COMPARISON OF FREQUENCY DEPENDENT EQUIVALENT LINEAR ANALYSIS METHODS

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ABSTRACT:

Equivalent linear one dimensional site response analysis, which approximates the nonlinear soil behavior within the linear analysis framework, is widely used in practice due to its simplicity and ease of use. However, the equivalent linear method has been known to be unreliable in case of propagating strong ground motion or in soft soil columns. Various frequency dependent algorithms have been proposed to improve the accuracy of the approximate solution and better simulate the wide range of shear strains within the duration of the seismic loading. However, the effectiveness of the modified schemes has not yet been fully verified. In this study, the results of the modified equivalent linear analyses are compared to evaluate the degree of improvement and the applicability of the modified algorithms. Results show that there is a significant difference between the developed procedures and they do not always provide improved estimates.

KEYWORDS:

Equivalent linear, Frequency dependent, Ground response analysis

1. INTRODUCTION

One-dimensional (1D) seismic site response analysis is routinely performed to characterize the site amplification effects under an earthquake ground motion. 1D analysis, which simulates the vertical propagation of horizontal shear waves through horizontally layered soil profile, has been known to give reasonable estimates of ground vibration under a seismic event (Idriss, 1990).

1D site response analysis is either performed in frequency or time domain. In a nonlinear analysis, which is performed in time domain, the dynamic equation of motion is integrated at each time step and the nonlinear soil behavior is accurately modeled. However, the non-linear site response analysis is not widely used due to difficulty in performing the analysis and high computational cost. Instead, the equivalent linear analysis, which approximates the nonlinear soil behavior within the linear framework, is widely used in engineering practice (Schnabel et al., 1972). The main limitation of the procedure is that a constant linear shear modulus and damping at a representative level of strain is used throughout the analysis.

Frequency-dependent equivalent linear algorithms have been proposed to overcome the limitation and better simulate the nonlinear hysteretic soil response under the seismic loading (e.g. Sugito et al., 1994; Yoshida et al., 2002; Kausel and Assimaki, 2002). It has been shown that the frequency-dependent equivalent linear methods result in improved estimates of the ground motion propagation. However, it is still now clearly understood on how the distinctively different schemes all result in improved solution, and whether the accuracy of the solution always increases universally for all cases. This study compares the computed responses using the schemes by Yoshida et al. (2002) and Kausel and Assimaki (2002) with those from equivalent linear and nonlinear analyses to evaluate whether the degree of improvement of the frequency-dependent procedures over a standard equivalent linear scheme.

2. FREQUENCY-DEPENDENT EQUIVALENT LINEAR ALGORITHM

Figure 1 shows a schematic plot of hysteretic loops under large (A) and small (B) shear strain amplitudes, respectively. Assuming that the strain increment at each time step is similar for both loops, the frequency of vibration of a large amplitude hysteretic loop will be lower than for a small amplitude loop. This indicates that the frequency of vibration is associated with the amplitude of shear strain. Figure 2, which shows the Fourier spectra of two shear strain time histories, demonstrates the strong dependence of the shear strain amplitude on the frequency. The amplitude of the shear strain decays quickly with increase in frequency. Figure 1 and Figure 2 show that the frequency-dependence of shear strain is a distinct phenomenon that needs to be incorporated in a site response analysis. Standard equivalent linear procedure ignores such dependence and applies constant shear modulus and damping throughout entire frequency range.

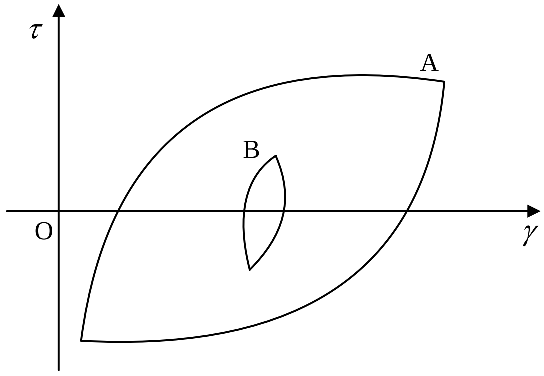

Figure 1 Comparison of hysteretic loops of large(A) and small(B) strain amplitudes (modified after Yoshida et al., 2002)

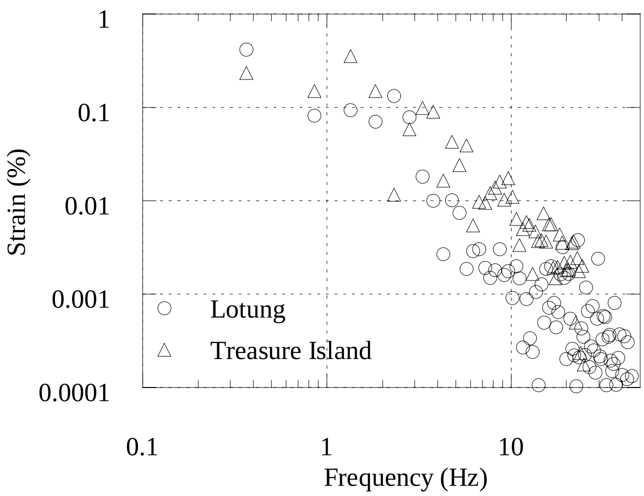

Figure 2 Frequency-dependence of shear strain Fourier spectra

The frequency-dependent equivalent linear procedures propose use of the shear strain Fourier spectrum, such as shown in Figure 2, in defying the frequency dependence of the shear strain in a site response analysis in characterizing. However, the direct use of the Fourier spectrum in a site response analysis can lead to numerical instability (Kausel and Assimaki, 2002) and hence, the Fourier spectrum is always smoothed. The difference in the frequency-dependent algorithms lie in how the Fourier spectrum is smoothed.

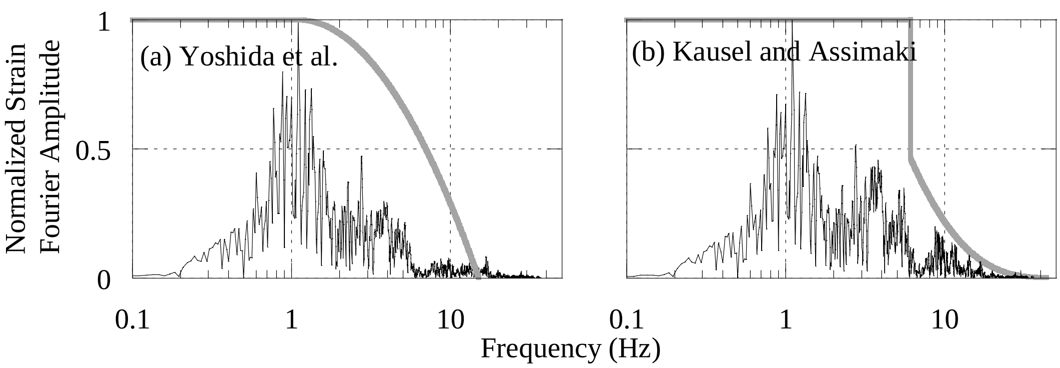

Figure 3 Smoothed Curves

Yoshida et al. (2002) proposed the following equation to characterize the smoothed frequency-dependent shear strain:

$$\begin{cases} \gamma_{eff} = \gamma_{\max} & f_p > f \\ \gamma_{eff} = \gamma_{\max}\left\{1 - \left(\dfrac{\log f - \log f_p}{\log f_e - \log f_p}\right)^m\right\} & f_p \le f \le f_e \\ \gamma_{eff} = 0 & f < f_e \end{cases} \tag{2.1}$$

where f_p = frequency at $\gamma_{\max}$, f_e = minimum frequency, m = constant. Yoshida et al., 2002 used f_e = 15 Hz and m = 2.

Kausel and Assimaki (2002) proposed the following equation:

$$\left|\frac{\gamma(\omega)}{\gamma_0}\right| = \begin{cases} 1 & \omega \le \omega_0 \\ \dfrac{\exp\left(-\alpha \dfrac{\omega}{\omega_0}\right)}{\left(\dfrac{\omega}{\omega_0}\right)^{\beta}} & \omega > \omega_0 \end{cases} \tag{2.2}$$

where ω_0 and γ_0 are defined as follows:

$$\omega_0 = \frac{\int_0^\infty \omega r(\omega)d\omega}{\int_0^\infty r(\omega)d\omega} \tag{2.3}$$

$$r_0 = \frac{1}{\omega_0}\int_0^{\omega_0} r(\omega)d\omega \tag{2.4}$$

α and β are constants that are determined by least square method. The smoothed curve is constant up to ω_0, and decreases at higher frequencies, as shown in Figure 3.

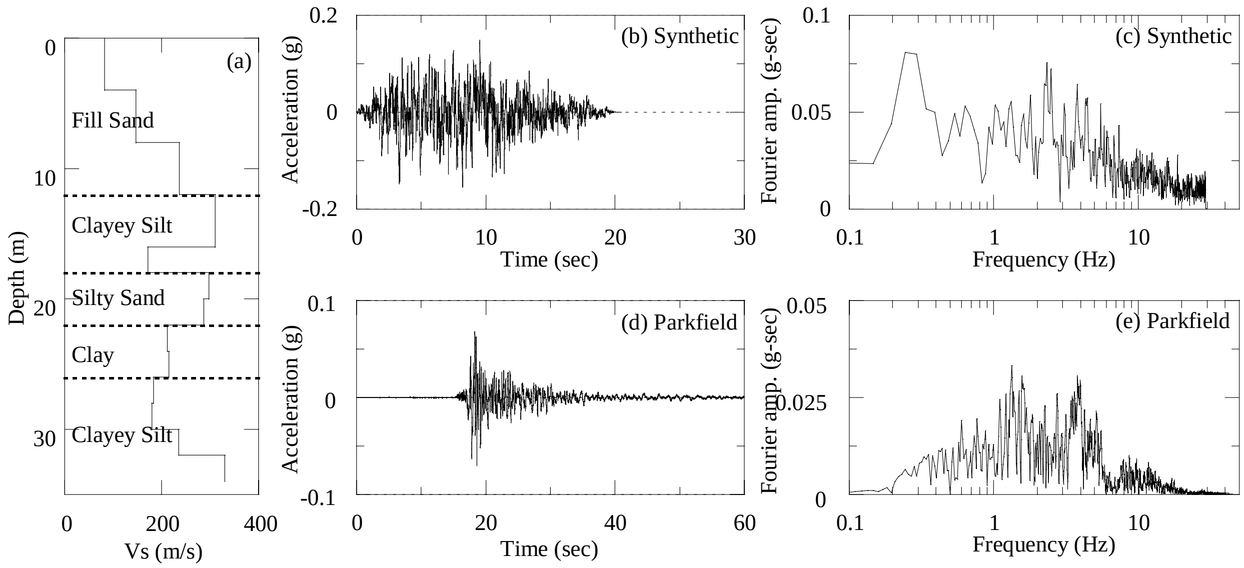

Figure 4 Shear wave velocity profile, Input motion acceleration time histories and Fourier spectrum

Yoshida et al. (2002) showed that the frequency-dependent algorithm results in lower peak ground acceleration (PGA) compared to the standard equivalent linear anlaysis, since maximum shear strain is applied up to f_p

and hence, resulted in a softer response. Kausel and Assimaki (2002) showed that the frequency-dependent algorithm resulted in improved match with the nonlinear solution, since low energy dissipation at high frequencies is modeled.

3. VERIFICATION OF ACCURACY OF FREQUENCY-DEPENDENT ALGORITHMS

This study performed a series of standard equivalent linear (EQL), nonlinear (NL), and frequency-dependent equivalent-linear analysis (FDEQL) at a selected site, as shown in Figure 4. The profile is 34 m in thickness, and is composed of layers of sands, silts, and clays. The curves by Seed and Idriss (1970) are used for sands, while the curves by Vucetic and Dobry (1991) are used for silts and clays. The shear wave velocity of the bedrock is assumed to be 760 m/sec. Two input motions were used: a synthetic motion and a recorded motion during Parkfield earthquake (M = 6.0, U.S.A.), also shown in Figure 4. The synthetic motion is rich in high frequency components, while the recorded motion displays strong energy concentration between 1 – 10 Hz and low high frequency content. All analyses are performed by newly developed 1D site response analysis program GeoSHAKE.

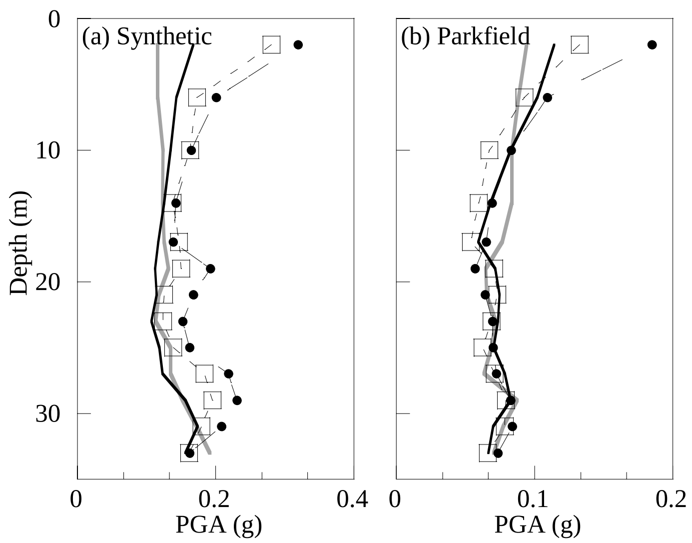

Figure 5 PGA profiles

Figure 5 to Figure 8 show the calculated responses. The PGA profiles calculated from all analyses show that the FDEQL resulted in higher estimates compared to the EQL and NL. The results contrast the findings of Yoshida et al. (2002), which showed that the FDEQL results in lower response than the EQL. Figure 6 shows the computed shear strain – stress relationships extracted from the surface layer using the synthetic motion. The grey lines represent the backbone curves. EQL, which uses the effective shear strain representative of 65% of maximum shear strain, showed stiffer response at maximum shear strain compared to the backbone-curve. The mean secant stiffness of the FDEQL compares better with the backbone curve. However, the application of lower shear strain amplitude for high frequencies resulted in highly irregular stress strain relationships. This phenomenon is especially evident using the equations by Kausel and Assimaki (2002), Figure 6d. Figure 7 shows the calculated shear strain Fourier spectra and the smoothed curves at the surface layer. EQL resulted in lowest spectrum, especially at high frequencies, due to overestimation of damping at high frequencies. On the contrary, the FDEQL resulted in higher response than the NL at high frequencies. The equations by Yoshida et al. (2002) applies minimum shear strain at frequencies higher than 15Hz, and hence abrupt amplification of the motion can be observed. Kausel and Assimaki (2002)'s equations applies low shear strain at frequencies higher than 5 Hz, and therefore highly overestimates the response.

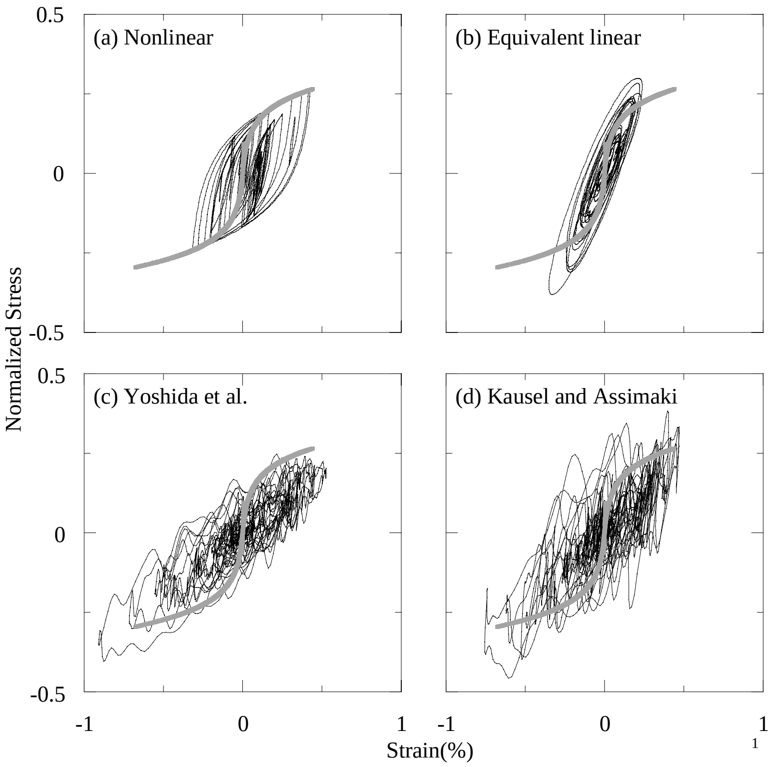

Figure 6 Stress – strain relationship and skeletons

Figure 8 shows the computed 5% damped acceleration response spectra at the surface using both input ground motions. Using the synthetic motion, rich in high frequency contents, the FDEQL highly overestimated the response compared to the EQL or NL. The unrealistic amplification at short period range between 0.01 – 0.4 sec can be observed. When using the recorded motion, use of the equations by Yoshida et al. (2002) resulted in a response similar to the EQL. Application of the equations by Kausel and Assimaki (2002) resulted in significant overestimation of the computed response.

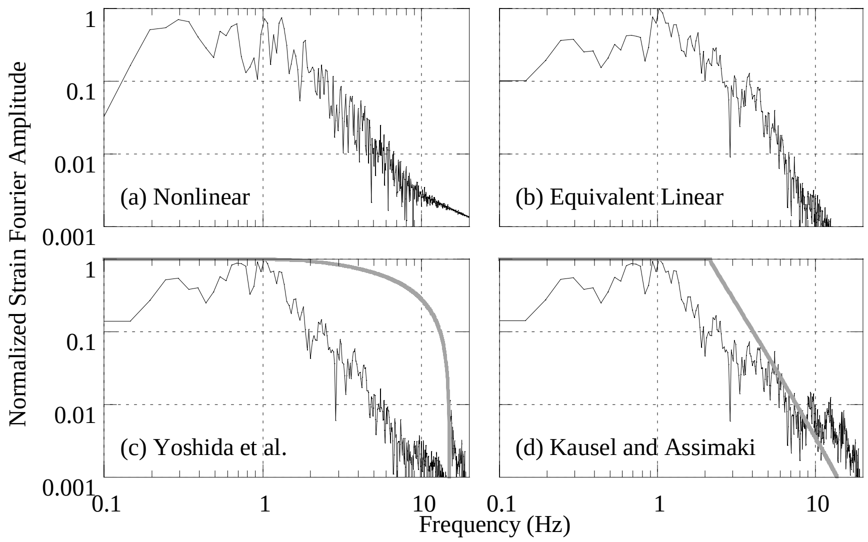

Figure 7 Normalized strain Fourier spectrum and smoothed curves

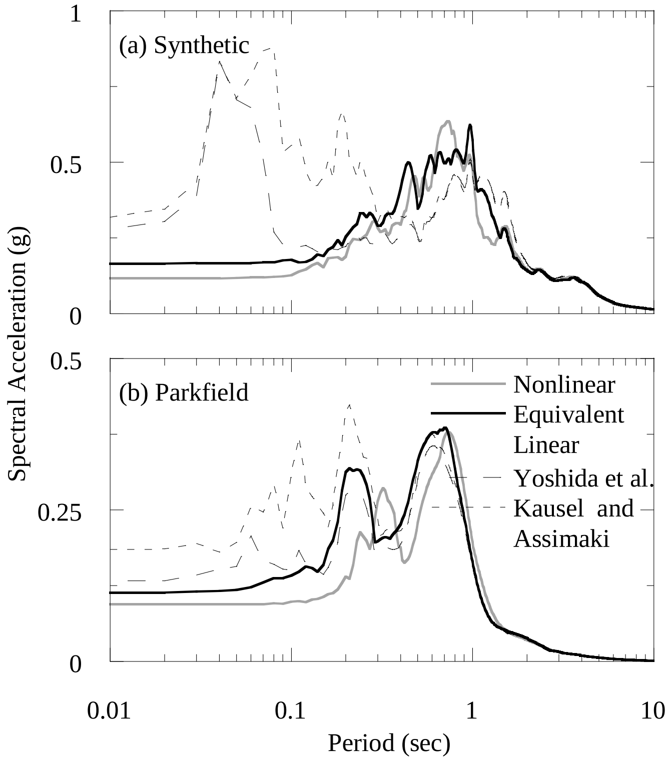

Figure 8 5% damped surface acceleration response spectrum

4. CONCLUSION

1D equivalent linear analysis is widely used in practice due to ease of use and low computational cost. The main deficiency of the procedure is constant application of the shear modulus and damping throughout the analysis selected at the effective shear strain. The equivalent linear analysis can overestimate the peak ground acceleration due to overestimation of the stiffness at maximum shear strain. Frequency-dependent equivalent linear procedures were proposed to overcome such limitations and better simulate the nonlinear soil behavior. The procedure is based on the observation that large strain amplitude is associated with large hysteretic loops and hence low frequency, while small strain amplitudes typically result in high frequency vibration. Such observation led to development of equations of frequency-dependent shear strain. The equations are essentially smoothed curves of shear strain Fourier spectrum. In this study, two equations that relate the frequency with the shear strain are implemented in an equivalent linear analysis and the accuracies of the equations were tested.

The results of the analyses showed that the results using two equations are highly different. It is demonstrated that the procedure of smoothing can significantly influence the propagated ground motion. At the soil profile selected in this study, the frequency-dependent equivalent linear analysis showed higher responses compared to both the equivalent linear and nonlinear analyses. The frequency-dependent procedure can especially result in unrealistic amplification of low period components using a ground motion rich in high frequency contents. The test analyses showed that the frequency dependent algorithm does not always improve the accuracy of the solution. More in-depth study is warranted to characterize the accuracy and applicability of the frequency-dependent procedure.

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