by the structure at the corresponding displacement levels. The target displacement is determined based on the intersection of energy demand and

capacity curves. The main feature of the proposed method is that the demand and capacity curves are produced in terms of energy. Hence, it avoids the arbitrary use of the base shear versus roof displacement representation of the capacity curve. The response can also be computed directly from the MDOF, even though the concept of equivalent simple oscillator plays an important role in the underlying process.

2. The accuracy of the proposed method was investigated for SDOF and MDOF systems. The results are compared with those from different analysis procedures. For these example systems and ground motions, the proposed method provided results that are accurate and comparable with established procedures.

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