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STOCHASTIC GROUND MOTION MODEL WITH TIME-VARYING INTENSITY, FREQUENCY AND BANDWIDTH CHARACTERISTICS

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ABSTRACT:

A recently developed nonstationary stochastic model of strong earthquake ground motion is described. The model employs filtering of a white-noise process, where nonstationarity is achieved by modulating the intensity and varying the filter parameters in time. The model is fitted to a target accelerogram by matching a set of statistical measures that characterize the evolution of the intensity, predominant frequency and bandwidth of the record. For performance-based earthquake engineering (PBEE) analysis, the model parameters are identified for a dataset of strong ground motion records, where each accelerogram is treated as a target ground motion with known earthquake and site characteristics. Statistical data analysis is then performed on the identified model parameters to investigate relations that will help predict the model parameters for a given set of earthquake and site characteristics. Preliminary results from this analysis, including fitted distributions of the model parameters, are the focus of this paper. Once these relations are fully developed, simulation algorithms will be used to generate an ensemble of artificial ground motion records for a given set of earthquake and site characteristics. Such simulated motions can be used in place of actually recorded accelerograms in PBEE analysis, thus avoiding the difficult and often questionable tasks of selecting and scaling ground motions recorded at different sites for a single project site.

KEYWORDS: accelerogram, earthquake ground motion, nonstationary process, simulation, stochastic model

1. INTRODUCTION

Stochastic models for characterization and simulation of earthquake ground motions are useful in the field of earthquake engineering. These models can be directly employed for probabilistic assessment of seismic demand by random vibration analysis [1], or they can be used to generate simulated ground motions for the evaluation of seismic demand on structures, foundations and soils by time-history dynamic analysis [2]. A recent paper by the first two authors (Rezaeian and Der Kiureghian [3]) provides a review of the literature on this topic.

A good stochastic ground motion model must be able to represent both the temporal and the spectral nonstationarity features of an earthquake. Whereas temporal nonstationarity can be easily modeled by modulating a stationary process over time, spectral nonstationarity is not so easy to model. Nevertheless, the spectral nonstationarity is of importance in nonlinear response analysis because of the moving resonant effect of nonlinear structures. For a stochastic ground motion model to be of practical use, it should be parsimonious and it should provide physical insight with its parameters relating to the characteristics of the earthquake and the site under consideration. Such a stochastic ground motion model can be especially useful in the current practice of PBEE, because it will enable engineers to generate artificial samples of ground motions for specified earthquake and site characteristics, thus avoiding dependence on actually recorded accelerograms that can be unavailable for the region and site of interest. This approach also avoids the controversial task of selecting ground motions recorded at various locations and modifying them to represent the seismic hazard of a project site, a task which in some cases might result in motions with unrealistic characteristics.

This paper considers a newly developed stochastic model for strong earthquake ground motion that is fully described in [3]. The model has a small number of parameters that can be related to the physical characteristics of an accelerogram, such as the variation of intensity and frequency content in time. One important advantage of the model is that separate parameters control the temporal and spectral nonstationarities of the process, thus simplifying parameter estimation. The model has a discrete form, which facilitates simulation of artificial records as well as nonlinear random vibration analysis by the tail-equivalent linearization method [4].

Many stochastic ground motion models have been proposed, but the selection of an appropriate set of model parameters for a given earthquake and site of interest remains unresolved. This issue can be addressed by constructing correlations between the model parameters and the earthquake and site characteristics. Such predictive relationships are created by statistically analyzing a dataset of the identified model parameters for a large number of recorded ground motions with known earthquake and site characteristics. This paper is a first attempt in that direction.

The paper begins with a brief summary of the stochastic ground motion model. The methodology of the statistical data analysis to be performed on the model parameters is explained in detail. A database of strong ground motions is selected and optimization schemes are proposed to estimate the parameters of the model that is fitted to each motion in the database. Preliminary results of the statistical analysis, including fitted distributions to the various model parameters, are presented.

2. STOCHASTIC GROUND MOTION MODEL

It is well known that earthquake ground motions have nonstationary characteristics both in time and frequency domains. The temporal nonstationarity refers to the variation of the intensity of the ground motion in time and can be easily modeled by any time-varying function that gradually increases from zero to achieve a nearly constant intensity, representing the strong-shaking phase of an earthquake, and then gradually decays back to zero. The spectral nonstationarity refers to the variation of the frequency content of the motion in time, which arises from the evolving nature of the seismic waves arriving at a site. This evolving frequency content of the ground motion must be modeled accurately since it can be critical to the response of degrading structures, which have resonant frequencies that tend to decay with time as the structure responds to the excitation.

A stochastic model for strong earthquake ground motions that accounts for both types of nonstationarities has been proposed by the first two authors in [3]. This model has the important advantage of separating the temporal and spectral nonstationary characteristics of the process and it can be written in a discrete format. The ability to separate the temporal and spectral nonstationarities allows flexibility and ease in modeling and parameter estimation.

The model for the ground acceleration is obtained by time-modulating a filtered white-noise process with the filter having time-varying parameters. In the continuous form, the model is defined by

$$x(t) = q(t)\left\{\frac{1}{\sigma_h(t)}\int_{-\infty}^{t} h[t-\tau, \boldsymbol{\theta}(\tau)]w(\tau)d\tau\right\} \tag{2.1}$$

In this expression, $q(t)$ is the (deterministic, non-negative) modulating function that also represents the standard deviation of the process; the expression inside the curved brackets is a unit-variance filtered white-noise process, where $h[t-\tau,\boldsymbol{\theta}(\tau)]$ denotes the impulse-response function (IRF) of a linear filter with time-varying parameters $\boldsymbol{\theta}(\tau)$; $w(\tau)$ denotes a white-noise process, and $\sigma_h(t)=\{\int_{-\infty}^{t} h^2[t-\tau,\boldsymbol{\theta}(\tau)]d\tau\}^{1/2}$ is the standard deviation of the process defined by the integral inside the curved brackets. Thus, $q(t)$ completely defines the temporal nonstationarity of the process, while the form of the filter IRF and its time-varying parameters completely define the spectral nonstationarity of the process.

In order to facilitate digital simulation, the stochastic model in (2.1) is discretized in the time domain into the form

$$\hat{x}(t)=q(t)\sum_{i=1}^{n}s_i(t)u_i \tag{2.2}$$

where $s_i(t)$, $i=1,\ldots,n$, are a set of deterministic basis functions controlling the frequency content of the excitation and u_i, $i=1,\ldots,n$, are a set of standard normal random variables representing random pulses at equally spaced time points t_i, $i=1,\ldots,n$, with t_n denoting the total duration of the motion. The basis functions are expressed in terms of the IRF of the filter and for $1\le k\le n-1$ take the form

$$\begin{aligned} s_i(t) &= \frac{h[t-t_i,\boldsymbol{\theta}(t_i)]}{\sqrt{\sum_{j=1}^{k} h^2[t-t_j,\boldsymbol{\theta}(t_j)]}} && t_k\le t<t_{k+1},\quad 0<i\le k \\ &= 0 && t<t_i \end{aligned} \tag{2.3}$$

The model described in Eqn. 2.1-2.3 is completely defined by specifying the forms and parameters of the modulating and IRF functions. For the current study, the “gamma” modulating function is selected:

$$q(t,\boldsymbol{\alpha})=\alpha_1 t^{\alpha_2-1}\exp(-\alpha_3 t) \tag{2.4}$$

The parameters $\boldsymbol{\alpha}=(\alpha_1,\alpha_2,\alpha_3)$ shape the modulating function and control its intensity. In the frequency domain, the properties of the process are influenced by the selection of the filter and its time-varying parameters. The parameters $\boldsymbol{\theta}(\tau)$ are typically the time-varying filter frequency, $\omega_f(\tau)$, and damping ratio, $\zeta_f(\tau)$, which respectively control the evolutionary predominant frequency and bandwidth of the process. In this study, we have selected an IRF that corresponds to the pseudo-acceleration response of a single-degree-of-freedom linear oscillator. As a simple approximation, and based on analysis of a large number of accelerograms, we have adopted a linearly decreasing function for the filter frequency and a constant value for the filter damping ratio. Thus,

$$\omega_f(\tau)=\omega_{Mid}+\omega_{Slope}(\tau-t_{Mid}) \tag{2.5a}$$
$$\zeta_f(\tau)=\zeta_f \tag{2.5b}$$

where t_{Mid} and ω_{Mid} represent the time and the filter frequency at the middle of the strong shaking phase of the ground motion, and ω_{Slope} represents the rate of change of the filter frequency over time. Thus, the three parameters $\boldsymbol{\theta}=(\omega_{Mid},\omega_{Slope},\zeta_f)$ completely define the evolutionary frequency content of the process.

3. PREDICTION OF MODEL PARAMETERS

The model parameters $(\alpha_1,\alpha_2,\alpha_3)$ and $(\omega_{Mid},\omega_{Slope},\zeta_f)$ control the statistical characteristics of the acceleration process, including the mean-square intensity, mean zero-level up-crossing rate, and the rate of cumulative count of negative maxima or positive minima. These statistical characteristics, all expressed as functions of time, are respectfully measures of the intensity, predominant frequency, and bandwidth of the process. The theoretical model is fitted to a target accelerogram by matching these statistical characteristics with those of the target motion, thereby determining the optimal values of the model parameters. Once the model parameters are determined, Eqn. 2.2 is used to simulate artificial accelerograms. The simulated process is eventually high-pass filtered to better model the long-period content of the motion. This allows the simulated motion to achieve zero residual velocity and displacement and improves the behavior in the long-period range of the corresponding response spectrum.

Once the model parameters for a target accelerogram are identified, it is easy to produce any number of ground motion realizations that match the statistical characteristics of the target accelerogram in an average sense. However, this ensemble of realizations is created from one specific set of model parameters that corresponds to the target accelerogram. Since earthquakes of similar characteristics (e.g. type of faulting, magnitude, and location) can produce vastly different accelerograms, the simulated motions with one set of parameters should be regarded as one realization of the possible ground motions resulting from such an earthquake. Therefore, in order to generate an ensemble of artificial records for an earthquake of known characteristics, it makes sense to treat the model parameters $(\alpha_1, \alpha_2, \alpha_3)$ and $(\omega_{Mid}, \omega_{Slope}, \zeta_f)$ as random variables conditioned on the characteristics of the earthquake and the project site. In order to construct such a predictive model, we have developed a dataset of the model parameter values by identifying them for each accelerogram in a dataset of recorded earthquake ground motions. Our eventual aim is to develop full regression formulas (including cross-correlations between the corresponding error terms) that will provide probabilistic estimates of the model parameter in terms of the earthquake and site characteristics.

To identify the parameters $(\alpha_1, \alpha_2, \alpha_3)$ of the modulating function, it is convenient to relate them to the physical characteristics of an accelerogram. Specifically, $(\alpha_1, \alpha_2, \alpha_3)$ are obtained in terms of three variables (I_a, D_{5-95}, t_{Mid}), as described below. I_a represents Arias intensity, a measure of the total energy defined as

$$I_a = \int_0^{t_n} a(t)^2 \, dt \tag{3.1}$$

where $a(t)$ denotes the recorded acceleration time history. D_{5-95} represents a measure of the "effective" duration of the motion, which is defined as the time interval between the instants at which the 5% and 95% of the Arias intensity are reached. As mentioned before, t_{Mid} is the time at the middle of the strong shaking phase. In this study we assume that the middle of strong shaking occurs at the 45% level of Arias intensity. Investigation of the ground motions in our database supports this assumption. Based on these definitions, and taking advantage of the fact that the function in Eqn. 2.4 is proportional to the gamma probability density function, we can write

$$D_{5-95} = t_{0.95} - t_{0.05} \tag{3.2a}$$

$$t_{Mid} = t_{0.45} \tag{3.2b}$$

in which t_p represents the $p \times 100$ th percentile of the gamma probability distribution having the parameter values $2\alpha_2 - 1$ and $2\alpha_3$. Furthermore, α_1 is directly related to the Arias intensity through

$$\alpha_1 = \sqrt{I_a \frac{(2\alpha_3)^{2\alpha_2 - 1}}{\Gamma(2\alpha_2 - 1)}} \tag{3.3}$$

where $\Gamma(\cdot)$ represents the well known gamma function. In the remainder of this paper, we only work with (I_a, D_{5-95}, t_{Mid}). Any simulated values of these parameters can be used in Eqns. 3.2-3.3 to back-calculate the corresponding values of $(\alpha_1, \alpha_2, \alpha_3)$.

Once a sample of observations is available for a model parameter, a probability distribution is assigned to its underlying population. The form of this distribution is inferred by visually inspecting the corresponding frequency diagram. The parameters of the chosen probability distribution are then estimated by the method of moments or the method of maximum likelihood. Finally, the validity of the specified distribution model is verified by a goodness-of-fit test, e.g. the Komogorov-Smirnov test. However, predicting probability distributions for the model parameters is not sufficient to generate a suite of ground motions that represents all the possible realization for a given set of earthquake and site characteristics. We also need to discover any existing relations between the model parameters and the earthquake and site characteristics, as well as any correlations between the parameters themselves. In the current study, the earthquake and site of interest are characterized by four factors (F, M, R_{rup}, $Vs30$). F corresponds to the type of faulting, including strike-slip, reverse, or normal mechanisms; M represents the moment magnitude of the earthquake; R_{rup} represents the closest distance from the recording site to the ruptured area, and $Vs30$ represents the shear wave velocity at the top 30 meters of the soil. We intend to develop regression formulas of the form

$$u_i = g([F, M, R, Vs30]) + \varepsilon_i \tag{3.4}$$

where u_i denotes a model parameter that has been transformed into the normal space ($i = 1,...,6$ since we have six parameters) and ε_i represents the zero-mean regression error that is normally distributed. For a parameter θ, the transformation to the normal space is achieved by $u = \Phi^{-1}[F(\theta)]$, where $\Phi^{-1}[\cdot]$ is the inverse of the standard normal cumulative distribution function and $F(\theta)$ is the cumulative distribution function of the parameter. We expect the error terms ε_i for different parameters to be correlated. This regression analysis is currently underway.

4. STRONG MOTION DATABASE

The strong motion database used in this study is a small subset of the PEER NGA (PEER Next Generation Attenuation of Ground Motion) database, and a subset of the data used in the development of the CB-NGA (Campbell-Bozorgnia NGA Ground Motion Relations) model [5]. The accelerograms in the database are representative of "free-field" ground motions recorded in shallow crustal events in tectonically active regions.

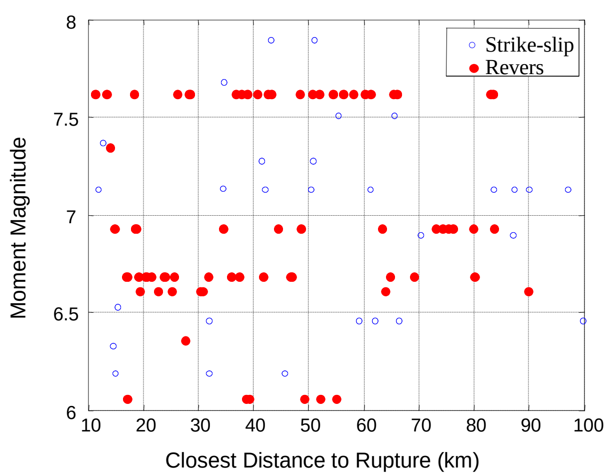

Figure 1 Distribution of the data used in this study with respect to earthquake magnitude and distance from rupture.

The selected earthquakes (see Figure 1) have a moment magnitude greater than or equal to 6.0, and a site with rupture distance range $10\,\text{km} \le R_{rup} \le 100\,\text{km}$. Furthermore, only stiff soil and rock site conditions with $Vs30 \ge 600\,\text{m/sec}$ are considered for the present phase of the study. These constraints reduced the dataset used in the analysis to 31 recordings from 12 earthquakes for strike-slip type of faulting, and 72 recordings from 7 earthquakes for reverse type of faulting. Each recording has two horizontal components, which double the sample size in the statistical analysis. The correlations between the two components of each recording are useful when constructing the predictive relationships.

5. SAMPLE OBSERVATIONS OF MODEL PARAMETERS

Figure 2 shows the normalized frequency diagrams for the parameters (I_a, D_{5-95}, t_{Mid}) for motions resulting from strike-slip type of faulting. The fitted probability density functions (PDFs) for D_{5-95} and t_{Mid} are superimposed in Figure 2 and their formulas and parameter values are listed in Table 5.1. A PDF is not needed for Arias intensity because existing empirical relations can be used to predict this parameter for given earthquake characteristics. It is observed that the duration parameter D_{5-95} varies between a few seconds up to about 40 seconds, with a mean of about 20 seconds. The parameter t_{Mid} assumes values up to about 50 seconds with a mean of 16 seconds. The fact that t_{Mid} is greater than D_{5-95} in some cases is due to long stretches of low intensity motion in some records. A strong positive correlation between these parameters has been observed.

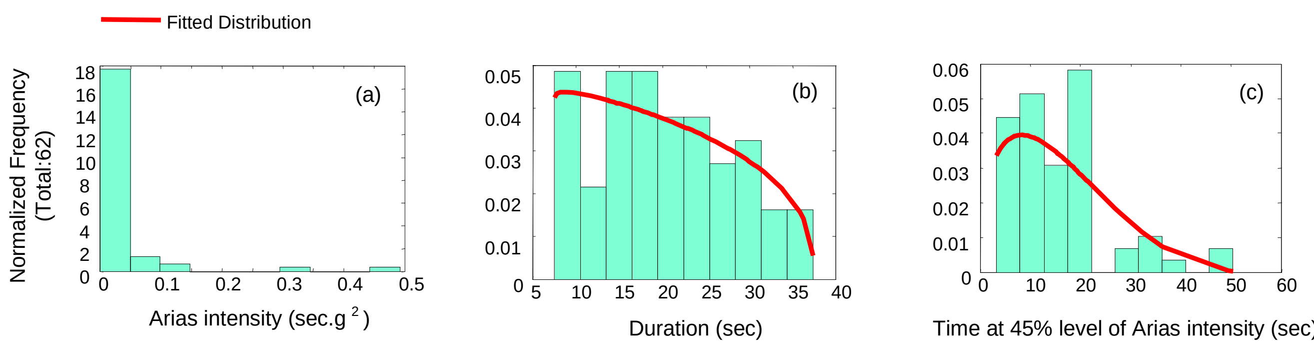

Figure 2 Normalized frequency diagrams for parameters I_a, D_{5-95}, t_{Mid} for earthquakes with strike-slip faulting; fitted PDFs for D_{5-95} and t_{Mid} are superimposed

One way to identify the filter parameters ($\omega_{Mid}, \omega_{Slope}, \zeta_f$) for a given accelerogram is to follow the optimization scheme provided in [3]. However, that method is ideal if the purpose is to calculate the filter parameters that give the closest match to the statistical characteristics of a target accelerogram. For the purpose of generating a sample of the filter parameters for a large number of earthquakes such high level of accuracy is not necessary. Instead, other estimation methods are used that are less demanding in terms of computational efforts while providing sufficient accuracy.

It is well known that the apparent frequency of the stationary response of the considered filter to a white-noise excitation is equal to the filter frequency. This motivates the idea of approximating the filter frequency by the rate of change of the cumulative count of zero-level up-crossings (see Figure 3a). Even though the excitation here is a nonstationary process, investigation of several accelerograms revealed that such an approximation is sufficiently accurate. In order to approximate the two parameters ω_{Mid} and ω_{Slope} for a given recording, a second order polynomial is fitted to the cumulative number of zero-level up-crossings of the accelerogram at Arias intensity percentages equal to 5%, 20%, 45%, 70% and 95%. The fitted polynomial is then differentiated to obtain a linear estimate of the apparent frequency as a function of time. The value of this line at t_{Mid} represents an estimate of ω_{Mid}, and its slope represents an estimate of ω_{Slope}.

To estimate the filter damping ratio, the cumulative number of negative maxima plus positive minima as a function of time for the target accelerogram is determined. This value, which is a measure of the bandwidth of the process, is compared with the estimated averages of the same quantity for sets of 20 simulations of the theoretical model with the already approximated filter frequency and the damping values $\zeta_f = 0.1, 0.2, ..., 0.9$ (see Figure 3b). Interpolation between the curves is used to determine the optimal value of ζ_f that best matches the curve for the target accelerogram. For this analysis, only the time interval between 5% to 95% levels of Arias intensity is considered, where it is more likely for ζ_f to remain constant [3].

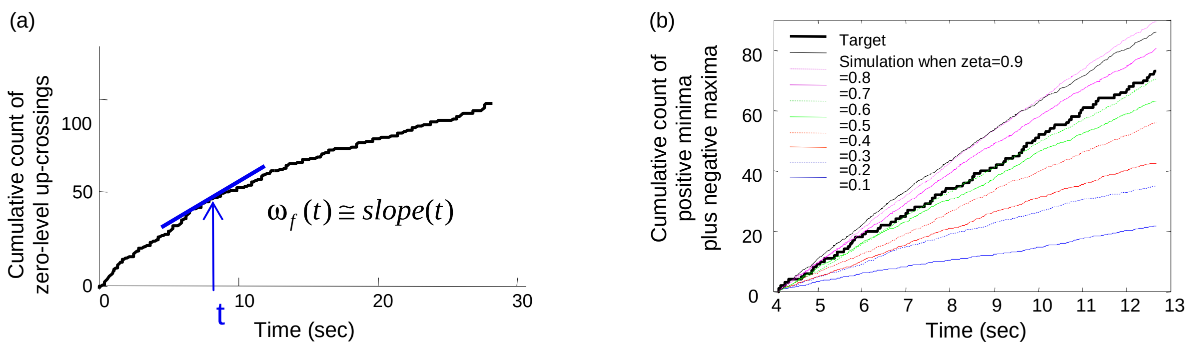

Figure 3 Identification of filter parameters: (a) filter frequency, (b) filter damping ratio

Figure 4 shows the normalized frequency diagrams for the filter parameters for earthquakes with strike-slip faulting. The fitted PDFs are superimposed and their formulas are listed in Table 5.1. It is interesting to note that the predominant frequency at the middle of strong shaking of the records in the dataset ranges from 2 to 12 Hz with a mean value of 6 Hz. The fact that only rock and stiff soils are considered is the reason for this high mean value. It is also interesting to note that ω_{Slope} is negative, i.e., the predominant frequency of the ground motion decreases with time. This is consistent with our expectation. Although small values of the slope are more likely, it is noted that the slope can be as large as a drop of 0.5 Hz per second or even larger. Finally, the filter damping ratio, which is a measure of the bandwidth of the process, is found to range from 0.05 to 0.7 with a mean of 0.26. This value is smaller than the typical value of 0.6 often assumed for the well known Kanai Tajimi model. Again we believe the fact that only rock and stiff soil sites are considered explains this narrow bandwidth of the ground motions in the dataset.

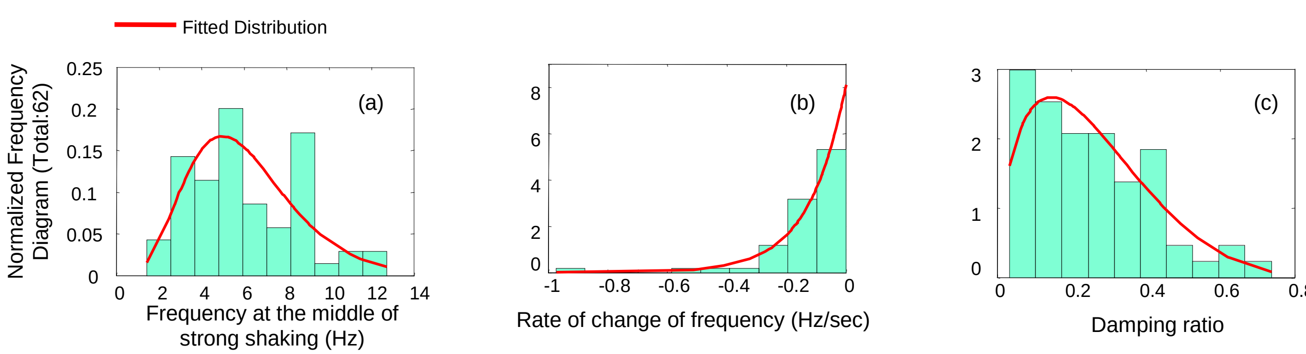

Figure 4 Normalized frequency diagrams and fitted distributions of the filter parameters ω_{Mid}, ω_{Slope} and ζ_f for earthquakes with strike-slip faulting.

As an example, Figure 5 shows a simulated accelerogram obtained by using a randomly selected set of model parameters in accordance to the distributions listed in Table 5.1 and a selected value of the Arias intensity. Cross-correlation coefficients between the model parameters (negative between ω_{Mid} and ω_{Slope}, positive between D_{5-95} and t_{Mid}), which were not discussed in this paper, are accounted for in the simulation. The total duration of the accelerogram is selected as $t_n = 2 \times \max(D_{5-95}, t_{Mid})$.

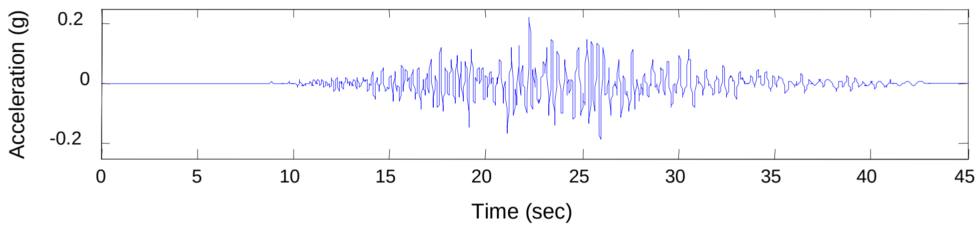

Figure 5 Simulated accelerogram with $\omega_{Mid}/(2\pi) = 3.56$ Hz, $\omega_{Slope}/(2\pi) = -0.07$ Hz/sec, $\zeta_f = 0.22$, $D_{5-95} = 16.36$ sec, $t_{Mid} = 22.59$ sec, and $I_a = 0.05\,\text{sec}.g^2$. Time steps chosen for simulation are 0.02sec. Post processing is done as described in [3] with a corner frequency of 0.05Hz.

Table 5.1 Distributions fitted to model parameters for earthquakes with strike-slip faulting

Model parameter	Fitted PDF	Parameter values	Mean	Standard deviation
$7.5 \le D_{5-95} \le 37.5$ (sec)	$f_X(x) = \frac{1}{\beta(q,r)} \frac{(x-a)^{q-1}(b-x)^{r-1}}{(b-a)^{q+r-1}}$	a=7.5, b=37.5 q=1.02, r=1.38	20.23	8.05
$0 < t_{Mid} \le 55$ (sec)		a=0, b=55 q=1.46, r=3.56	15.95	10.17
$0 < \zeta_f < 1$	$\beta(q,r) = \frac{\Gamma(q)\Gamma(r)}{\Gamma(q+r)}$	a=0, b=1 q=1.66, r=4.84	0.26	0.16
$\omega_{Mid}/(2\pi)$ (Hz)	$f_X(x) = \frac{x^{a-1}}{b^a\Gamma(a)}\exp(-x/b)$	a=5.53, b=1.10	6.09	2.59
$\omega_{Slope}/(2\pi) \le 0$ (Hz/sec)	$f_X(x) = \mu^{-1}\exp(-x/\mu)$	$\mu = -0.12$	−0.12	0.15

6. CONCLUDING REMARKS

A fully nonstationary stochastic model for strong ground motion having statistical characteristics similar to those of a target accelerogram is described. A recorded strong motion database is selected and the model parameters are estimated for each of the recordings in the database. This provides a set of sample observations for each model parameter, which facilitates performing statistical analysis that eventually will help to construct predictive relations for the model parameters in terms of earthquake and site characteristics. Future work will focus on expanding and completing this data analysis.

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