POROELASTIC STOCHASTIC MULTILAYERED SOIL PROFILE AMPLIFICATION

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ABSTRACT :

In this paper, dynamic response of a multi-layered poroelastic soil with stochastic parameters is studied through an exact stiffness matrix formulation based on Biot's theory. Shear modulus as well as permeability and saturation degree are simulated via Monte Carlo technique. Whereas shear modulus follows a log normal distribution, porosity and saturation degree simulations are done using a beta distribution. Some poroelastic multilayered soil profile amplifications as well as a parametric study are presented in order to analyze the effect of the parameters stochasticity on the site response. The results show that the effects of shear modulus and saturation degree variations are as significant as those of the porosity in the case of inclined P_1 or SV wave incidence.

KEYWORDS: Porous media, Simulation, Wave propagation, Earthquake, Stochasticity

1. INTRODUCTION

The soil is a porous media by its own nature and its porosity becomes more significant in the case of completely or partially water pore filled strata. The well known Biot's theory, which revealed the existence of a second compressive wave propagating through a porous media, has been extensively used during this last five decades in several formulation treating the soil dynamic response to ground shaking. Some of these formulations were purely analytical (Bo and Hua 2000, Rajapakse and Senjuntichai 1995, Simon *et al* 1984, Tabatabaie *et al* 1994) whereas others were numerical like the finite elements method (Akiyoshi *et al* 1998, Tabatabaie *et al* 1994, Zienkiewicz and Shiomi 1984) , the thin layer elements method (Nogami and Kazama 1992) and the boundary elements method (Dominguez 1993).
In earlier studies (Mehiaoui and Hadid 2003, Mehiaoui and hadid 2005), amplification functions of multilayered poroelastic soil profile have been computed for various geomechanic parameters. The results revealed that whatever is the soil behaviour i.e. linear or nonlinear; some parameters are more influent than other ones.
In this paper, shear modulus, porosity and saturation degree are simulated via Monte Carlo technique in order to analyze the effect of their stochasticity on the site response. Whereas shear modulus follows a log normal distribution, permeability and saturation degree simulations are done using a beta distribution. The soil profile is modelled as a two-phase porous medium system consisting of a viscoelastic skeleton and an incompressible fluid phase. The soil nonlinearity is taken into account via the viscoelastic linear equivalent model which is a well recognized model. Elsewhere, the input accelerations are carried out using the shinuzuka technique (Shinuzuka et al 1987) and are compiled from the 2003 Boumerdès earthquake.
An analytical approach based on the exact stiffness matrix method (Rajapakse & Senjuntichai 1995) is used herein in order to study the dynamic response of a multi-layered poroelastic medium to wave propagation. Amplification function and response spectrum are calculated for various coefficient of variation of shear modulus, saturation degree and porosity.

2. EQUATIONS OF MOTION AND GENERAL SOLUTION

Biot (1962) gives the equations governing the dynamic behaviour of porous media. The equations of motion can be written as below (Yang et al 1998)

$$\mu\nabla^2 u + (\lambda + \alpha^2 M + \mu)\nabla e - \alpha M \nabla\zeta = \rho\ddot{u} + \rho_f \ddot{w} \tag{2.1}$$

$$\alpha M \nabla e - M\nabla\zeta = \rho_f \ddot{u} + \frac{\rho_f}{n}\ddot{w} + \frac{\eta}{k}\dot{w} \tag{2.2}$$

where u and w are, respectively, the displacement vectors of solid skeleton and pore fluid with respect to solid phase; e=divu and ζ=-divw denote, respectively, the volumetric strain of the solid skeleton and the increment of fluid content; η is fluid viscosity and k is permeability (with the unit m²); ρ is total density and ρf is the density of pore fluid; λ and μ are Lame constants of solid skeleton; α and M are Biot (1962) constants which are accounting for the compressibility of grains and fluid.
Using Helmholtz representation for a vector field and applying Fourier integral transform with respect to the x-coordinate to equations (2.1) and (2.2), the general solutions (Rajapakse et al 1995) to the problem of wave propagation in poroelastic media are expressed in the frequency domain in the following matrix form

$$v(k,z) = R(k,z)\, Y(k) \tag{2.3}$$

$$f(k,z) = S(k,z)\, Y(k) \tag{2.4}$$

with k the wave number and

$$v(k,z) = \{ iu_x, u_z, P\}^T ; f(k,z) = \{ i\tau_{xz}, \sigma_z, w_z\}^T ; Y(k) = \left\{ A_{p_1}, B_{p_1}, A_{p_2}, B_{p_2}, C_{SV}, D_{SV} \right\}^T \tag{2.5}$$

A_{pi} and B_{pi} i=1, 2 are the reflected and refracted wave potentials associated, respectively, with P_1 and P_2 waves. C_{SV} and D_{SV} are SV wave associated potentials (Rajapakse et al 1995).
Let us consider a N-layered soil profile resting on a poroelastic half-space. Using the equations (2.3) and (2.4), the displacements and stress fields can be expressed for each layer. Inversing the matrix R, the potential vector can be expressed in function of displacement vector. The substitution of this expression into the relation (2.4) for each layer, leads (Rajapakse et al 1995)

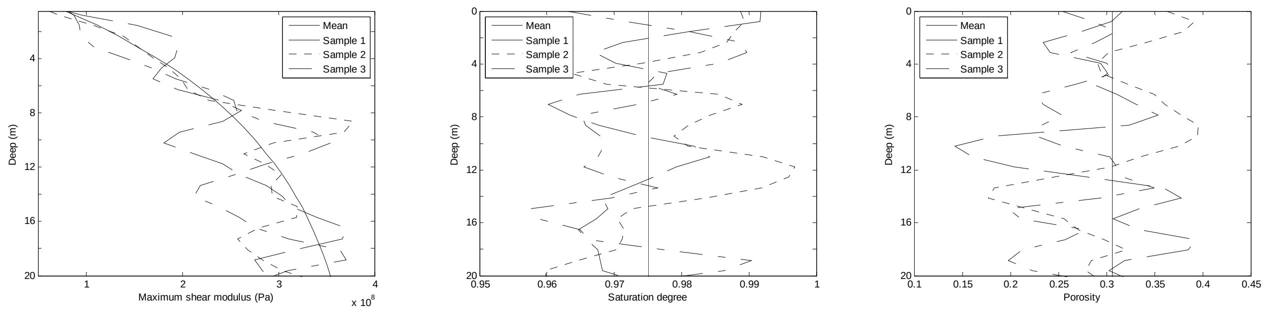
$$\sigma^{(n)} = K^{(n)} u^{(n)}, \quad n = 1, 2, \ldots, N \tag{2.6}$$

$K^{(n)}$ is the stiffness matrix describing the relationship between the generalized displacement vector $u^{(n)}$ and the force vector $\sigma^{(n)}$ for the n^{th} layer. The explicit expression of the stiffness matrix for a multilayered soil-half-space system is given by Rajapakse et al (1995).

3. SOIL PROPERTIES SIMULATION

The maximum shear modulus random field is compiled using the probability function of the lognormal distribution. Once obtained, these realisations are used in the viscoelastic linear equivalent model.
Because the porosity is bounded in practice between two extreme values, its random field is obtained using the Beta distribution. Likewise, the saturation degree is bounded herein between 95 and 100% and hence is obtained using the same distribution as the porosity.
In this study, the Beta field is obtained by a mapping technique on the probability distribution function diagram, and by solving a non-linear equation. However, mean and variance are unchanged through the mapping operation. Because fraction porosity and saturation degree are positive parameter, one prefers to perform the mapping operation via the probability function of the lognormal distribution (Nour et al 2003).

The figure 1 shows some typical realisation of maximum shear modulus, saturation degree and porosity for the respective following coefficients of variation: 0.2 ; 0.01 and 0.2. The vertical correlation length is equal to 1.5 m for all soil characteristics parameters simulation.



(a) (b) (c)

Figure 1 Typical realisation of (a) maximum shear modulus C_{vG} = 0.2 (b) saturation degree C_{vSr} = 0.01 and (c) porosity C_{vn}

4. SEISMIC GROUNG MOTION

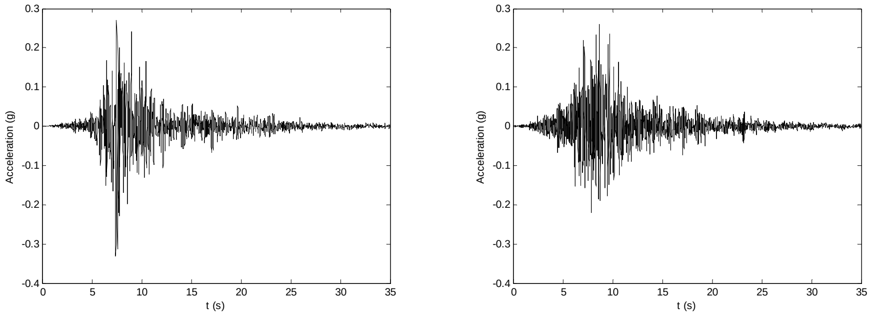
The earthquake ground motion and the resulting response are nonstationary stochastic processes. Because seismic ground motion exhibits a beginning and an ending, it cannot be truly stationary, even thought, for practical purposes, it is assumed stationary for the majority of its duration. Also as stated in Gupta et al (1998) even for a stationary input acceleration, the structural response is also nonstationary in nature. Nonstationary of motion shows three stages (i) the motion increases rapidly from weak to strong (ii) the motion maintains its average strength (iii) the motion gradually decreases. The model considering these features may be generally written as suggested by Amin and Ang [17] as follows:

$$\ddot{x}_g(t) = A(t)\ddot{x}_{sta}(t) \tag{4.1}$$

with

$$A(t) = \begin{cases} (t/t_1)^2 & 0 \le t \le t_1 = 3s \\ 1 & t_1 < t \le t_2 = 13s \\ \exp(0.6(t - t_2)) & t_2 < t \le t_3 = 20.48s \end{cases} \tag{4.2}$$

A(t) is a deterministic envelope function which is modulating a uniformly stationary process. The stationary process $\ddot{x}_{sta}(t)$ is simulated as recommended by shinozuka et al (1987). For this study, the power spectral density function is calculated from a real seismic ground acceleration records namely the Boumerdès (Algeria) earthquake that occurred in may 21st 2003 (figure 2). Figure 3 shows a typical realisation of the ground motion simulation. The PGA is fixed to 0.2g.



(a) (b)

Figure 2 Boumerdès (Algeria) earthquake records (may 21st 2003) (a) Est-West component (b) North-South component

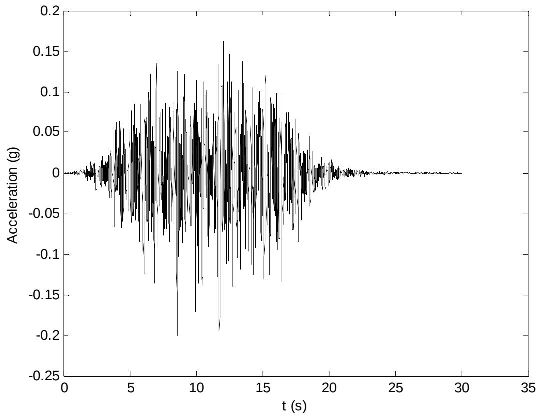


Figure 3 Typical seismic ground simulation (PGA=0.2g)

5. APPLICATION

Some poroelastic multilayered soil profile amplifications as well as a parametric study are presented hereafter in order to analyze the effect of the parameters stochasticity on the site response. For this purpose, a soft stochastic saturated single soil layer overlaying a half space is considered (see Figure 4).

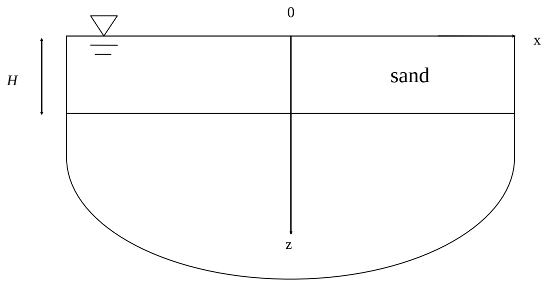


Figure 4 Geometry of the soil profile

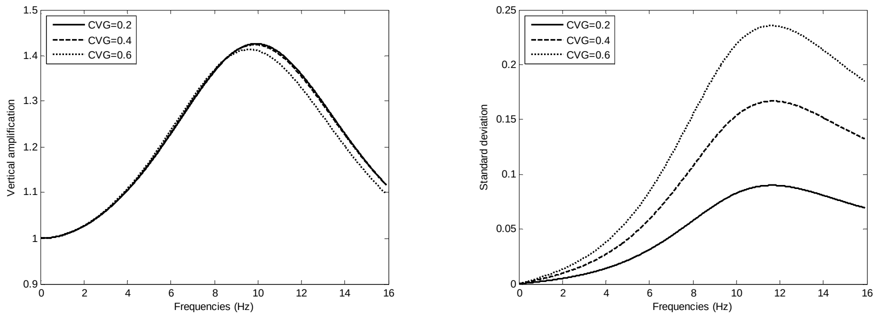
The geomechanic characteristics of the sand and the half space are summarized in table 5.1.

Table 5.1 Characteristics of soil layers

Characteristics	Sand	Half space(Soft rock)
Solid grains density (kg/m3)	2600	2610
Bulk modulus of solid skeleton (Mpa)	83.33	1300
Bulk modulus of solid grains (Gpa)	36	36
Bulk modulus of pore water (Gpa)	2.2	2.2
Fluid viscosity (Ns/m²)	10^{-3}	10^{-3}
Permeability (m²)	10^{-11}	10^{-12}
Dumping (%)	5	0

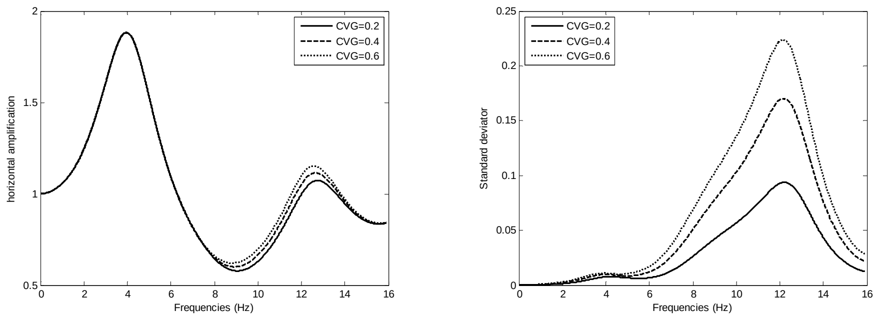
5.1 Shear modulus stochasticity influence

The maximum shear modulus coefficients of variation are varied in order to investigate its effects on the soil amplification (fig. 5 and 6). It appears that this stochasticity has a significant effect on the soil layers amplification standard deviator in both P_1 and SV wave incidence case.



(a) (b)

Figure 5 Influence of shear modulus stochasticity (a) mean amplification (b) standard deviator – Incidence of vertical P_1 wave

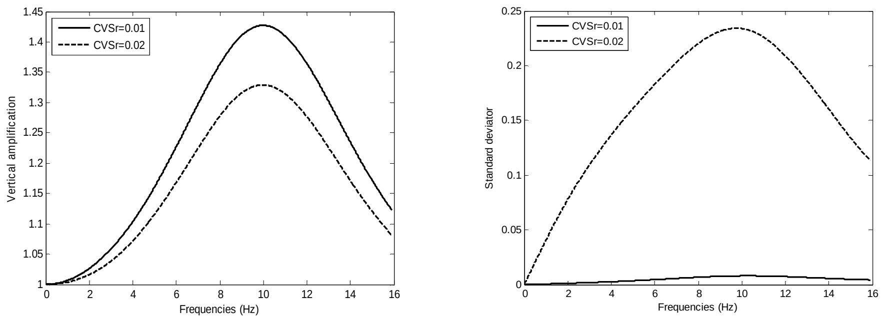


(a) (b)

Figure 6 Influence of shear modulus stochasticity (a) mean amplification (b) standard deviator – Incidence of 20° inclined SV wave

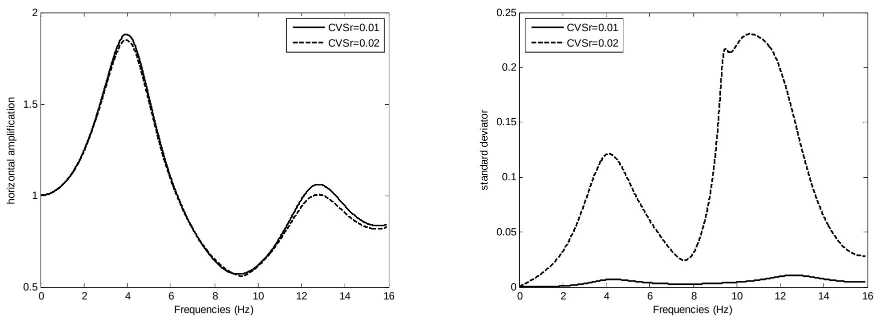
5.2 Saturation degree stochasticity influence

The saturation degree coefficients of variation are varied in order to investigate its effects on the soil amplification (fig. 7 and 8). It appears that this stochasticity has a significant effect on the soil layers amplification and particularly on its standard deviator. This is true in both case of P_1 and SV wave incidence. Earlier studies (Yang and Sato 1998, Yang 2000, Mehiaoui and Hadid 2003, 2005) has pointed that the saturation degree variation has a significant influence on the soil response since it varies slightly from 100%. The figures 7 and 8 show that for coefficients of variation as small as 0.01 and 0.02, the soil amplification standard deviator vary considerably.



(a) (b)

Figure 7 Influence of saturation degree stochasticity (a) mean amplification (b) standard deviator – Incidence of vertical P_1 wave

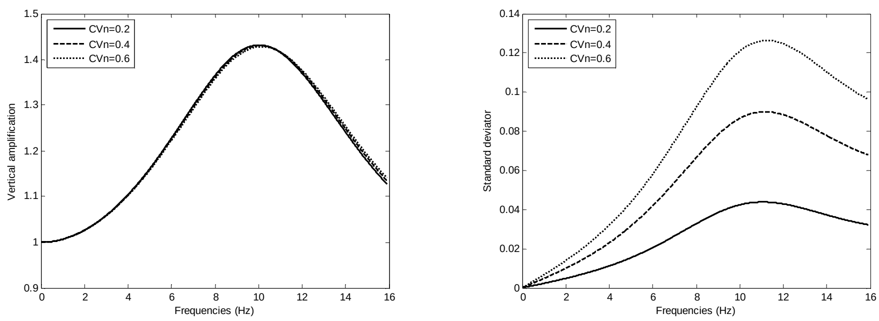


(a) (b)

Figure 8 Influence of saturation degree stochasticity (a) mean amplification (b) standard deviator – Incidence of 20° inclined SV wave

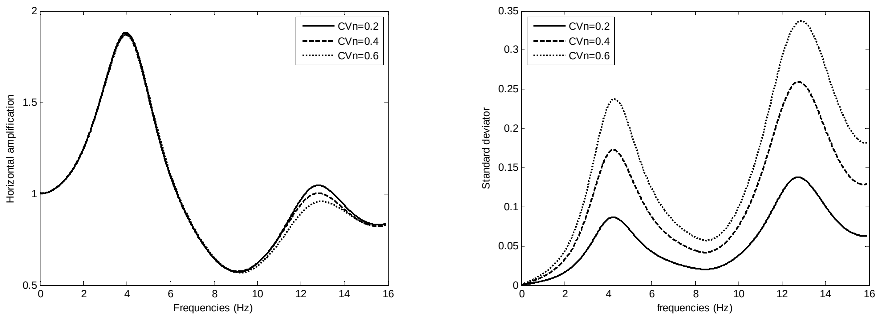
5.3 Porosity stochasticity influence

The porosity coefficients of variation are varied between 0.2 and 0.6. This stochasticity seems to affect the amplification standard deviator in the case of inclined SV wave more than in the case of vertical P_1 wave.



(a) (b)

Figure 9 Influence of porosity stochasticity (a) mean amplification (b) standard deviator – Incidence of vertical P_1 wave



(a) (b)

Figure 10 Influence of porosity stochasticity (a) mean amplification (b) standard deviator – Incidence of 20° inclined SV wave

6. CONCLUSION

In this paper, dynamic response of a multi-layered poroelastic soil with stochastic parameters is studied through an exact stiffness matrix formulation based on Biot’s theory. Shear modulus as well as permeability and saturation degree are simulated via Monte Carlo technique. Whereas shear modulus follows a log normal distribution, permeability and saturation degree simulations are done using a beta distribution. The soil profile is modelled as a two-phase porous medium system consisting of a viscoelastic skeleton and an incompressible fluid phase. The soil nonlinearity is taken into account via the viscoelastic linear equivalent model which is a well recognized model. Elsewhere, the input accelerations are carried out using the shinuzuka technique (Shinuzuka et al 1987) and are compiled from the 2003 Boumerdès earthquake records.
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