RESEARCHES OF STANDIND WAVES IN STRUCTURES AS THE BASE OF THEIR PHYSICAL STATE DIAGNOSTIC

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ABSTRACT:

The paper presents a new approach of detailed seismological investigation of engineering structure by the method of standing waves using low-channel thin-route instruments. Conversion of standing waves from a point to another, which is possible to a predetermined accuracy, allows obtaining simultaneous records of standing waves on the basis of microseisms recorded at different times at each point of an array of any density.

KEYWORDS: standing waves, seismic resistant, engineering seismology

Engineering structures are closed volumes in constructions, and seismic vibrations there induce standing waves forming. As early as 19th century, the well-known theorem on independent solution on standing waves from initial conditions was proved. It means that standing waves are independent from a source of oscillations.
Being guided by this theorem, we can say that detail and accuracy of research of standing waves in engineering structures must be equal whether we research microseismic oscillations or use high-stable man-made sources of oscillations.

A method of microseism observations in an engineering structure is proved in the paper, there is originated a mathematic model of wave field, algorithms, which allow to select standing waves in pure form and transform them from nonsimultaneous observations to simultaneous records of standing waves at dense system of observation in an engineering structure. The paper analyses possibilities of qualitative interpretation of detail information on standing waves for physical state diagnostic and seismic stability determination, there are also touched on problems of preparation of methods of detail observations quantitative interpretation.
Oscillations for each normal mode is described by Eqn.1 [Pein, 1979]

$$\Psi(x,y,z,t) = A(x,y,z)\cos(w_m t + j_{x,y,z}), \qquad (1)$$

where $A(x,y,z)$ is the geometrical form of a standing wave, $j_{x,y,z}$ - is wave phase at a given point for a specific normal mode, w_m is proper frequency.

The geometrical form of a standing wave, as well as the phase and the frequency of proper oscillations, depend on the structure of a building and on the properties of its material. High-resolution measurements of oscillations at each normal mode inside an engineering structure allows to investigate its construction and detect its defects and weak sites. Such investigations can be referred to seismic crack detection of engineering structures.

The standard way to assess seismic stability of engineering constructions is based on estimation of oscillations amplification in the building at frequencies of the highest-intensity normal modes [Medvedev, 1962]. Detailed investigation of standing waves in an engineering construction is not only detection of defects in the construction, but a new level of accuracy and detail in seismic stability assessment.

Important **property of coherence in time in standing waves** is followed from the above mentioned theorem. Linear connection between oscillations record of two points does not depend on time. This connection is correct both for amplitudes and phases of standing waves. The property of coherence in time will be used for creation of method of detailed research of standing waves characteristics in engineering structures.

There are two key problems in the detailed investigation of standing waves. If we put a seismometer in a building, we can record a sum of standing and traveling waves. *The first question is: how to pick out standing waves from the recording oscillations?* Detailed investigation of standing waves in an engineering structure demands dense systems of observation. Testing large would require simultaneous recording at hundreds or thousands of points. The second question: how to carry out investigation of standing waves in an engineering structure at dense system of observation using thin-route instruments?

An ideal solution would be to deploy a dense system of simultaneous recording of oscillations for detailed investigation of a building using microseisms, but it is impossible. We consider another system of observations and it's possibilities. Simultaneous recording of oscillations in a building being under effect of microseisms is carried out in a reference point and in an i-point (or a group of points); then the i-point (or the group of points) is moved, and recording of seismic oscillations is carried out again simultaneously with recording at the reference point. Thus observations can cover an investigated object using thin-route equipment.

The main purpose is to convert observations at different points taken at different times into a simultaneous record of standing waves for the entire system of observation. A mathematical model of oscillations must be used for solving this problem. Oscillations at each point of the structure are presented as a sum of two components. First component is standing waves, second one is traveling waves.

We put the following suppositions as a base of the model of seismic oscillations of engineering structures:
1. At influence of microseismic oscillations on the building (engineering structure), differences of standing waves, recorded in two points, are described by a linear system, characteristic of which doesn't depend on time.
2. At influence of microseismic oscillations on the building, differences of traveling waves, recorded in two points, can not be described by a linear system, characteristic of which doesn't depend on time.
3. Linear connections in changes of standing waves, recorded in two points, exist for each pair of analogous components of recording oscillations.
4.

The third supposition simplifies the purpose. Instead of vector signal with nine connection characteristics, we use record data on each component of oscillations, independently of one another; three connection characteristics will be enough.

The model of connection of oscillations, recorded in two different points of the engineering structure (building) is described ин Eqn. 2

$$\overline{F}_0(t) = F_0(t) + W_0(t), \quad \overline{F}_i(t) = F_0(t) * h_{0i}(t) + W_i(t), \qquad (2)$$

where $h_{0i}(t)$ is impulse response of the linear system describing connection between simultaneous records of standing waves in 0 and i points at the investigated object, $W(f)$ are traveling waves, $F_0(t)$ is the record of a standing wave in the reference point.

The approach to engineering-seismological data processing, presented in the paper, is development of the previous papers [Emanov, 1995, Emanov and other, 2002].

For a model of oscillation process in the building, it is possible to get simultaneous records of standing waves from nonsimultaneous, sequential observations with a reference point. The processing procedure in this case consists of the following operations.

1. To determine frequency characteristic of the linear systems $h_{0i}(w)$.

2. To record or form an independent realization of oscillation process of the reference point at seismic influence on the investigated object.

3. To convert standing waves from the reference point to all points of the investigated structure, using $h_{0i}(w)$.

The key issue of this scheme is determination of $h_{0i}(w)$ with necessary precision according to microseismic oscillations records. Considering the model of signals recorded at two points on the same components, we can see that determination of $h_{0i}(w)$ is reduced to determination of the linear system characteristic through input and output signals recorded against the background of noise. In fact, it is necessary to determine a filter characteristic to provide the best conversion of standing waves from the reference to the i-point. Therefore, we search $h_{0i}(w)$ as an optimum Wiener filter [Bendat and Pirsol, 1971] that converts the signal $\overline{F_0}(t) = F_0(t) + W_0(t)$ into a record of a standing wave at i-point. For this, the mathematical expectation of squared difference between the filtered signal $\overline{F_0(t)}$ and $F_i(t)$ signal, which is the output of the linear system [1], is minimized with respect to $h_{0i}(w)$*:*

$$\mathrm{M}\left|F_i(t) - \overline{F_0}(t) * h_{0i}'(t)\right|^2 = \min \tag{3}$$

Equating $h_{0i}'(t)$ to zero, we obtain

$$\mathrm{M}\left[F_i(t)\overline{F_0}(t-q)\right] - h_{0i}'(t) * \mathrm{M}\left[\overline{F_0}(t-t)\overline{F_0}(t-q)\right] = 0$$

Taking into account that $\mathrm{M}\left[F_i(t)\overline{F_0}(t-q)\right] = R_{0i}(q)$ is a cross-correlation function of signals F_t *(t)* and *F_0(t)*, and $\mathrm{M}\left[\overline{F_0}(t-t)\overline{F_0}(t-q)\right] = R_{00}(t-q)$ is the autocorrelation function of oscillations recorded at the reference point, we obtain

$$R_{0i}(q) - h_{0i}'(t) * R_{00}(t-q) = 0 \tag{4}$$

We make Fourier transform, pass on to frequency domain and obtain

$$h_{0i}'(w) = \frac{R_{0i}(w)}{R_{00}(w)} = \frac{\mathrm{M}\left|F_i(w)\overline{F_0}^*(w)\right|}{\mathrm{M}\left|\overline{F_0}(w)\right|^2} \tag{5}$$

The denominator of Eqn. 5 contains an averaged square spectrum modulus of oscillations at the reference point. We can calculate it finding a way of averaging. The numerator of (5) contains averaged cross spectrum of oscillations at the reference point and the standing wave in the i- point. $F_i(t)$ cannot be measured experimentally. Assuming that $W_0(t)$ $W_i(t)$ are stationary stochastic processes that do not correlate with each other and with the records of the standing waves in the reference and i-points, it is easy to prove that [1]

$$\mathrm{M}\left|F_i(w)\overline{F_0}^*(w)\right| = \mathrm{M}\left[F_i(w)F_0^*(w)\right] = \mathrm{M}\left|\overline{F_i}(w)\overline{F_0}^*(w)\right| \tag{6}$$

Eqn. (6) allows us to substitute into the nominator of Eqn. (5) the averaged cross spectrum of oscillations recorded simultaneously in the reference and i-points.

Averaging of spectrums can be carried out in this case, if we divide oscillations, recording simultaneously, to n blocks. Dividing the record to non-overlapping blocks, we get realization set, which can be used for averaging. Formula for Wiener filter, converting oscillations from te reference point to the i-point, will be the following

$$\boldsymbol{h}_{0i}'(w) = \frac{\sum_{j=1}^{n} \overline{F_i}(w)\overline{F_0}^{*}(w)}{\sum_{j=1}^{n} \left|\overline{F_0}(w)\right|^2} \tag{7}$$

Differences in the frequency characteristic of Wiener filter from the characteristic of the linear system (which describes differences in the standing waves at two points of the investigated object) are described by the bias of estimation, which is determined by the deviation of the mathematical expectation of the determined frequency characteristic from its true value.

$$\mathrm{M}\left[\boldsymbol{h}_{0i}'(w)\right] = \frac{\boldsymbol{h}_{0i}(w)}{1 + \left|\boldsymbol{W}_0(w)\right|^2 / \left|\boldsymbol{F}_0(w)\right|^2} \tag{8}$$

According to Eqn. (8) frequency characteristic of the filter is the biased estimation of the linear filter characteristic, which describes connections of oscillations at any two points of the investigated object. The bias of estimation is expressed in regularizing of the filter in ratio noise/signal. Estimation of faze characteristic of the filter is not biased.

Eqn. (7) allows to calculate the filter characteristic with error, which depends on the following parameters of observations: sampling interval - Δt, unit block length – T, number of blocks in a record – *n*. Selection of first two parameters does not cause difficulties. Sampling interval correlates with frequency range which studies response of the structure to seismic effects. Unit block length is related to spectral resolution $\Delta f = 1/T$. A question on block quantity is more complicated. Calculation according to Eqn. (7) is an evaluation of the filter characteristic, its error depends on quantity of blocks *n* and ratio of noises energy and useful signals in the model (2).

Spectrum of coherence $g(w)$ will be useful for evaluation of block quantity, which is necessary for the given error of the filter. This spectrum can be calculated from records of microseismic oscillations in two points of the investigated object according to Eqn. (9)

$$g^2(w) = \frac{\left|\boldsymbol{R}_{0i}(w)\right|^2}{\boldsymbol{R}_{00}(w)\,\boldsymbol{R}_{ii}(w)} \tag{9}$$

where $\boldsymbol{R}_{00}(w)$ is the spectrum of auto-correlation function of oscillations recorded at the reference point, $\boldsymbol{R}_{ii}(w)$ is the same in i-point.

Applying model (2), we obtain

$$\boldsymbol{R}_{0i}(w) = \boldsymbol{h}_{0i}(w)\mathrm{M}\left|\boldsymbol{F}_0(w)\right|^2,\ \boldsymbol{R}_{00}(w) = \mathrm{M}\left|\boldsymbol{F}_0(w)\right|^2 + \mathrm{M}\left|\boldsymbol{W}_0(w)\right|^2,$$

$$\boldsymbol{R}_{ii}(w) = \left|\boldsymbol{h}_{0i}(w)\right|^2 \mathrm{M}\left|\boldsymbol{F}_0(w)\right|^2 + \mathrm{M}\left|\boldsymbol{W}_i(w)\right|^2.$$

These equations are obtained under an assumption of independence of input and output noise in the linear system and its non-correlation with useful signal. Substitution of the obtained equations into Eqn. (9) gives the following

$$g^{2}(w)=\frac{1}{(1+a_{0}(w))(1+a_{i}(w))} \qquad (10)$$

where $a_{0}(w)=\frac{\mathrm{M}\left|W_{0}(w)\right|^{2}}{\mathrm{M}\left|F_{0}(w)\right|^{2}}$, $a_{i}(w)=\frac{\mathrm{M}\left|W_{i}(w)\right|^{2}}{\mathrm{M}\left|F_{i}(w)\right|^{2}}$ are ratios of weighted average square of noise/useful signal in the reference and i-points.

It follows from Eqn. (10) that spectrum of coherence depends on ratio of traveling waves energy to standing waves energy only. Spectrum of coherence can be calculated by simultaneous records in any pair of points of the investigated object. The $g(w)$ values lie in the range of 0 to 1. When $g(w)$ =1, only standing waves are registered in two point. When $g(w)$ =0, there are only traveling waves can be registered in two points of the building.

The experiments proved [Emonov and others, 2002] that the values of coherence spectrum at proper frequencies of engineering constructions are high: 0.8-0.99, and in the space between them they correspond to values of 0.1-0.3.

The obtained algorithm of the filters characteristics calculation is symbiosis of Wiener filter, property of coherence of standing waves and methods of mathematical statistics. Mathematical statistics makes it possible both to create filters for determination of coherent in time standing waves and to determine accuracy of the obtained wave field by dispersion of characteristics evaluation. The detailed description of the algorithms of accuracy estimation can be found in the paper /4/. The mean-square error of phase response of Wiener filter for conversion of standing waves from the reference point into i- point will be equal to

$$s_{q}(w)\approx\frac{\sqrt{1-g_{oi}^{2}(w)}}{\left|g_{oi}(w)\right|\sqrt{2n}}.$$

This formula makes it possible to calculate an error of phase response depending on frequency in the radians. The relative error of amplitude characteristic of the filter is equal to

$$e\left[\left|h_{0i}'(w)\right|\right]=\frac{s\left[\left|h'_{oi}(w)\right|\right]}{\left|h_{oi}(w)\right|}\approx\frac{\left[1-g_{0i}^{2}(w)\right]^{1/2}}{\left|g_{0i}(w)\right|\sqrt{2n}}$$

where $s\left[\left|h'_{oi}(w)\right|\right]$ is mean-square distance of evaluation of frequency characteristic of the filter. A quantity of blocks determines the length of realization, recorded at each point of the investigated object, and it substantially influences on productivity of works. Optimum productivity is achieved by sequential record of oscillations at the points of the object at length of the realization of 5-10 minutes and it is substantially reduced at the lengths of the realization measured by hours. At high coherence of oscillations - 0.8-0.99, observed at engineering constructions at normal mode frequencies, it is possible to obtain an error of 5% and less for conversion of standing waves, productivity of investigation can be reached to several days. Such object as Sayan-Shushenskaya hydroelectric power station can be observed in two weeks (sampling interval is 5-10m).

An example of the standing waves field is given in Figure 1. There are given the nonsimultaneous observations in a gallery of Sayan-Shushenskaya hydroelectric power station (Figure 1a), traces of oscillations, converted to the same points of the system of observation (Figure 1b), the result of two-dimensional filtration of the converted traces (Figure 1c). Instead of irregular oscillations we obtained the regular wave field, where reflections from boards of the dam are clearly visible.

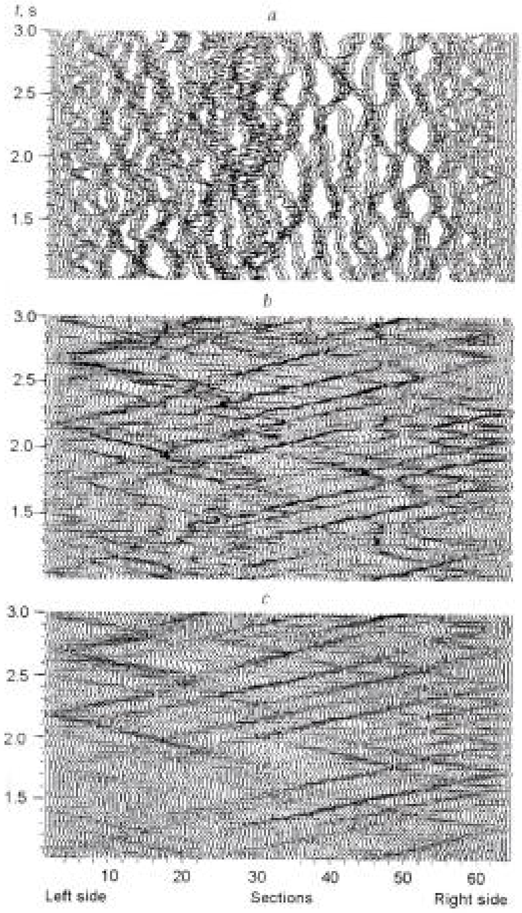

Figure 1 Example of standing waves transformation
from the reference point to other points of the system of observation

Detailed investigation can be started according to these records. The first question of the investigation is determination and research of normal modes of an engineering structure. There is given a section of traces spectrums along a gallery of the dam of Sayan-Shushenskaya hydroelectric power station (radial oscillations) in Figure 2. Amplitude modulation, obtained from one trace to the next by spectrums, makes it possible to immediately determine multiplicity of the detected mode. For such objects as dams of hydroelectric power stations the proposed method allows to reveal and to study the special features of amplitude distribution and phases of standing waves in the space at several tens of normal modes.

The determined standing waves are easy filtered, maps of amplitudes and oscillation phases are constructed by each of the normal modes in the investigated object. The special features of each of the standing waves are studied by them. When investigating buildings, basic information is contained in geometric shape defect of standing wave, caused by the physical state of the constructions. Analysis of defects makes it possible to carry out diagnostics of buildings physical state and to reveal the elements of the most probable destruction under seismic influences.

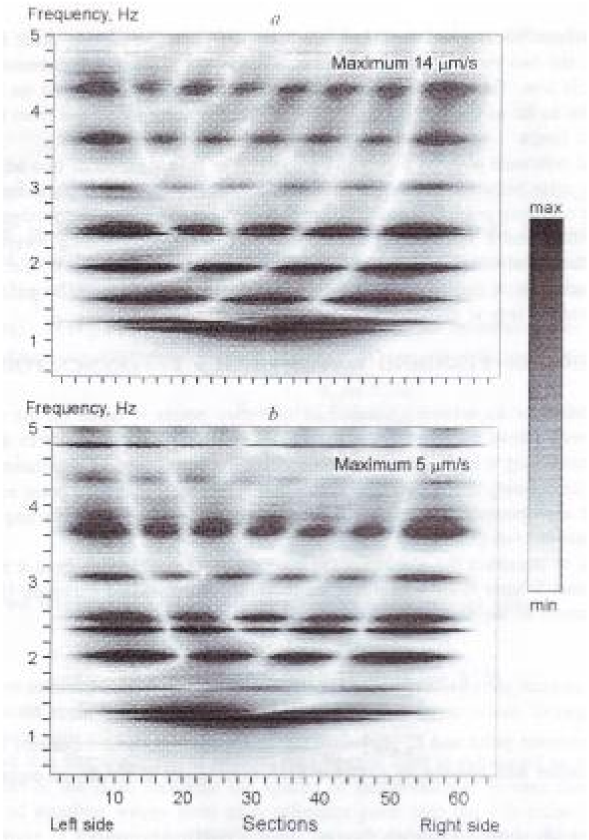

Figure 2 The maps of amplitude spectrums of radial velocity of oscillations displacement to a gallery of Sayan-Shushenskaya hydroelectric power station

CONCLUSION

- There are developed the algorithms of obtaining of simultaneous records of standing waves in engineering constructions by nonsimultaneous measurements of microseismic oscillations with records of reference oscillations at one or several points, and also the algorithm of accuracy estimation. The basis of these algorithms is the methods of Wiener filtration, the coherence property of standing waves in time and the methods of statistical evaluation of characteristics.
- There are experimentally proved high accuracy and detail of standing waves obtaining in engineering constructions both by three-dimensional measurements and according to the oscillation frequency scale.
- The developed method of detailed investigation of standing waves in engineering constructions is the basis of diagnostics of buildings and constructions physical states and research of their seismic stability at the level of the construction elements.

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