DYNAMIC RESPONSE AND IMPEDANCE FUNCTIONS OF FOUNDATION RESTING ON SANDY SOIL USING PHYSICAL MODEL TESTS

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ABSTRACT:

Experimental results to investigate the dynamic behavior and impedance function of foundations resting on a sand layer are presented in this paper. Physical model block vibration tests were carried out in a cubic steel container of size 1.0 *m*×1.0 *m*×0.8 *m* using foundations in three shapes, circular, square, and rectangular. The radius of the circular one was 7.5 *cm* and the two other types had the same radius of the equivalent circle, where the aspect ratio of the rectangular foundation was $L/B = 2$. The employed vertical force generator was a vibration exciter system, driven by a power amplifier. Using available sea sand, a sandy sample with specific density was prepared, and tests were carried out for three different footing shapes, by varying contact pressure and causing embedment for footing. The impedance functions for the different above conditions were calculated and compared with the preceding numerical and experimental works. It was observed that the stiffness and damping of the supporting soil are both functions of the vibration frequency, which are mainly because of boundary conditions especially side walls. Also two resonance frequencies were observed in the test results, where one is because of the wave's reflection (waves emanating from the oscillating foundation reflect at the soil-container interface and return back to their source at the surface) and the other is because of the system's inertia and the system's natural frequency.

KEYWORDS: Footing vibration, impedance functions, Stiffness, Damping, Dynamic response.

1. INTRODUCTION

One of the fundamental problems in dynamic soil-structure interaction is the characterization of the dynamic response of surface foundations resting on a soil medium under time-dependent loads. Despite its importance to various soil dynamics and earthquake applications, a clear understanding of the problem has yet to be established owing to the complexity of real soil behavior and its constitutive modeling, the in-situ and stress-induced heterogeneity in the soil's modulus, and the three-dimensional nature of the underlying wave propagation phenomenon. To provide a basis for improvement, a comprehensive experimental data base that can permit a direct identification of key physical parameters for the problem would clearly be valuable. For such purpose, one should note the dynamic field tests by Dobry & Gazetas (1986), who studied vertical, torsional, horizontal and rocking vibrations of circular and rectangular surface foundations. Some information on the effect of foundation embedment and shape can be found in researches of Stokoe & Richart (1974). In addition to providing important insights into the complex nature of the subject, these field data have been helpful as a direct check of the applicability of relevant theories to practice. As to the use of the field studies for basic research and scientific generalization, however, such an approach is inherently limited due to the significant expenses and technical difficulties in performing systematic parametric variations and characterization of aspects such as soil properties and site formations in the prototype environment. Scaled modeling on the other hand doesn't present such problems. Owing to the small scale of models, numerous tests with well-defined can be performed at a fraction of the cost of full-scale tests.
One of the key steps in the current methods of dynamic analysis of foundation soil system under seismic or machine type loading is to estimate the dynamic impedance functions (spring and dashpot coefficients) associated with rigid

but massless foundations. However literature presents experimental results from studying variation of the dynamic impedance function is scanty.
In scaled modeling simulation of dynamic problems, however, it is well known that one of the major difficulties is the boundary/box effect caused by desirable wave reflections from the finite boundary of the soil model. Without proper treatment, the latter can contaminated the response to the extent that it may become meaningless.
In what follows, a series of physical model tests on the vertical dynamic characterization of surface and embedded foundations in circular, rectangular and square shapes on sand will be presented. The paper includes an analytical synthesis of the results of an extensive parametric study on the effect of foundation contact pressure, footing shape, and footing embedment on the dynamic foundation stiffness as a function of frequency.
The subject of vibratory response of foundations has attracted the attention of many researchers since the classical work. The significant advances during the last three decades in developing the theoretical solution to problems of foundation vibrations resting on a thick homogeneous soil deposit has been achieved (e.g. Lysmer & Richart 1966; Gazetas 1983, 1991).

2. TEST SETUP AND PROGRAM

In this investigation, the experiments were conducted in a container with rectangular geometry which was chosen to avoid the usual severe wave-focusing effects caused by cylindrical boundaries. The size of the container at the bottom was fixed as 1.0 *m* × 1.0 *m* where it was 0.8 *m* deep. To realize the non-wave-reflecting characteristics of a semi-infinite medium more closely for container, a layer of plastofoam with 5 *cm* thickness was placed at the side-walls. Dynamic excitation system consists of a permanent magnetic shaker, a power amplifier and a signal generator. A Brüel and Kjær 4814 vibration exciter was used in this study. The unit can provide acceleration, velocity, and displacement over the frequency rang of 1-10,000 Hz. The exciter is supported by a steel adjustable frame. The excitation signal was provided by a Brüel and Kjær 2035 signal analyzer.
To monitor the loading and the response, all model footings were instrumented with an Analog Devices ADXL 105EM-1 accelerometer, a pair of PY3 C 50 LVDTs and a Kyowa load cell. The instrumentation configuration is shown in Fig. 1. The soil tested was a uniformly graded, air dried (moisture less than 1%), medium-fine Babolsar sand. The properties for sand for this loading condition were shear modulus 4700 kN/m^2, unit weight (dry) 17.80 kN/m^3, Relative density 55%, and Poisson's ratio 0.33 (assumed). Shear modulus of the sand was obtained based on static stiffness of foundation and equation $K = 4GR_0/(1-\nu)$, where R_o and ν are equivalent radius of model and Poisson's ratio of soil respectively. Magnitude of the average shear strain during the tests of the homogeneous material was on the order of 10^{-4}. The shear modulus of the soil becomes strain dependent when the associated shear strain magnitude exceeded 10^{-4}.
To construct a homogeneous sample, the method of pluviation through air was employed with satisfactory results. Existing raining devices were calibrated to yield relative density of 55% by controlling the raining height and the flux of the sand. To permit adjustment of the foundation contact pressure (foundation weight), which is a parameter of critical interest, two steel disks were also fabricated that can be attached to the vibrating assembly. These two steel disks, henceforth referred to as "mass #1" and "mass #2," had masses of 1.1 *kg* and 2.3 *kg*; respectively.

Table 1 Basic properties of model footing

Mass (kg)	Aspect ratio (L/B)	Equivalent radius (m)	Base shape	Footing number
3.5	1	0.075	circle	1
3.6	1	0.075	square	2
3.6	2	0.075	rectangle	3

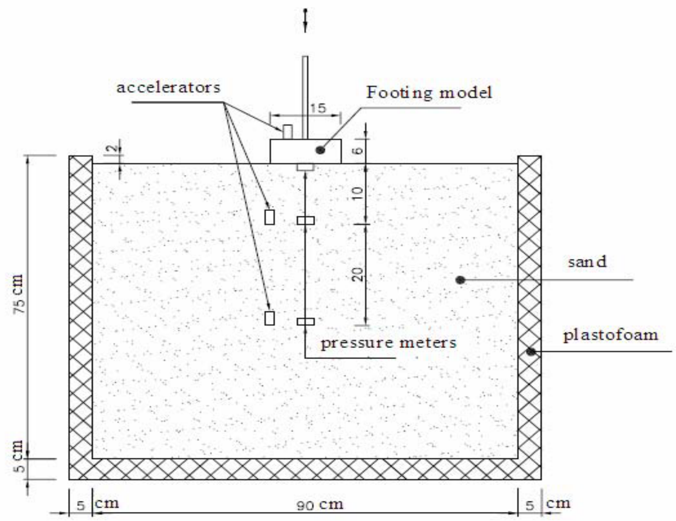

Figure 1 Typical footing design with transducer location

Table 2 Properties of sand tested (Babolsar sand)

Unified Gradation	Gs	Minimum density (kg/m3)	Maximum density (kg/m3)	Mass density ρ (kg/m3)
SP	2.691	1.646	1.915	1.78

3. MEASUREMENTS AND DATA PROCESSING

In this study, the dynamic characteristics of surface foundations undergoing small-amplitude motion are of primary interest. For such problems, the soil-foundation system can be conveniently characterized in the frequency domain in terms of its frequency response function (FRF). Mathematically, the FRF is defined by:

$$FRF(\omega) = Y(\omega)/X(\omega) \tag{3.1}$$

where X(ω) and Y(ω) are Fourier transforms of the analog input [x(t)] and output [y(t)] signals; respectively. In general, the FRF is a complex-valued function that can be represented in terms of its real and imaginary parts. Usually, the vertical dynamic force applied to the footing is taken as the input to the system while the vertical displacement of the foundation is taken as the output. In this investigation, instead of applied load the total interfacial load at the soil-foundation interface experimentally is being measured and used. So the dynamic foundation characteristic is characterized by this complex-valued dynamic interfacial compliance function commonly referred to as the “massless compliance” function which is defined as the ratio of the displacement response to the total interfacial load at the soil-foundation interface as a function frequency. Inverse of the compliance function is called impedance function. Results of experiments in this investigation are shown in the form of impedance function. Subsequently minimums in results represent the maximum values in pertinent compliance function. The use of the term “interfacial compliance” (or impedance) is strongly preferred for the foundation problems (Pak & Guzina 1995). To examine the linearity of the dynamic soil-foundation system in small-amplitude vibration, the compliance function was determined for different forcing levels, which yielded displacement amplitudes ranging from $10^{-6} \times a$ to $10^{-5} \times a$, where a is the foundation radius. At this level of vibration amplitudes, compliance function was found to be virtually invariant with respect to the magnitude of dynamic load.

4. ANALYTICAL FRAMEWORK

For each particular harmonic excitation with frequency ω, the dynamic impedance function is defined as the ratio between the steady-state force and the resulting displacement at the base of the massless foundation.

$$K_v = \frac{R_v(t)}{v(t)} \tag{4.1}$$

In which $R_v(t) = R_v \exp(i\omega t)$ is the harmonic vertical force applied at the base of the disk, and $v(t) = v\exp(i\omega t)$ is the uniform harmonic displacement of the soil-foundation interface. It is evident that is the total soil reaction against the foundation; it's made up of the normal R_v stresses against the basement plus, in the case of embedded foundations, the shear stresses along the vertical side walls. As dynamic force and displacement are generally out of phase, impedance may also be written in the form:

$$K_v(\omega) = K_v(0)[K_{v1}(\varpi) + iK_{v2}(\varpi)] \tag{4.2}$$

where K_{v1} and K_{v2} are dimensionless frequency functions associated with the uniform half-space solution; and

$$K_v(0) = 4Ga/(1-\nu)\;; \qquad \varpi = 2\pi af \big/ \sqrt{G/\rho} \tag{4.3}$$

For some appropriate shear modulus *G*, Poisson's Ratio *v*, and soil density *ρ*. K_{v1} reflects the stiffness and inertia of the supporting soil; its dependence on frequency is attributed solely to the influence which frequency has on inertia, since soil properties are essentially frequency independent. The imaginary component reflects the radiation and material damping of the system. The former, being the result of energy dissipation by waves propagating away from the foundation, is frequency dependent; the latter, arising chiefly from the hysteretic cyclic behavior of soil, is practically frequency independent.
Combining Eqs. 3.1 and 4.1 reveals correlation between impedance function and frequency response function (FRF) of a system as below:

$$H(\omega) = \frac{1}{K_v - m\omega^2} \tag{4.4}$$

where m is mass of footing and ω is excitation frequency. If we write the impedance function in the form of $K_v(\omega) = K + iC\omega$, then magnitude of a system's FRF is equal to:

$$|H(\omega)| = \frac{A_z}{Q_o} = \frac{1}{K_o\sqrt{\left[1-\left(\omega^2/\omega_n^{\,2}\right)\right]^2 + 4\beta^2\left(\omega^2/\omega_n^{\,2}\right)}} \tag{4.5}$$

where A_z is amplitude of displacement, Q_0 is exciting force amplitude, $\beta = C/(2K_0/\omega_n)$ is damping ratio, ω_n is systems natural frequency equals to $\sqrt{K_0/m}$, and K_0 is system's static stiffness. The ratio of $A_z/(Q_0/K_o)$ is called dimensionless amplitude.

5. OBSERVATION ON EXPERIMENTAL RESULTS

In this section, some typical results from seven series of tests will be presented. The first three series of tests, named A, B, and C were performed using the circular block on sand samples with a uniform relative density Dr = 55%. Series, namely, D, E and G were performed with square block. Seri F was performed using rectangular foundation. In some specific frequencies footing model undergoes harmonic excitation and system dynamic characteristics (Impedance value) were specified for that specific frequency. By doing this procedure for a specific frequency rang, system's characteristics will be defined for the relevant range. Program of tests is mentioned in Table 3. These series consist of 185 harmonic excitation experiment with a variety of footing shape and static bearing pressure.
Real (in-phase) and imaginary (out-of-phase) parts of the Impedance functions results from first three experiments are shown in Fig. 2. Real part represent stiffness characteristic where imaginary part represents system's damping characteristics. Also experimental Impedance result of the test G is shown in Fig. 3.

Table 3 Experimental program

Block weight (kg)	Embedment ratio δ=D/R0	Footing base shape	Model
3.5	0	Circle R=7.5cm	A
4.6	0	Circle R=7.5cm	B
5.7	0	Circle R=7.5cm	C
4.7	0	Square B=L= 6.63 cm	D
5.8	0	Square B=L= 6.63 cm	E
4.7	0	Rectangle L/B=2 B=4.7cm L=9.4cm	F
4.7	0.8	Square B=L= 6.63 cm	G

According to Eqn. 4.4 and experimental evaluation of impedance functions, where results for series A, B, C, and F are shown in Figs. 2 and 3, magnitude of frequency response function for each of test series are driven. Frequency response functions were plotted for the modes with different shapes. Fig. 4 presents the plot for response of circular footing model. It can be seen from Fig. 4 that two resonances occur during examined frequency range. Frequency of the first resonance is independent of foundation's mass where the second one depends on this parameter. In other words the first resonance is not a consequence of foundation inertia and seems that relevant undulations about it's frequency in impedance function associate with the natural frequency of the soil layer. In other words, the observed fluctuation is the outcome of resonant phenomena: waves emanating from the oscillating foundation reflect at the boundaries of the model soil (interface of soil and plastofoam) and return to their source at the surface. As a result, the amplitude of foundation motion may significantly increase at specific frequencies of vibration, are close to the natural frequency of deposit.
Despite plastofoam's high material damping, the very low-density-and-stiffness sawdust layer created a boundary of sharp "acoustic impedance" contrast, thereby sending part of the arriving wave energy to the oscillating footing. Also another phenomenon is revealed through the variation with frequency of the imaginary part which reflects the damping characteristics of the system. At low frequencies, below the first resonance, damping is much less than upper frequencies. This is due to the fact that no surface waves can be physically created in a soil stratum at such frequencies and thus geometrical spreading of wave decreases.
This resonance however can hardly be predicted by the simple one-dimensional wave propagating since apparently they involve a mixture of *P* and *S* waves reflecting from all sides. Also it can be seen from Fig.4 that with increase in footing mass, FRF decreases and dimensionless amplitude increase which are in agreement with *SDOF* systems concept. Obtained by bandwidth method, damping of all three tests is around 5%. This observation reveals that the radiation damping is little (about half of this amount can be assumed as material damping and the remaining amount is negligible compared to radiation damping of vertical vibrating systems) because of the presence of boundaries.

Fig.5 presents comparisons for response of square and rectangle footings with the same equivalent radius ($a_0 = \sqrt{A/\pi}$; *A* being area of the footing base) in the range of the second resonance frequency, where rectangular footing's aspect ratio is *L/B*=2. It can be seen from this figure that response of these two type models coincides reasonably. From comparison of Figs. 4 and 5 it is evident that equivalent radius method result is accurate, at least for footings with aspect ratios less than 2. From comparison of Figs. 2 (a&b) with Fig. 3 it can be seen that embedment decrease the effect of boundaries and smoothen the impedance function. Notice also that the influence of embedment on damping depends on the particular frequency of oscillation. The stiffness coefficient is only slightly affected by embedment, at least at lower frequencies (examined range).

6. CONCLUSION

In this study the influence of shape, mass and embedment of foundation on the dynamic response of a foundation-soil system was investigated experimentally. It was observed that reflection of waves cause resonance in system in addition to inertia effect. Radiation damping of tests around resonance was little because of the presence of boundaries.
Equivalent circular footing yield reasonably good estimate of the response for rectangular foundation with values of L/B at least less than 2. Also it has been seen that embedment decrease sharpness of impedance function and also its value depends on particular frequency. On the other hand embedment overly increases damping of the system but it does not affect stiffness coefficient strongly.

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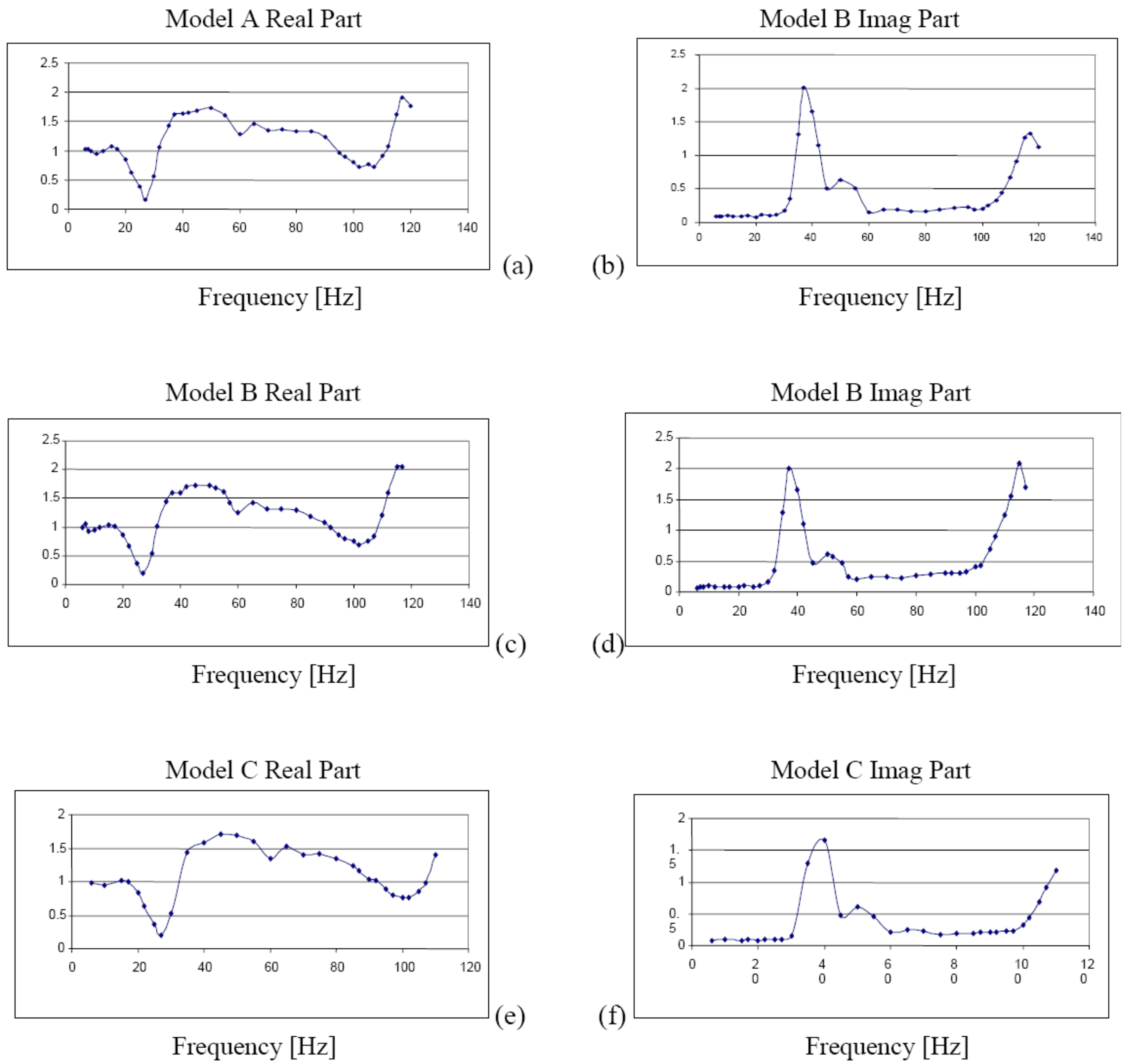

Figure 2 Experimental impedance functions for models A, B and C

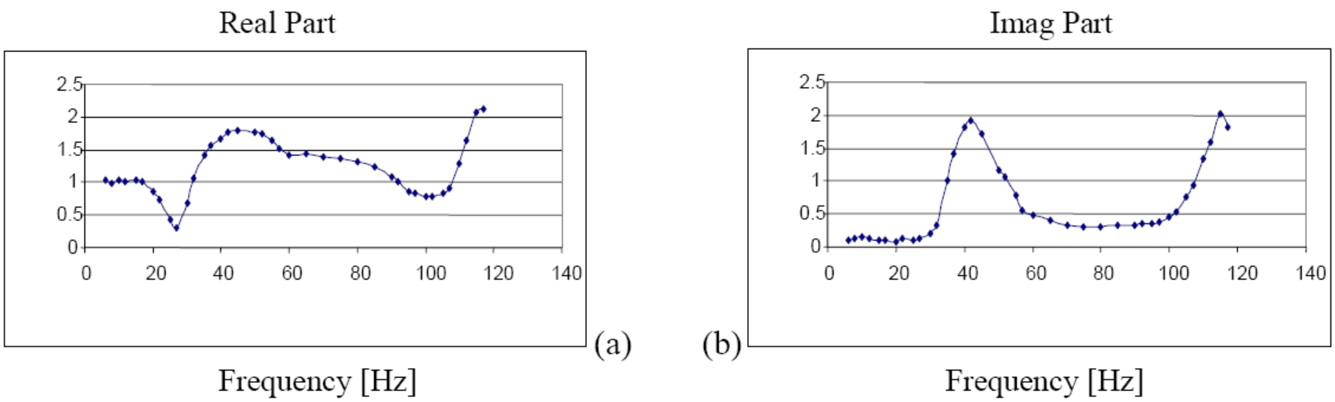

Figure 3 Experimental impedance functions for model F

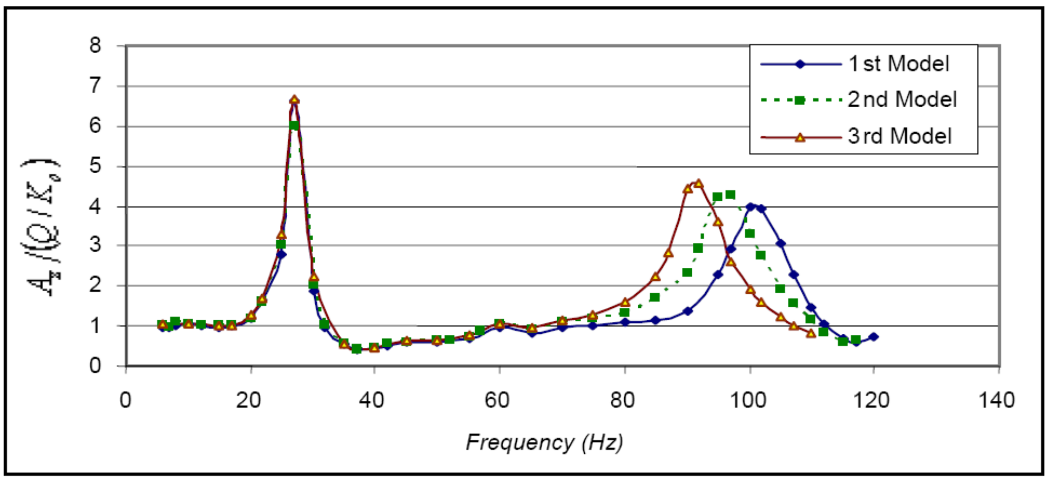

Figure 4 Comparison of (FRF Magnitude) dimensionless amplitude for three first tests A, B and C

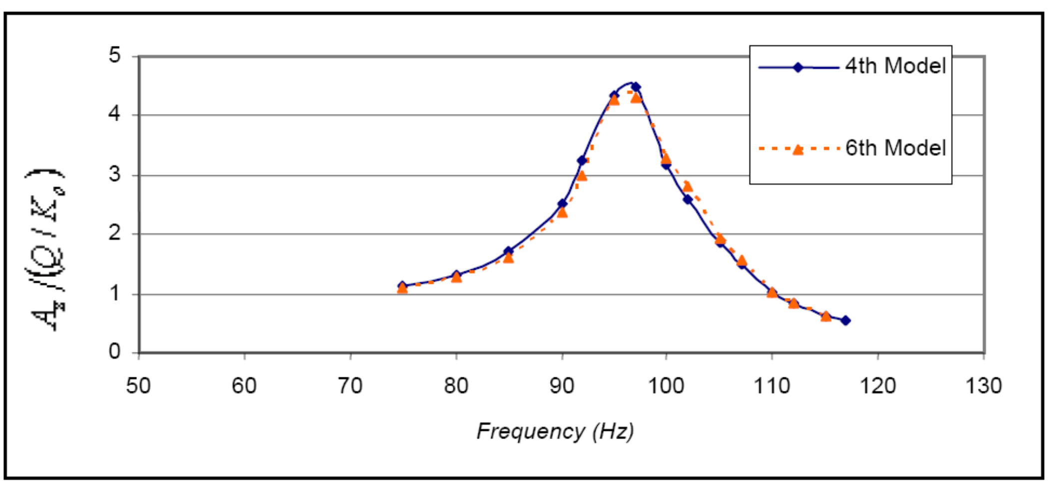

Figure 5 Comparison of dimensionless amplitude for test series D and F