STATIC EQUIVALENT METHOD FOR THE KINEMATIC INTERACTION ANALYSIS OF PILE FOUNDATIONS

F. Dezi[1] S. Carbonari[2] and G. Leoni[3]

[1] *PhD, Dept. of Materials,Environment Engineering and Physics, Università Politecnica delle Marche, Italy*
[2] *Dr, Dept. of Architecture, Constructions, Structures, Università Politecnica delle Marche, Italy*
[3] *Professor, Dept. ProCAm, Università di Camerino, Ascoli Piceno, Italy*
Email: francesca.dezi@tin.it

ABSTRACT :

This work presents a simplified procedure for the evaluation of the kinematic stress resultants in single piles subjected to earthquake soil displacements. The procedure is applicable to single piles embedded in homogeneous as well as horizontally layered soil profiles overlaying a rigid formation. By neglecting radiation damping the evaluation of the effects induced in the piles by the propagation of seismic waves in the soil is studied by means of a static equivalent procedure, without the use of dynamic analyses. Piles are modelled as elastic beams on Winkler restraints and are pushed by the earthquake soil displacements provided by a response spectrum analysis of the soil profile oscillating. The soil is assumed to be homogeneous with constant shear modulus and density and to behave elastically. A validation of the method is carried out comparing results with those furnished by more sophisticated dynamic soil-pile interaction analyses.

KEYWORDS: Kinematic interaction, pile foundations, simplified method, soil-structure interaction

1. INTRODUCTION

Pile failures, as consequence of seismic events, have been observed after earthquake. When liquefaction problems may be excluded, the damages may be generally associated to two main causes: the first is related to the inertial interaction between the soil-foundation system and the superstructure while the second is due to the kinematic interaction between the soil and the foundation system and concerns the actions induced in the piles by the propagation of the seismic waves in the soil. Damages induced by kinematic interaction localize mainly at the interface between layers with high impedance contrast. Modern seismic codes have acknowledged these aspects and prescribe the kinematic interaction effects to be taken into account in the foundation design process. Many authors during the years have proposed simplified procedures and analytical solutions for the evaluation of the kinematic bending moments along the piles and at the interface between layers characterized by different shear moduli. By assuming that long piles follow the deformation of the soil, in 1975 Margason suggested to compute kinematic pile bending moments starting from the evaluation of the free-field soil curvatures by means of a finite difference approach and without accounting for soil-pile interaction and radiation damping. In 1983 Dobry & O'Rourke presented a simple model for the evaluation of the kinematic bending moment at the interface between two soil layers by modelling the pile as a beam resting on Winkler foundation. Assuming simplified strains distribution within the layers and postulating that the pile is long with respect to the thickness of the two layers, they developed a solution for the pile bending moment at the interface as function of the Winkler parameter, the soil strain at the interface and the ratio between the shear moduli of the two layers. The procedure proposed by Dobry & O'Rourke allows accounting for soil-pile interaction but requires the computation of a free-field one-dimensional site analysis for the evaluation of the soil strain at the interface. In 2005 Dente proposed an alternative expression for the evaluation of the shear strain at the interface as function of the maximum acceleration at the free-field ground surface and the dynamic properties of the surface layer. Nikolaou et al. (1995-1997) have performed a parametric investigation on the bending strains in a pile embedded in a two layered soil deposit subjected to harmonic steady-state shear waves. By assuming the pile length to be greater than the active pile length (Randolph, 1981; Velez et al., 1983) they derived a closed-form expression for the evaluation of the maximum bending moment at the interface. Furthermore Nikolaou et al.

(2001) have performed a series of analyses using real accelerograms and real soil profiles in order to find a correlation between the steady-state and a real transitory response.
A simplified procedure for the evaluation of stress resultants in single piles subjected to earthquake soil displacements has been developed in this work. The procedure allows obtaining not only the peak bending moment at the interface between soil layers with impedance contrast but also the complete distribution of the stress resultants along the pile. The evaluation of the effects induced in the piles by the propagation of seismic waves in the soil is studied by means of a static equivalent procedure can be easy implemented in commercial finite element computer codes for structural analysis or simple spreadsheets.

2. PROPOSED METHOD

A single pile embedded in a layered soil is considered. Under the simplifying assumption that the soil deposit motion due to the seismic excitation is not influenced by the presence of the foundation, the pile stress resultants due to kinematic interaction are evaluated by assuming the pile as an elastic beam resting on a Winkler foundation subjected to the earthquake soil displacements evaluated by means of a dynamic response spectrum analysis of the soil deposit.

2.1. Free-field displacement profile

In this section a procedure for the evaluation of the free-field displacement profile is presented with reference to a generic soil deposit overlying a rigid bedrock and constituted by n homogeneous horizontal layers of thickness h_j (Figure 1). The soil is assumed to behave as a linear elastic shear deformable column with constant elastic modulus and density within each layer and the bedrock is subjected to a seismic motion. By considering a reference system frame $\{0; z_j\}$ for each layer as in Figure 1 the equation of motion for the column is

$$\rho_j \ddot{u}_s(z_j,t) - G_j u_s''(z_j,t) = -\rho_j \ddot{u}_b(t) \qquad \text{for } j = 1, \ldots n \qquad (2.1)$$

with the relevant boundary and continuity conditions

$$u_s'(z_1,t) = 0 \qquad \text{for } z_1 = 0 \qquad (2.2a)$$
$$u_s(z_j,t) = u_s(z_{j+1},t) \qquad \text{for } j = 1, \ldots n\text{-}1,\ z_j = h_j \text{ and } z_{j+1} = 0 \qquad (2.2b)$$
$$G_j u_s'(z_j,t) = G_{j+1} u_s'(z_{j+1},t) \qquad \text{for } j = 1, \ldots n\text{-}1,\ z_j = h_j \text{ and } z_{j+1} = 0 \qquad (2.2c)$$
$$u_s(z_n,t) = 0 \qquad \text{for } z_n = h_n \qquad (2.2d)$$

where G_j and ρ_j are the elastic modulus and density of the j-th soil layer, respectively. In Eqn. 2.1 $u_s(z_j,t)$ denotes the horizontal relative soil displacement while $u_b(t)$ is the soil displacement at the bedrock level. Assuming that the associated eigenvalue problem have been solved, the soil displacement may be expressed by a linear combination of the modes as

$$u_s(z_j,t) = \sum_{r=1}^{\infty} U_r(z_j) q_r(t) \qquad \text{for } j = 1, \ldots n \qquad (2.3)$$

where the r-th term of the series is the contribution of the r-th mode of vibration to the response. Soil displacements expressed by means of Eqn. 2.3 satisfy boundary and continuity conditions (2.2). Substituting Eqn. 2.3 into Eqn. 2.1 yields

$$\sum_{r=1}^{\infty} \rho_j U_r(z_j) \ddot{q}_r(t) + \sum_{r=1}^{\infty} G_j U_r''(z_j) q_r(t) = -\rho_j \ddot{u}_b(t) \qquad \text{for } j = 1, \ldots n \qquad (2.4)$$

Furthermore, by multiplying each term by U_m, integrating over the length of the soil column and by considering the orthogonality properties of the modes, Eqn. 2.4 transforms into

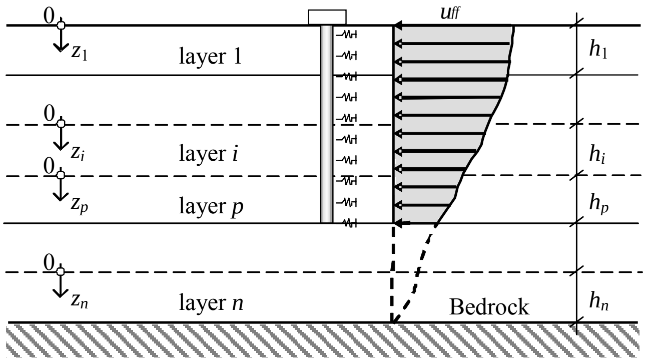

Figure 1. Horizontally layered soil profile overlying a rigid formation and pile subjected to the free-field soil displacements

$$\ddot{q}_m(t) + \omega_m^2 q_m(t) = -\Gamma_m \ddot{u}_b(t) \tag{2.5}$$

with

$$\omega_m^2 = \frac{K_m}{M_m} \qquad \Gamma_m = \frac{\sum_{j=1}^{n} \int_0^{h_j} \rho_j U_m(z_j) dz}{M_m} \tag{2.6a-b}$$

where

$$M_m = \sum_{j=1}^{n} \int_0^{h_j} \rho_j \left[U_m(z_j)\right]^2 dx \qquad K_m = \sum_{j=1}^{n} \int_0^{h_j} G_j U_m(z_j) U''_m(z_j) \tag{2.7a-b}$$

In Eqns. 2.6 ω_m is the natural frequency and Γ_m is the modal participation factor associated to the m-th mode of vibration while M_m and K_m in Eqns. 2.7 are the generalized mass and stiffness for the m-th mode of vibration, respectively. It is worth noticing that Eqn. 2.5 is the well known equation of motion for a SDF system. For classically damped system Eqn. 2.5 transforms into

$$\ddot{q}_m(t) + 2\xi_m \omega_m \dot{q}_m(t) + \omega_m^2 q_m(t) = -\Gamma_m \ddot{u}_b(t) \tag{2.8}$$

where ξ_m is the damping ratio of the m-th mode. Solution of Eqn. 2.8 assumes the form

$$q_m(t) = \Gamma_m D_m(t) \tag{2.9}$$

where D_m(t) is the response of the m-th SDF system. The peack soil displacement for the m-th vibration mode can be directly computed through the earthquake response spectrum by means of the expression

$$u_{ff,m}(z_j) = \Gamma_m \frac{S_a(\omega_m, \xi_m)}{\omega_m^2} U_m(z_j) \tag{2.10}$$

where $u_{ff,m}(z)$ is the maximum free-field displacement and $S_a(\omega_m, \xi_m)$ is the ordinate of the earthquake pseudo-acceleration response spectrum associated to the modal damping ratio ξ_m and corresponding to the vibration frequency ω_m.
The damping ratio ξ_m represents the damping of the soil that is generally dependent on the strain level to which the ground is subjected. A constant value for the modal damping ratio, associated to the m-th mode of the soil profile, may be determined to represent the material hysteretic damping of the whole system. If energy dissipation occurs differently in each layer a weighted value of the damping ratio may be determined by

empirical approaches. For systems where the energy dissipation mainly develops from the hysteretic behaviours of its components it can be assumed that the modal damping ratio is proportional to the strain energy stored in the system. The modal damping ratio may be obtained from the weighted sum of the damping of each soil layer constituting the system with the weight being the normalized elastic energy stored in each layer for a deformed shape corresponding to the *m*-th mode. Modal damping ratio corresponding to the generic *m*-th mode may thus be computed as follows:

$$\xi_m = \frac{\sum_{j=1}^{n} \xi_{m,j} G_j \int_0^{h_j} U_m'^2 (z_j) dz}{\sum_{j=1}^{n} G_j \int_0^{h_j} U_m'^2 (z_j) dz} \tag{2.11}$$

where $\xi_{m,j}$ is the damping ratio of the *j*-th soil layer for the *m*-th mode.
The procedure needs the evaluation of the natural frequencies and modes of the layered soil profile in order to expressed the soil displacement according to Eqn. 2.3. The eigenvalue problem associated to Eqn. 2.1 can be easily solved separating variables *z* and *t* for the definition of the horizontal displacement $u_s(z_j,t)$ as follows:

$$u_s(z_j, t) = U(z_j) e^{i\omega t} \tag{2.12}$$

where $U_j(z)$ is the modal displacement function and ω is the circular natural frequency. The general solution of the problem assumes the form

$$U(z_j) = A_j \cos\left(\frac{\omega}{V_{s\,j}} z_j\right) + B_j \sin\left(\frac{\omega}{V_{s\,j}} z_j\right) \tag{2.13}$$

where

$$V_{s\,j} = \sqrt{\frac{G_j}{\rho_j}} \tag{2.14}$$

is the shear wave velocity within the *j*-th soil layer and A_j and B_j are the integration constants depending on the conditions at the boundaries of each layer. Introducing the boundary conditions together with the continuity conditions at the interface the following system of 2*n* homogeneous equations in the constants A_j and B_j can be obtained

$$\boldsymbol{H}(\omega)\boldsymbol{x} = \boldsymbol{0} \tag{2.15}$$

where $\boldsymbol{H}$ is the matrix of the system coefficients and $\boldsymbol{x}$ is the vector grouping the integration constants A_i and B_i, respectively. The natural frequencies of the system can finally be computed from equation

$$\det[\boldsymbol{H}(\omega)] = 0 \tag{2.16}$$

2.2. Effects of the kinematic interaction for the single mode

The pile is assumed to be a Euler-Bernoulli beam with flexural rigidity *EI* resting on a Winkler-type medium embedded into *p* soil layers with $1 \le p \le n$ as in Figure 1. Coefficients of distributed springs are given by the following expression, according to the formulations proposed by Makris and Gazetas (1992):

$$k_j = 2.4 \rho_j V s_j^2 (1 + \nu_j) \tag{2.17}$$

where ν_j is the Poisson's ratio of the *j*-th soil layer. By considering a constant flexural rigidity for the pile, the equilibrium condition of a generic pile element subjected to the soil displacement may be written in the form

$$EI\frac{d^4u_m(z_j)}{dz_j^4}+k_j\left[u_m(z_j)-u_{ff,m}(z_j)\right]=0 \tag{2.18}$$

where $u_m(z_j)$ is the pile displacement. Solution of Eqn. 2.18 is achieved by standard mathematics. Taking Eqn. 2.10 into account, the procedure allows obtaining the following general expression for the displacement

$$\begin{aligned}u_m(z_j)=&e^{-\alpha_j z}\left(C_{1,j}\cos\alpha_j z_j+C_{2,j}sen\alpha_j z_j\right)+e^{\alpha_j z}\left(C_{3,j}\cos\alpha_j z_j+C_{4,j}sen\alpha_j z_j\right)+\\&\frac{4S_m\alpha_j^4}{\beta_j^4+4\alpha_j^4}\left(A_j\cos\beta_j z_j+B_j sen\beta_j z_j\right)\end{aligned} \tag{2.19}$$

where

$$\alpha_j^4=\frac{k_j}{4EI}, \qquad \beta_j=\frac{\omega_m}{Vs_j} \quad \text{and} \quad S_m=\Gamma_m\frac{S_a(\omega_m,\xi_m)}{\omega_m^2} \tag{2.20 a-b-c}$$

By substituting the boundary conditions at the pile head and at the pile base, and the continuity conditions at the interface between the soil layers a system of 4*p* homogeneous equations in the constants $C_{1,j}$, $C_{2,j}$, $C_{3,j}$, $C_{4,j}$ may be obtained. The solution of the system allows calculating the displacements of the pile sections and consequently the stress resultants at each depth of interest for each mode of the free-field motion.
If the pile is embedded into the bedrock formation the supports of the springs relevant to the pile sections below the bedrock location are subjected to a null prescribed motion. The equilibrium condition for the generic pile element below the bedrock surface is expressed by Eqn. 2.18 where $u_{ff,m}(z_j)$ is made zero and the general solution is that of the homogeneous problem. Alternatively the problem may be solved by considering a fixed restraint for the pile at the bedrock level; this leads to a conservative evaluation of the pile stress resultants that may be accepted for design purposes.

2.3. Final Remarks
Since the response to earthquake is primarily due to the lower modes of vibration only the first few natural frequencies and modal shapes have to be evaluated and applied to the piles to compute the kinematic response with a good level of precision. When higher modes have to be considered the effects need to be combined with the modal Complete Quadratic Combination. It is worth noticing that the evaluation of the free-field displacement profile and the pile response pushed by the free-field soil displacement may both be achieved by means of standard structural analysis finite element codes or simple spreadsheets.

3. VALIDATION OF THE METHOD

A validation of the proposed method of analysis is presented comparing results with those obtained by dynamic soil-pile kinematic interaction analyses. Single piles of two diameters are considered together with two different soil profiles. The model proposed by Dezi et al. (2007) is used for the evaluation of the kinematic interaction of the selected case studies taking into account the soil-pile interaction and the radiation damping. The concrete piles have a Young modulus of $3\cdot10^7$ kPa and a density ρ_p of 2.5 Mg/m³. They are characterized by a length of 24 m and by a circular cross section of diameter *d* equal to 600 and 800 mm.
The seismic action defined at the outcropping bedrock consists of an artificial accelerogram matching the EC8 Type 1 elastic response spectrum for ground type A. For each investigated soil profile the acceleration time history at the bedrock level is obtained by linearly deconvoluting the artificial accelerogram at the bedrock level by means of a one-dimensional local site response analysis. A constant soil Poisson's ratio $\nu = 0.4$ and a constant material hysteretic damping ξ = 10%, compatible with the strain level in the soil, are considered for all

the case studies. The bedrock formation is characterized by a shear wave velocity of $Vs_b = 800$ m/s and a density of $\rho_b = 2.5$ Mg/m³.

3.1 Single layered soil

A soil profile constituted by a single horizontal soil layer overlying a rigid formation is firstly analysed. The dynamic properties of the soil are reported in Figure 2a with the accelerogram at the bedrock level and the corresponding earthquake response spectrum. Only the contribution of the first vibration mode of the soil is considered; the modal response parameters of the soil column and the maximum soil displacement profile, computed as in Eqn. 2.10, are shown in Figure 2b. When subjected to the soil displacement profile, both piles undergo similar displacements that are shown in Figure 2c. Figure 3a-b show comparisons between the results obtained from the proposed method and those derived by the dynamic analyses performed according to the procedure developed by the authors. Results are presented in term of bending moment and shear force along the piles; the maximum absolute values of the stress resultants descending from the dynamic analyses have been mirrored with respect to the pile axis in order to make comparisons with the static case easier. Results obtained from the simplified procedure agree very well with those achieved through the dynamic analyses.

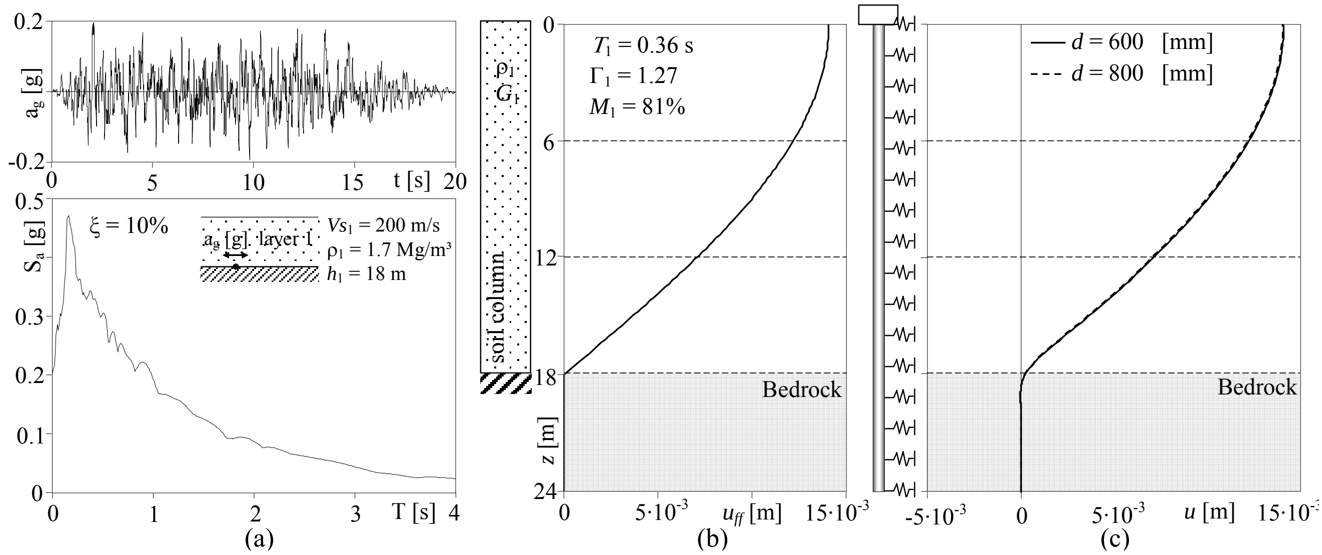

Figure 2. (a) Accelerogram and response spectrum at the bedrock level, (b) free-field maximum displacement profile, (c) pile displacements when subjected to the free-field prescribed motion

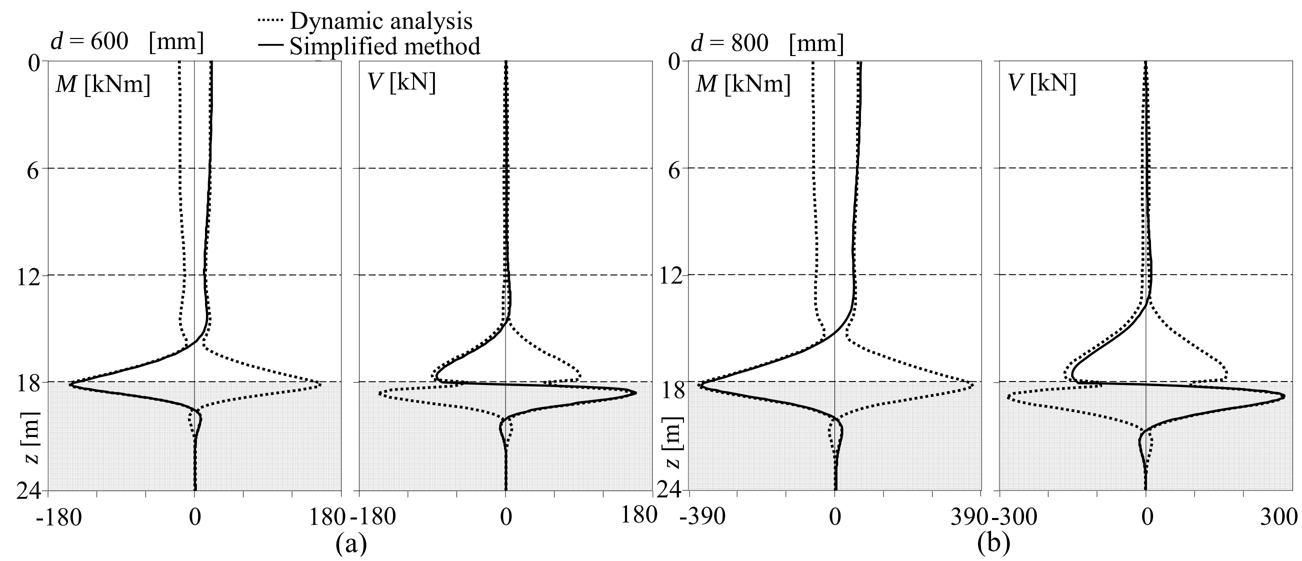

Figure 3. Comparisons between the proposed simplified method and the model proposed by Dezi et al. (2007) in terms of bending moment and shear force: (a) pile of diameter 600 mm and (b) pile of diameter 800 mm

3.2 Multiple layered soil

A soil profile constituted by three horizontal soil layers, having properties as in Figure 4a and overlying a rigid bedrock is also investigated. The accelerogram and the relevant response spectrum at the bedrock level is presented in the same figure. The first three modes have been considered for a total effective modal mass equal to the 86% of the total. The contributions of each mode to the pile response are independently evaluated and then combined. The modal response parameters of the soil column are shown in Table 3.1.

Table 3.1 Modal response parameters

Mode	*T* [s]	Γ	M [%]
1	0.44	1.50	46
2	0.20	0.16	28
3	0.12	0.17	12

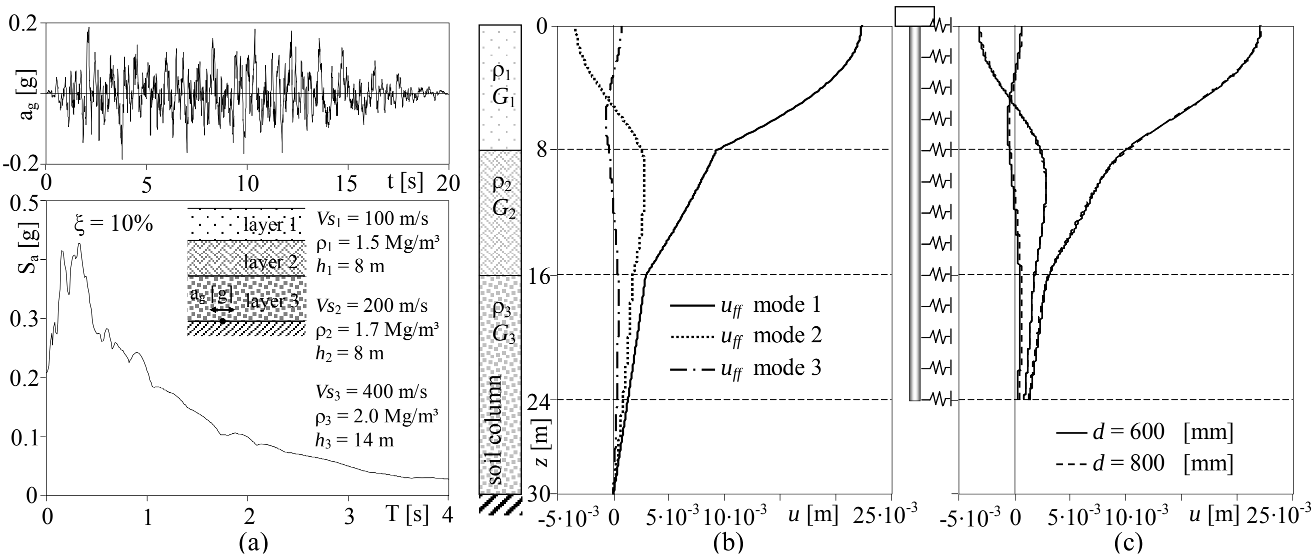

Figure 4. (a) Accelerogram and response spectrum at the bedrock level, (b) free-field maximum displacement profile, (c) pile displacements when subjected to the free-field prescribed motion

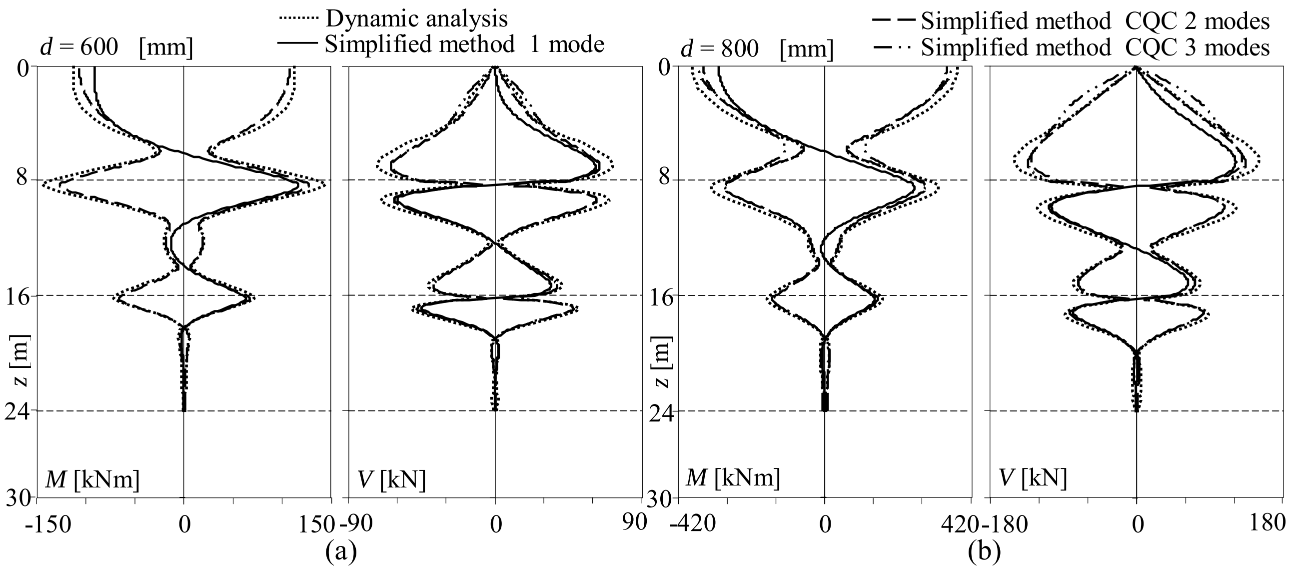

Figure 5. Comparisons between the proposed simplified method and the model proposed by Dezi et al. (2007) in terms of bending moment and shear force: (a) pile of diameter 600 mm and (b) pile of diameter 800 mm

Figure 4b shows the soil displacement obtained from Eqn. 2.10 while in Figure 4c the deformed shapes of the piles are presented. Figure 5a-b shows comparisons between the results obtained by the use of the proposed simplified method and that obtained by the dynamic analyses. The effects induced by the contribution of only the first mode and the modal Complete Quadratic Combinations of the effects produced by the first two and three modes are presented.
As expected, by increasing the number of mode contributing to the response the stress resultants obtained from the dynamic analyses are reproduced closer even if in this case the simplified method underestimate the actual stress resultants obtained from the dynamic soil-pile interaction analyses of about 20%.

4. CONCLUSIONS

In this paper a simplified method for the evaluation of kinematic bending moments in single piles has been presented. The method descends from the assumption that during a seismic excitation the motion of the soil is not influenced by that of the pile. The soil is considered to behave as a shear deformable column and the maximum free-field displacement profile in the soil is computed by means of a response spectrum analysis. The pile is modelled as a Euler-Bernoulli beam resting on a Winkler foundation and is subjected to the free-field soil displacements. The following conclusions may be drawn:

- the potential of the method is demonstrated by comparing results in terms of kinematic stress resultants along the piles with those achieved by complex dynamic soil-pile interaction analysis;
- only the contributions of few modes need to be considered for a good estimation of the kinematic effects in the piles;
- the method furnishes a practical tool for the evaluation of the kinematic bending moments and shear force in piles to structural and geotechnical engineers;
- commercial finite element computer codes for structural analysis or simple spreadsheets may be used for the computation of the input soil displacements and for the analysis of the pile response.

REFERENCES

Dezi, F., Carbonari, S. and Leoni, G. (2007). A model for 3D kinematic interaction analysis of pile groups in layered soils. *Submitted to Earthquake Engineering and Structural Dynamics*.

Dezi, F., Carbonari, S. and Leoni, G. (2008). Kinematic interaction in pile foundation. *14th World Conference on Earthquake Engeneering.*

Dobry, R. & O'Rourke, M. J. (1983). Discussion on "Seismic response of end-bearing piles" by Flores-Berrones, R. & Whitman, R. V. *J. Geotech. Engng Div., ASCE*, 109.

Flores-Berrones R. & Whitman R. V. (1982). Seismic response of end bearing piles. *J. Geotech. Engng Div., ASCE* **108, No. 4**, p.554-569.

Margason, E. (1975). Pile bending during earthquakes. Lecture, 6 March 1975, ASCE-UC/Berkeley Seminar on Design Construction and Performance of Deep Foundations (unpublished).

Nikolaou, A. S., Mylonakis, G. & Gazetas, G. (1995). Kinematic bending moments in seismically stressed piles. Report NCEER-95-0022, National Center for Earthquake Engineering Research. State University of NY.

Nikolaou, A. S., Gazetas, G. (1997). Seismic design procedure for kinematically loaded piles. *Proc. 14th Int. Conf. Soil Mech. Found. Engng, Hamburg,* Special volume, ISSMFE TC4 Earthquake geotechnical engineering, 253-260.

Nikolaou, S., Mylonakis, G., Gazetas, G. & Tazoh T. (2001). Kinematic pile bending during earthquakes: analysis and field measurements *Géotechnique* **51, No. 5**, 425-440

Randolph, M. F. (1981). The response of flexible piles to lateral loading. *Géotechnique* **31, No. 2**, 247-259.

Velez, A., Gazetas, G. & Khrishnan, R. (1983). Lateral dynamic response of constrained-head piles. *J. Geotech. Engng Div., ASCE* **109, No. 8**, 1063-1081.

Makris, N. & Gazetas, G. (1992). Dynamic pile-soil-pile interaction. Part II: Lateral and seismic response. *Earthquake Engng & Struct. Dynamics* **21, No. 2**, 145-162.