

# THE EFFECT OF CONSOLIDATION RATIOS ON DYNAMIC SHEAR MODULUS OF SOIL

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# **ABSTRACT :**

The dynamic shear modulus (DSM) is the most basic soil parameter in earthquake or other dynamic loading conditions and can be obtained through testing in the field and in the laboratory. By using the resonant column tests, the effect of the consolidation ratios on the maximum DSM of the soil is investigated and an incremental formula for calculating the maximum DSM for the cases of the anisotropic consolidation is presented. The silty clay, silt and sludgy soil of China are employed in the tests to establish the basic formula in the paper. The form of the formula is a power function to describe the relation between the ratio of the maximum DSM's due to anisotropic and isotropic consolidations and the increment of consolidation ratio,  $G_{max}/G_0=1+C\cdot(k_c-1)^B$ . The new formula indicates that the maximum DSM is the power of  $k_c$ -1 instead of the nearly linear relation in Hardin and Black's and the increasing degree of the maximum DSM due to  $k_c>1.0$  is significantly larger than that described by Hardin and Black's formula. Compared with sand, the increment of the maximum DSM for silty clay due to anisotropic consolidation is much larger than that of sand.

**KEYWORDS:** consolidation ratio, dynamic shear modulus, undisturbed cohesive soil, resonant column test

# **1. INTRODUCTION**

The dynamic shear modulus (DSM) is the most basic parameter in soil property and can be obtained through testing in the field and in the laboratory (Hardin and Black, 1968, 1969; Yu et al, 1988; He, 1997; Chen et al, 2002; Yuan et al, 2005). The most advanced apparatus for testing the dynamic shear modulus in small strain range in the laboratory now is the resonant column device because of its advantages in the simply mechanical principle, clear stress condition and convenient operation and the small deviation of testing results. However, the most existing resonant column devices are only suitable for specimens under isotropic consolidation. Considering that the consolidation ratio,  $k_c$ , varies in the range of 1.4-3.0 in actual subsoil, some researchers and engineers tried to use Hardin and Black's formula (1968, 1969) to predict the maximum DSM of soils under a different consolidation ratios. The maximum DSM for  $k_c=1.5$  and  $k_c=2.0$  respectively are 8% and 15% larger than that for  $k_c=1.0$  by the Hardin and Black's formula. Some researchers (He, 1997) questioned the accuracy of Hardin and Black's formula in predicting the maximum DSM of soil under anisotropic consolidation in terms of the dynamic triaxial tests. However, these results are only a few and the dynamic triaxial tests are basically suitable for the moderate and large deformation of soils. Therefore, the effect of consolidation ratios on the maximum DSM still needs to be studied further.

# 2. EQUIPMENT AND SOIL SPECIMEN

### 2.1. Equipment

To investigate the DSM of soil under anisotropic consolidation, a new resonant column device is developed at the Institute of Engineering Mechanics, China Earthquake Administration, in 2002. The device includes two special designs for performing deviatoric stress tests. One is a special transmission mechanism that applies vertical static deviatoric force to the soil specimen without eccentric force. Another is placing a manipulator inside the pressure vessel to avoid the instability problem encountered in the process of specimen fitting and



applies vertical deviatoric stress to the soil specimen. The device is the fixed-free resonant column. The diameter and the height of the specimen are 3.91cm and 8.00cm, respectively. The resonant column device is shown in Figure 1.



Figure 1 The resonant column device used for tests

### 2.2. Soil Specimen

Three types of undisturbed cohesive soil, the silty clay, silt and sludgy soil in China are tested. Fourteen specimen of three fields which is Bridge of Taoerhe, University City of Changqing and Tunnel of Huanghe in Shan Dong, respectively, are used for silty clay test. Three specimen of one field, Which is Bridge of Taoerhe in Shan Dong are used for silt test. Two specimen of one field, Which is Bridge of Hangzhou in Zhe Jiang are used for sludgy soil test. The physical specifications of undisturbed cohesive soil are listed in Table 2.1.

Speciality	Serial Number	Deep / m	Density / g/cm <sup>3</sup>	Water Content
Silty Clay	SC-Tao-1	25.3-25.5	2.161	17.36
	SC-Tao-2	30.7-30.9	1.989	30.83
	SC-Tao-3	34.6-34.8	2.175	17.18
	SC-Tao-4	36.6-36.8	2.063	23.50
	SC-Chang-5	9.8-10.0	2.140	19.90
	SC-Chang-6	16.2-16.4	2.078	23.37
	SC-Chang-7	20.6-20.8	1.974	27.84
	SC-Chang-8	26.2-26.4	2.099	20.69
	SC-Chang-9	17.6-17.8	2.057	19.73
	SC-Chang-10	32.2-32.4	1.974	26.21
	SC-Huang-11	16.4-16.6	2.094	21.23
	SC-Huang-12	29.6-29.8	2.014	22.62
	SC-Huang-13	15.4-15.6	2.135	19.25
	SC-Huang-14	23.1-23.3	2.167	18.55
Silt	S-Tao-1	15.4-15.6	2.238	15.00
	S-Tao-2	19.0-19.2	2.146	17.58
	S-Tao-3	22.2-22.4	2.156	19.79
Sludgy Soil	SS-Hang-1	29.8-30.1	1.863	33.44
	SS-Hang-2	47.8-48.0	1.972	34.29

Table 2.1 Physical specifications of three types of undisturbed cohesive soil



### 3. THE RECOMMENDED FORMULA OF MAXIMUM DSM

#### 3.1. Results of the Silty Clay

In the tests, fourteen specimens of three fields of Bridge of Taoerhe, University City of Changqing and Tunnel of Huanghe, are made for the silty clay. One confining stresses and five consolidation ratios,  $k_c = 1.0, 1.2, 1.5, 1.7$  and 2.0, are employed. For each confining stress, five 'identical' samples are used respectively for the tests of five different consolidation ratios. After finishing the consolidation, the graded loads of the torsional moment are conducted to the soil specimen. DSM is obtained by using the free vibration technique.

The relations of DSM and dynamic shear strain, G- $\gamma$ , are conducted for different specimen of silty clay. The hyperbolic equation, G=1/(a+b $\gamma$ ), is used to obtain the regression curve of G- $\gamma$ . The maximum DSM is obtained from the regression equation by  $\gamma \rightarrow 0$ . According to the relations of the maximum DSM and the confining stress, the increment of the maximum DSM,  $G_{\text{max}}$ , is formed as Eqn. 3.1.

$$\frac{G_{\max}}{G_0} = 1 + C \cdot (k_c - 1)^B$$
(3.1)

where  $G_0$  is maximum DSM for  $k_c=1.0$  and  $G_{max}/G_0$  is the relative increment of the maximum DSM due to  $k_c>1.0$ . By averaging the coefficients of *C* and *B* from fourteen specimens, the coefficients *C* and *B* are 1.25 and 0.76, respectively. The comparison between the calculation and test data is shown in Figure 2 and the average error is 8.8%.



Figure 2 Comparison between the presented formula and test data of silty clay

#### 3.2. Results from The Silt

In the tests, three specimens of Bridge of Taoerhe, are made for the silt. One confining stresses and five consolidation ratios,  $k_c=1.0$ , 1.2, 1.5, 1.7 and 2.0, are used for the analysis. Using the same procedure for the silty clay, the relative increment of the maximum DSM for the silt due to  $k_c>1.0$  is expressed in the same form, provided the coefficients are C=1.20 and B=1.05. The comparison between the calculation and test data is illustrated in Figure 3 and the average error is only 8.7%.



Figure 3 Comparison between the presented formula and test data of silt



### 3.3. Results from the Sludgy Soil

In the tests, two specimen of Bridge of Hangzhou, are made for the sludgy soil. One confining stresses and four consolidation ratios,  $k_c$ =1.0, 1.25, 1.5 and 2.0, are used for the analysis. Using the same procedure for the silty clay and silt, the relative increment of the maximum DSM for the sludgy soil due to  $k_c$ >1.0 is expressed in the same form, provided the coefficients are *C*=1.85 and *B*=0.47. The comparison between the calculation and test data is illustrated in Figure 4 and from which it can be concluded that the errors are quite small.



Figure 4 Comparison between the presented formula and test data of sludgy soil

### 3.4. Recommended Formula in the Paper

By comparing the experimental results of three types of undisturbed cohesive soil tested, three unique expression of the relative increment of the maximum DSM under the different consolidation ratios  $k_c$  for the silty clay, silt and sludgy soil, respectively, are developed as

Silty clay: 
$$\frac{G_{\text{max}}}{G_0} = 1 + 1.25 \cdot (k_c - 1)^{0.76}$$
 (3.2)

Silt:

$$\frac{G_{\text{max}}}{G_0} = 1 + 1.20 \cdot (k_c - 1)^{1.05}$$
(3.3)

Sludgy soil: 
$$\frac{G_{\text{max}}}{G_0} = 1 + 1.85 \cdot (k_c - 1)^{0.47}$$
 (3.4)

In Eqn. 3.2. - Eqn. 3.4., the coefficient 1.25, 1.20 and 1.85 represents the increment of the maximum DSM for  $k_c$ =2.0, which is 125%, 120% and 185%, respectively, larger than that for  $k_c$ =1.0. The results also indicate that the effect of consolidation ratios on the maximum DSM of undisturbed cohesive soil is quite remarkable. The increasing degree of the maximum DSM for the sludgy soil is significantly larger than that of the silty clay and silt.

### 4. COMPARISON WITH OTHER RESULTS

In terms of  $\sigma_2 = \sigma_3$  and  $\sigma_1 = k_c \sigma_3$ , the simplified formula for calculating  $G_{\text{max}}/G_0$  was recommended by Hardin and Black (1968) as shown in Eqn. 3.5.

$$\frac{G_{\max}}{G_0} = \left(\frac{2+k_c}{3}\right)^{0.5} \tag{3.5}$$

The comparison of Eqn. 3.2 with Eqn. 3.5 proposed in this paper is illustrated in Figure 5 and it shows that the difference becomes larger and larger with the increasing of  $k_c$ . According to Hardin and Black's formula,

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 $G_{\text{max}}/G_0$ - $k_c$ , there is a nearly linearly increases in the maximum DSM, while the formula proposed in this paper,  $G_{\text{max}}/G_0$ - $k_c$ -1, increases in exponential form in the interval of  $k_c$ =1.0 to 2.0. It also can be seen from Figure 5 that the effect of consolidation ratios on the maximum DSM of soil is quite remarkable. The increasing degree of the maximum DSM from the formula proposed in the paper is significantly larger than that Hardin and Black's formula.

For  $k_c=1.5$  and  $k_c=2.0$  in Figure 5, the estimated values of  $G_{\text{max}}/G_0$  from Eqn. 3.2 are 1.85 and 2.25 respectively, however, the values estimated from Hardin and Black's formula are only 1.08 and 1.15. It means the  $G_{\text{max}}/G_0$  will be underestimated by the Hardin and Black's formula.



Figure 5 Comparison of  $G_{\text{max}}/G_0 - k_c$  for three formulas

The effect of the consolidation ratios on the maximum DSM for two types of sand had been investigated by the author (Sun, 2006) and an incremental formula for calculating the maximum DSM under the anisotropic consolidation had been recommended as shown in Eqn. 3.6.

$$\frac{G_{\max}}{G_0} = 1 + 0.66 \cdot (k_c - 1)^{0.54}$$
(3.6)

The comparison between Eqn. 3.2 and Eqn. 3.6 proposed in the paper is also illustrated in Figure 5. By comparing with the formula of sand, it shows that the increasing degree of the maximum DSM for the silty clay proposed is significantly larger than that of sand. For instance, in case of  $k_c=2.0$  in Figure 5, the value of  $G_{\text{max}}/G_0$  is around 2.25 according to the Eqn.3.2 but only 1.66 estimated from the formula of sand.

He (1997) conducted dynamic triaxial tests for disturbed the silty clay under the deviatoric stresses. It can be deduced that  $G_{\text{max}}/G_0$  for the cases of  $k_c$ =2.0 are about 1.4 and 2.0 which is compatible with the result obtained in the paper.

# 5. CONCLUSIONS

The conclusions of the paper are summarized as follows:

- 1. The effect of the anisotropic consolidation on the maximum DSM of soil is quite remarkable and the recommended incremental formula for calculating the effect is developed.
- 2. For silty clay, silt and sludgy soil, respectively, the recommended formula are  $G_{max}/G_0=1+1.25 \cdot (k_c-1)^{0.76}$ ,  $G_{max}/G_0=1+1.20 \cdot (k_c-1)^{1.05}$  and  $G_{max}/G_0=1+1.85 \cdot (k_c-1)^{0.47}$ , respectively. The results also indicate that the increasing degree of the maximum DSM for the sludgy soil is significantly larger than that of the silty clay and silt.
- 3. The increasing in the maximum DSM due to  $k_c>1.0$  is significantly larger than that predicted by Hardin and Black's formula, e.g., the increasing of  $k_c$  from 1.0 to 2.0 according to the silty clay is about 125%, while it is only 15% according to Hardin and Black's formula.
- 4. By comparing to the formula of sand, the increasing degree of the maximum DSM for silty clay is



significantly larger than that of sand.

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### REFERENCES

Hardin, B.O., and Black, W.L. (1968). Vibration modulus of normally consolidated clay. *Journal of the Soil Mechanics and Foundations Division. ASCE* **94:2,** 353-369.

Hardin, B.O., and Black, W.L. (1969). Vibration modulus of normally consolidated clay (closure). *Journal of the Soil Mechanics and Foundations Division. ASCE* **95:6**, 1531-1537.

He, C.R. (1997). Dynamic triaxial test on dynamic modulus and damping ratio. Chinese Journal Geotechnical Engineering, **19:2**, 39-48 (in Chinese).

Yu, P.J., and Richart, F.E. (1984). Stress ratio effects on shear modulus of dry sands. Journal of the Soil Mechanics and Foundations Division. ASCE, **110:3**, 331-345.

Yu, P.J., Liang, Y.X., and Qin, W.Q. (1988). Dynamic modulus and damping ratio of disturbed sands. Chinese Journal Geotechnical Engineering, **10:4**, 57-63 (in Chinese).

Chen C.L., Hu Z.Q., Xie D.Y. and Feng Z.Y. (2002), Research on dynamic characteristic of sand in foundation of Xiabandi Dam. *Proc. 6th Chinese National Conference on Soil Dynamics, Nanjing, China*, 698-704. (in Chinese)

Xiaoming Yuan, Jing Sun and Sun Rui (2005). Effect of Consolidation Ratios on Maximum Dynamic Shear Modulus of Sands. *Earthquake Engineering and Engineering Vibration* **4:1**, 59-68. (in Chinese)

Jing Sun and Xiaoming Yuan (2006). The Effect of the Consolidation Ratio of Sands on Dynamic Shear Modulus and Response Spectrum of Soil layer. *Geotechnical Special Publication, ASCE*, 437-443.