COMPARISONS OF 5 TYPES OF SOIL DYNAMIC NONLINEAR CONSTITUTIVE MODELS IN SEISMIC RESPONSE OF SITE

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ABSTRACT :

Firstly, research evolution, merits and disadvantages of soil nonlinear constitutive models are reviewed, and needs of nonlinear analysis methods in time domain are pointed out. Secondly, the whole equation and application key of 5 types models, which are Pyke method, damping degradation coefficient model, non-Masing rule model ONE and model TWO, implicit stress damping equivalent model, are presented. Thirdly, by detailed comparisons of nonlinear response of clay layer mantled base rock site and Taiwan Lotung DHB borehole array site, nonlinear performance, hysteretic energy, difference of dynamic response, calculating efficiency and application condition of each model are expounded. At last, discrepancy of synthetic ground motion of each model and record of strong motion observation are discussed, and the improvement problems, which need to be done in future, are suggested.

KEYWORDS: 5 types, dynamic constitutive model, nonlinear seismic response of site, comparisons

1. INTRODUCTION

The nonlinear method of site response in time domain has attracted more and more attention for it can actually reflect the primary characteristics of site response, such as "along with the seismic motion increases, the nonlinear response strengthen, the response amplitude decrease, the predominant period moves to long period range" etc. However, the site response results of equivalent linearization method with the questions of "false resonating", when it processes "the strong motion input", "the soft layer" cases, are not consistent with the observation records.

In the 1920s, the most basic equation Masing rule(Masing, 1926)which reflect dynamic stress - strain nonlinear constitutive relations of soil in time domain, was proposed firstly. Later, based on soil experiment achievements of strain-dependent shear modulus and damping ratio according to skeleton curve equation, many kinds of soil dynamic nonlinear constitutive models, such as the General Masing rule, Pyke method(Pyke, 1979), Hardin-Drnevich(Hardin and Drnevich, 1972a, 1972b) hyperbolic curve model, Ramberg-Osgood(R-O) model(Ramberg and Osgood, 1943), Martin-Davidenkov(M-D) model(Martin, 1975), Iwan model(Iwan, 1967)and Revised Iwan model (ZHENG Da-tong and WANG Hui-chang, 1983; LI Xiao-jun and LIAO Zhen-peng, 1989) and so on, have been established. But, R-O model will exceed ultimate stress in very big strain, and its constitutive formula meeting the demands of test damping ratio is quite complex. The theoretical damping function of M-D model is an implicit and complex expression. The accuracy of Iwan model is influenced by the quantity of mechanics elements. As a result, the above several models have affected engineering application in such certain extent. Anyway, many scholars still widely explored and beneficially attempted to these models(LUAN Mao-tian and LIN Gao, 1992; CHEN Guo-xing and ZHUANG Hai-yang, 2005).

The H-D hyperbolic skeleton curve is widely accepted in geotechnical earthquake engineering for its many merits, which have the merits of simple form, few parameters, the clear physical meaning and easy to fit test results and so on. Simultaneously, the thoughts of "the damping ratio degeneration coefficient"(WANG Zhi-liang and HAN Qing-yu. 1981), "the mobile skeleton curve"(LI Xiao-jun, 1993), "partly analytic solution, partly soil test fitting"(LUAN Mao-tian and LIN Gao, 1992)etc. are proposed in aspect of fitting test damping ratio. To improve the complex rules of "on the great-circle" in general Masing rule, many kinds of soil dynamic nonlinear constitutive relations are produced. The commonly used model includes: Pyke model, "damping ratio degeneration coefficient" model(WANG Zhi-liang and HAN Qing-yu. 1981), based on non-Masing rule model ONE and model TWO(ZHANG Ke-xu, LI Ming-zai et al. 1997), and implicit stress damping equivalent model constructed by "the mobile skeleton curve"(LI Xiao-jun, 1993). Whereas, above these five kinds of models are the empirical relationship models

The type of soil dynamic nonlinear constitutive model is very complex, which kind of nonlinear model is chosen in analyzing nonlinear site response that usually has certain subjectivity. Comparisons study on each kind of model's results on the same condition are very few in China. It is necessary to provide certain standard and the basis for the engineering application after detailed examining the nonlinear simulation ability of each kind of model, the difference of hysteresis energy, the interrelation and the suitable condition. Limited to the condition in China, the related report has not been discovered until now.

2. FUNCTION EXPRESSION OF 5 TYPES OF SOIL DYNAMIC NONLINEAR CONSTITUTIVE MODELS

According to 5 types of models, in initial load (or unloading) process, it is supposed that shear stress - shear strain skeleton curve follow the H-D hyperbolic curve $f(\gamma)$. The following irregular unloading and the reverse load process, the principle of each kind of model is different.

2.1. Pyke Method(Pyke, 1979)

Pyke (1979) proposed that the following unloading (or load) curve and the original skeleton curve maintain "C time" the relations, namely the following unloading (or load) curve point to ultimate stress - strain point (+∞, τ_{ult}) and (- ∞, - τ_{ult}), its whole expression is:

$\tau = f(\gamma) = \dfrac{G_{\max}\gamma}{1+|\gamma|/\gamma_r}$, Initial load (or unloading) case;

$\tau = \tau_C + \dfrac{G_{\max}(\gamma-\gamma_C)}{1+|\gamma-\gamma_C|/(C\gamma_r)}$, Other cases

And, G_{max} and γ_r respectively are the biggest shear modulus and reference shear strain. γ_C and τ_C was the corresponding strain and the stress of "the inflexion point" in recently load (or unloading) process. $C = 1-\tau_C/(\pm\tau_{ult})$, soil ultimate stress $\tau_{ult} = G_{\max}\gamma_r$. "±"symbol mean, when $d\gamma > 0$, "+" is used, on the contrary, $d\gamma < 0$, "-" is used. This model is very simple, because only the coordinate of recently inflexion point of unloading (or reverse load) needs to memory.

2.2. Damping Degradation Coefficient Model(WANG Zhi-liang and HAN Qing-yu. 1981)

Wang Zhi-liang and Han Qing-yu (1981) adopted "the damping ratio degeneration coefficient" K(γ) to adjust the shape of theory unloading curve, which cause that the area of hysteretic loop satisfy the experimental damping ratio, its function expression is equation(2.1),

$$\tau(\gamma) = \begin{cases} \tau_C + K(\gamma_0)\left[\dfrac{G_{\max}(\gamma-\gamma_C)}{1+|(\gamma-\gamma_C)|/2\gamma_r'} - \dfrac{\pm\tau_M-\tau_C}{\pm\gamma_M-\gamma_C}(\gamma-\gamma_C)\right] + \dfrac{\pm\tau_M-\tau_C}{\pm\gamma_M-\gamma_C}(\gamma-\gamma_C) & |\gamma| < \gamma_M \\ \dfrac{G_{\max}\gamma}{1+|\gamma|/\gamma_r} & |\gamma| \ge \gamma_M \end{cases} \tag{2.1}$$

In above equation, $\tau_0 = \left|\dfrac{\pm\tau_M-\tau_C}{2}\right|$, $\gamma_0 = \left|\dfrac{\pm\gamma_M-\gamma_C}{2}\right|$, $(\gamma_r')^{-1} = \dfrac{2G_{\max}(\pm\gamma_M-\gamma_C)}{|\pm\gamma_M-\gamma_C|(\pm\tau_M-\tau_C)} - \dfrac{2}{|\pm\gamma_M-\gamma_C|}$,

$K(\gamma_0) = \dfrac{\pi\gamma_0^2\lambda_T(\gamma_0)}{2\gamma_0(2\gamma_r'+\gamma_0) - 4\gamma_r'(\gamma_r'+\gamma_0)\ln(1+\dfrac{\gamma_0}{\gamma_r'})}$. (τ_M, γ_M) is the point of the biggest stress and the biggest strain in history, the biggest stress and the biggest strain are always positive. $\lambda_T(\gamma_0)$ means experimental damping ratio.

2.3. Non-Masing rule model ONE and model TWO(ZHANG Ke-xu, LI Ming-zai et al. 1997; LUAN Mao-tian and LIN Gao, 1992)

Zhang Ke-xu, Li Ming-zai and Wang Zhi-kun etc. (1997) proposed model ONE and model TWO based on non-Masing rule which can directly apply in site dynamic analysis. The rule is supposed that, the unloading and the reverse load curve are, $\tau - \tau_C = n_1 G_{\max}\dfrac{\gamma-\gamma_C}{1+|\gamma-\gamma_C|/(n_2\gamma_r)}$, and $n_1 > 0$, $n_2 > 0$. On the condition of the symmetrical principle and the reverse load (or unloading) no exceeding soil ultimate shear stress, model ONE parameters are obtained.

$$\begin{cases} n_1 = \dfrac{1}{n_2}\left(1 - \dfrac{\tau_C}{\pm G_{\max}\gamma_r}\right) \\ n_2 = \dfrac{\pm\gamma_M - \gamma_C}{\pm\tau_M - \tau_C}\left(G_{\max} - \dfrac{+\tau_M}{\gamma_r}\right) \end{cases} \tag{2.2}$$

According to the symmetrical principle and the condition of hysteretic energy satisfying real damping characteristics of soil, 2 parameters simultaneous equations of model TWO may be obtained.

$$n_1 = \frac{1}{G_{\max}}\frac{\pm\tau_M - \tau_C}{\pm\gamma_M - \gamma_C}\left(1 + \frac{\left|\pm\gamma_M - \gamma_C\right|}{n_2\gamma_r}\right), \quad \lambda_T(\gamma_0) = \frac{1}{\pi}\left\{\frac{n_1 G_{\max} n_2^2\gamma_r^2}{\tau_0\gamma_0}\left[\frac{2\gamma_0}{n_2\gamma_r} - \ln\left(1 + \frac{2\gamma_0}{n_2\gamma_r}\right)\right] - 2\right\}$$

Through hypothesis initial value and the iterative method, n_1, n_2 values are gotten. In order to avoid the iterative computation, Luan Mao-tian etc. (1992) proposed the method of “partly analytic solution, partly soil test fitting (including shear modulus and damping ratio)” to recover it.

2.4. Implicit Stress Damping Equivalent Model(LI Xiao-jun, 1993)

Li Xiao-jun (1993) proposed one kind of nonlinear model which take the stress as independent variable. Linear stress item and quadratic stress item are introduced. The function of stress is the implicit expression for shear strain. Therefore, this model is suitable for velocity and stress recursion equation with “the space and time overlap”. Through the definition of mobile skeleton curve and the damping equivalence, shear stress - strain relationship of this model in the entire irregular load process is,

$$\gamma(\tau) = \begin{cases} \gamma_C + \dfrac{\tau - \tau_C}{G_{\max}}\left(\dfrac{1}{1 - \left|\dfrac{\tau - \tau_C}{2}\right| \Big/ \tau_{ult}} + c\left|\dfrac{\tau - \tau_C}{2}\right| + d\left|\dfrac{\tau - \tau_C}{2}\right|^2\right) & |\gamma| < \gamma_M \\ \dfrac{\tau}{G_{\max}}\left(\dfrac{1}{1 - |\tau|/\tau_{ult}}\right) & |\gamma| \ge \gamma_M \end{cases} \tag{2.3}$$

In above equation, $c = B - 3A$, $d = (4A - B)/\tau_0$, $A = \dfrac{G_{\max}\gamma_0}{\tau_0^2} - \dfrac{1}{(1 - \tau_0/\tau_{ult})\tau_0}$,

$$B = \frac{3G_{\max}\gamma_0}{\tau_0^2}\left(2 - \pi\lambda_T(\gamma_0)\right) + \frac{12\tau_{ult}^2}{\tau_0^3}\left[\frac{\tau_0}{\tau_{ult}} + \ln\left(1 - \frac{\tau_0}{\tau_{ult}}\right)\right]$$

In the following, using above five types of soil dynamic nonlinear constitutive models, nonlinear site response of single clay layer mantled base rock site and Taiwan Lotung DHB borehole array site are analyzed.

3. COMPARISONS OF NONLINEAR SEISMIC RESPONSE OF COMPLEX SITE

3.1. Single clay layer mantled base rock site

The goal of choosing this simple site is avoiding affecting the principal characteristics of soil dynamic constitutive relations in complex site nonlinear response. Similar to soil dynamics test, the artificial double harmonious acceleration history is taken as input as shown in Figure 1. Thickness of clay layer is 8m, the density is 2000kg/m^3, shear velocity is 300m/s; Bedrock shear velocity is 500m/s, density is 2080 kg/m^3, the seismic motion is inputted at position of below bedrock surface 6m, the condition of bottom boundary is 3 nodes 2-order Multi-Transmitting Formula(LIAO Zhen-peng, 2002). The strain-dependent shear modulus and damping ratio parameters of the clay are shown in Table1.1. The consolidation pressure is 1.0kgf/cm^2.

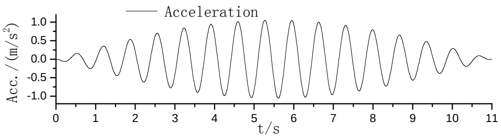

Fig.1 The acceleration history as input

Table3.1 Strain-dependent shear modulus and damping ratio for clay

Strain $\gamma/10^{-4}$	0.05	0.1	0.5	1	5	10	50	100
shear modulus ratio G/G_{max}	0.9847	0.9698	0.8656	0.7631	0.3918	0.2436	0.0605	0.0312
Damping ratio λ/%	0.7240	1.0900	2.6940	3.7960	6.7150	7.6620	8.7360	8.8950

Table3.2 The maximum and minimum of site acceleration response

Position	PGA/(m/s²)	Damping degradation coefficient model	Implicit stress damping equivalent model	Non-Masing rule model ONE	Non-Masing rule model TWO	Pyke method
Bedrock	Maximum	1.80851	1.80800	1.80353	1.80843	1.79750
	Minimum	-1.81366	-1.81279	-1.80878	-1.81341	-1.80212
Middle	Maximum	1.88495	1.87805	1.89707	1.88518	1.91447
	Minimum	-1.89021	-1.88332	-1.90195	-1.89040	-1.92136
The surface	Maximum	1.92023	1.90819	1.94118	1.92053	1.97177
	Minimum	-1.92560	-1.91368	-1.94604	-1.92535	-1.97958

Note: "Middle" means the position of above the surface of bedrock layer 2m.

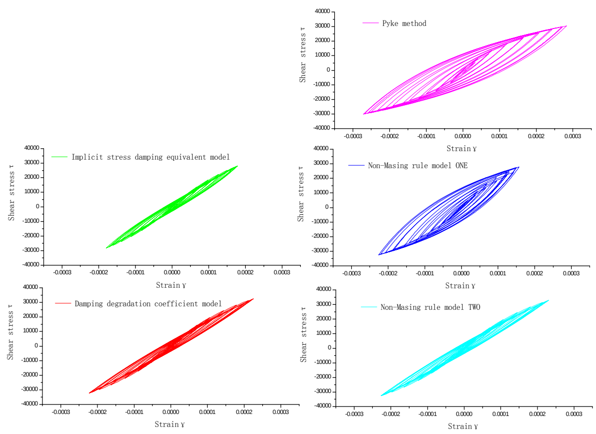

Fig.2 Nonlinear constitutive curves of the first sublayer mantled on base rock of each model

From 5 types of soil dynamic nonlinear constitutive curves and site acceleration responses (Figure 2 and Table3.2), the results indicated that, the response of Pyke method is the biggest, non-Masing rule model ONE's is the second, the damping degeneration coefficient model's and non-Masing rule model TWO's is the third, the response of implicit stress damping equivalent model is the smallest. The response of non-Masing rule model TWO is very consistent with the damping degeneration coefficient model, which showed that the method of "the damping degeneration coefficient" adjusting unloading shear modulus and the method of iterative solving parameters n_1, n_2 that influence unloading shear modulus and the reference strain are the same. Although both function expression are different, its essential meaning and the computed results are completely consistent. Because the non-Masing rule model TWO need to iterate, the calculating efficiency is appreciably lower.

Damping degeneration coefficient model, non-Masing rule model TWO and implicit stress damping equivalent model, although they satisfy symmetrical principle, no exceeding ultimate stress in very big strain, damping equivalent principle, because the equations of mobile skeleton curve of unloading (or reverse load) are different, in addition, the increase explicit step-by-step integration formula in time domain is complex, finally it causes that the shapes of hysteretic constitutive curves are different. The principles of non-Masing rule model ONE and Pyke method are no doubt simple, but they had not considered that the characteristic of soil actual hysteretic damping, and the Pyke method have the question of asymmetrical constitutive curves in constant-amplitude load history. From this case, both cause the computed result to be obviously big. It is suggested that the hysteretic damping ratio of soil test needs to consider in complex nonlinear site response.

Next, we take borehole array observation records as the standard finally, analyze and compare the characteristics of each method.

3.2. Taiwan Lotung DHB borehole array test site

Taiwan Lotung borehole array LSST obtained many strong motion records, according to the research results of Lotung site model parameters and strain-dependent shear modulus and the damping ratio of soil in Huang Huey-chu, Shieh Chie-song, et al. (2001); Borja, Blaise, et al. (2002) about Lotung site research (Figure3, Table3.3, Table3.4). Synthetic seismograms of 5 types of soil dynamic constitutive models are calculated, simultaneously, have carried on the comparison with actual observation records of the DHB borehole array site.

In the Huang Huey-chu, Shieh Chie-song, et al. (2001) paper, the figure of seismic strong motion history underground 47 m at Lotung DHB site is provided(as shown in Figure 8). We digitized seismic strong motion history at -47m and take this digitized record as the input of this site. In computation analysis of this site, rigid boundary condition is assumed. (Space limited, The corresponding displacement history, velocity history, permanent displacement and soil dynamic constitutive hysteretic curves are omitted.)

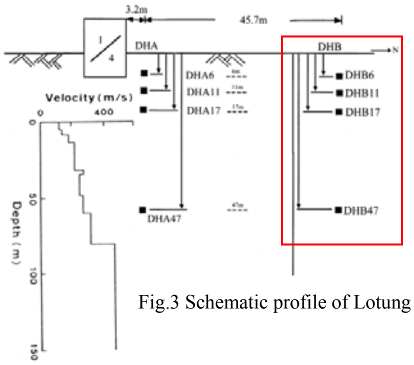

Fig.3 Schematic profile of Lotung DHB site

Table 3.3 Model parameters of Lotung DHB site

Layer No.	Thickness /m	Vs/ m/s	Density / kg/m^3	Soil type
1	5	120	1870	1
2	3	140	1870	1
3	5	190	1870	2
4	18	220	1870	3
5	3	280	1900	4
6	13	250	1900	4

Table3.4 Strain-dependent shear modulus and damping ratio of different soil in Lotung DHB site

Soil type	Strain γ / %	1e-4	3e-4	1e-3	3e-3	0.01	0.03	0.1	0.3	1.0	3.0	10.0
1. Sand (Depth<100m)	shear modulus ratio G/G_{max}	1.00	1.00	0.99	0.96	0.85	0.64	0.37	0.18	0.08	0.05	0.035
	Damping ratio λ /%	0.24	0.42	0.80	1.40	2.80	5.10	9.80	15.50	21.00	25.00	28.00
2. Silty sand (Depth=10m)	shear modulus ratio G/G_{max}	1.000	0.995	0.978	0.923	0.805	0.653	0.413	0.240	0.100	0.060	0.040

	Damping ratio λ /%	1.20	1.21	1.48	1.90	3.15	5.10	10.30	16.00	21.00	25.00	28.00
3. Silty sand (Depth=25m)	**shear modulus ratio G/G_{max}**	1.000	0.995	0.990	0.915	0.770	0.580	0.330	0.150	0.050	0.025	0.020
	Damping ratio λ /%	1.60	1.60	1.60	2.60	5.00	8.80	14.40	18.60	22.00	25.00	28.00
4. Silty clay (Depth=45m)	**shear modulus ratio G/G_{max}**	1.000	1.000	0.995	0.960	0.825	0.610	0.370	0.180	0.080	0.050	0.035
	Damping ratio λ /%	1.96	1.96	1.96	2.31	4.03	8.30	12.50	16.00	20.00	22.70	25.00

Note: Parameters of shear modulus and damping ratio of Sand (Depth<100m) are the experimental achievement of Seed et al. (1984). Silty sand(Depth=10m), Silty sand(Depth=25m) and Silty clay (Depth=25m) have used the KAJIMA\ROSINE experimental achievement.

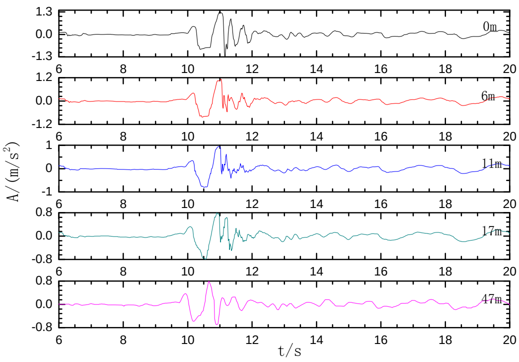

Fig.4 Calculated acceleration response of each layer using damping degradation coefficient model and non-Masing model TWO

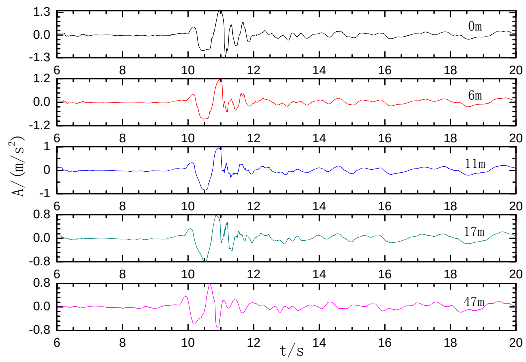

Fig.5 Calculated acceleration response of each layer using implicit stress damping equivalent model

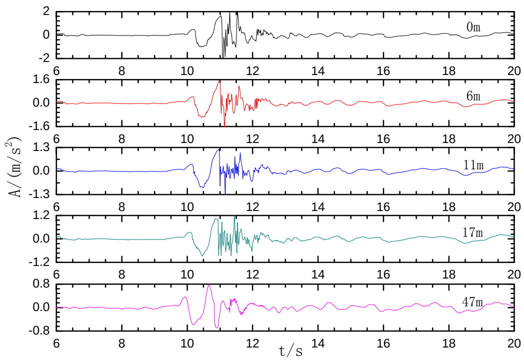

Fig.6 Calculated acceleration response of each layer using Pyke method

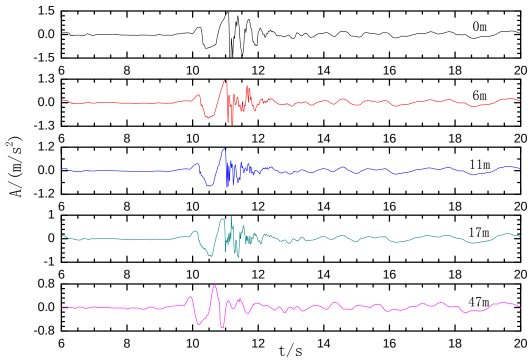

Fig.7 Calculated acceleration response of each layer using non-Masing model ONE

Looking from Figure 5-8, the overall waveforms and its variation characteristic of synthetic acceleration in different depths which are computed by the damping degeneration coefficient model and the non-Masing rule model TWO, that very close to actual earthquake records. Waveform of the surface seismic motion of implicit stress damping equivalent model is much closer to the real records than the damping degeneration coefficient model and the non-Masing rule model TWO. However, the results of these three methods have the question of peak ground acceleration are a little small, the following waveform is flat than the real records. As a result, these three types of models already satisfied for reflecting overall characteristic and the tendency of engineering demand. However, it also needs to improve and to further explore for serious analyzing the mechanism of nonlinear site response and precise revealing the soil nonlinear dynamic performance.

Except that the surface PGA of Pyke method is much close to the real observation, site nonlinear acceleration responses of the Pyke method and non-Masing rule model ONE are bigger than the real records, moreover, the waveform difference is also big, high frequency of soil layer response is richer. Therefore, these two types of models are not suitable for evaluating this site. Because these two types of models are only considered theory hysteretic damping ratio, they are more suitable for analyzing the condition of soil test damping ratio of site nearly soil theory hysteretic damping ratio. Moreover, Pavlenko(2001) and Field(1998) etc. theoretical analyses and the site research indicated that the nonlinear response sometimes will have the branch phenomenon, causing frequency transform and producing the high-frequency component. But regarding high-frequency component of this site, further research is needed.

4. DISCUSSION AND CONCLUSION

The researches of soil dynamic nonlinear stress - strain relationship are developing now. The areas of research needs based on the results of the test, but can not rigidly adhere to soil test, because of many differences of the soil test condition and prototype real engineering site condition. Due to soil complexity and the uncertainty of stress condition, stress history, stress path, soil composition and structure, temperature and so on, the major factors affecting site dynamic response, which are shear modulus degeneration and soil experimental damping ratio, are only considered in nonlinear site response, this research technique is practical and feasible. In this paper, although strong motion input of Lotung DHB site is the digitized acceleration record, the overall strong motion vibration tendency of each model in site response and the differences between synthetic seismograms and observation are still might reflect. Therefore, any soil dynamic nonlinear constitutive model which base on the achievement of soil experiment, is theoretically reasonable, whose simulation result of engineering project close to the fact, are to be worth inquiring and practicing.

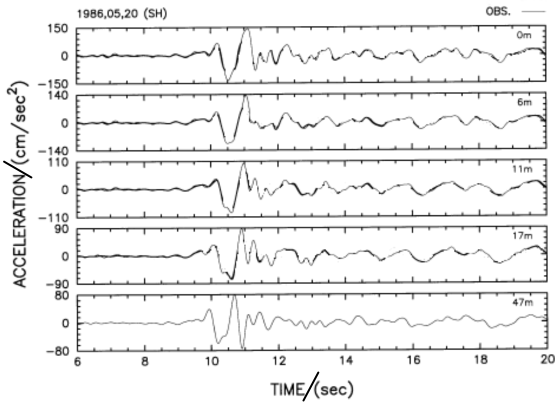

Fig.8 Acceleration records of DHB in Lotung site (From Huang etc.2001)

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