

# GENERATION OF ARTIFICIAL EARTHQUAKE MOTION USING OBSERVED EARTHQUAKE MOTIONS

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# **ABSTRACT :**

In probabilistic seismic hazard analysis, severer earthquake motions than those used in usual design is very important to estimate a damage frequency of structures and components. An artificial earthquake motion of such a severer earthquake is required to have nonlinear response spectral properties similar to those of real motions. In this paper we propose a new convenient non-parametric method to generate an artificial earthquake motion which conforms to a target response spectrum. In this method, the artificial earthquake motion is generated in the form of the orthonormal wavelet transform, in which the wavelet coefficients of the artificial earthquake motion are given by weighted sums of wavelet coefficients of observed earthquake motions in each frequency band. The weights for observed earthquake motions are adjusted to reduce the difference between the response spectrum of the artificial earthquake motion and the target response spectrum, to a tolerance level. We apply this method to generate an artificial earthquake motion subject to the Ohsaki spectrum, from randomly selected several observed earthquake motions in a given band of magnitude and epicentral distance. Each of these several observed earthquake motions is found to contribute appreciably to the generated motion in all frequency bands. The average and standard deviation of group delay time of the generated motions are, respectively, nearly at the central values of those for these observed earthquake motions. These results show that the artificial earthquake motions have average time-frequency characteristics of those for the observed earthquake motions.

**KEYWORDS:** Artificial Earthquake Motion, Wavelet Transform, Group Delay Time, Time-Frequency Characteristics, Response Spectrum, Observed Earthquake Motion

# **1. INTRODUCTION**

Artificial earthquake motion is used in seismic proof structural design and seismic hazard analysis to structure and machinery. In the circumstances, the fundamental and important task is that the setting of the seismic hazard.



Although there are various methods of making artificial earthquake motions, the method which conforms to a target response spectrum by superimposing sine waves is widely used. In other words, the artificial earthquake motion is composed to multiply stationary motions which are calculated with phase given by uniform random numbers and envelope function given in advance. It is also possible to use the phase of specific observed earthquake motion, instead of the phase mentioned above. However those methods which provide a phase at random or use a specific phase characteristics make it difficult to consider the change of phase characteristics. Therefore those methods do not reflect a large portion of phase characteristics of observed earthquake motion. Nuclear power plants and skyscrapers are in need of response analysis in case response structure reaches strong nonlinear area even though such an earthquake rarely occurs. In this situation, there are two important points. One is what kind of characteristics the motion has and another is when and how long the motion comes. In other words, not only amplitude but also time-frequency characteristics that are to say, the relation with phase is more important. From this point of view, modeling of artificial earthquake motion included phase is proceeding these days.

In this paper, we propose a non-parametric method which we called the Synthesis Method of Earthquake Motions. The artificial earthquake motion made by our method cover time-frequency characteristics and amplitude characteristics including several observed earthquake motions and satisfy target response spectrum.

# 2. SYNTHESIS METHOD OF EARTHQUAKE MOTIONS

In this part, we explain the details of the formulation of the Synthesis Method of Earthquake Motions.

#### 2.1 Wavelet transform of observed earthquake motions

In the Synthesis Method of Earthquake Motions, observed earthquake Motion x(t) is transformed to time frequency domain by orthonormal wavelet transform, and then divided into particular wavelet circular frequency bands.

Let  $\varphi_{ik}(t)$  be a mother wavelet. Then, orthonormal wavelet transform is defined by Eqn. 2.1.

$$\alpha_{j,k} = \int_{-\infty}^{\infty} x(t) \overline{\varphi_{j,k}(t)} dt$$
(2.1)

Where,

 $\{\varphi_{ik}(t) \mid j,k \in \mathbb{Z}\}\$  are wavelets and defined by Eqn. 2.2, where  $\mathbb{Z}$  is the set of all integers.

$$\varphi_{j,k}(t) = 2^{1/2} \varphi(2^{j} t - k)$$
(2.2)

And  $\alpha_{j,k}$  is the orthonormal wavelet coefficient  $(j,k \in \mathbb{Z})$ . The divided motions  $x_j(t)$  in the each of scale parameters j are defined by Eqn. 2.3.

$$x_j(t) = \sum_k \alpha_{j,k} \varphi_{j,k}(t)$$
(2.3)

Where,

$$x(t) = \sum_{j} x_{j}(t)$$
. (2.4)

In this paper, we use Meyer wavelet as a mother wavelet. Meyer wavelet has the compact support in frequency domain. The circular frequency band of Mayer wavelet is as follows.



$$\{\omega \mid 2\pi/3 \le \omega \le 8\pi/3\} \tag{2.5}$$

Therefore, let T be a duration time of transformed motion. The observed earthquake motion x(t) is divided into each circular frequency bands.

$$\{\omega \mid 2^{j+1}\pi/3T \le \omega \le 2^{j+3}\pi/3T\}$$
(2.6)

# 2.2 The Flow chart of the Synthesis Method of Earthquake Motions

The outline of the Synthesis Method of Earthquake Motions (Figure 2.1) is shown as follows:



Figure 2.1 Illustration of flow chart of the Synthesis Method of Earthquake Motions

Step 1 Select the observed earthquake motions

Search the observed earthquake motions which satisfy certain adopted conditions.

Let,

N be a number of searched earthquake,

R be a number of selected observed earthquake motions, and

L be a number of selected observed earthquake motions from R at random.

If  $L \le N$ , select L motions from each of different N, or if L > N, select at least one motion from each of different N. Selected motions are given number i = 1...L.

Step 2 Wavelet transform and inverse wavelet transform in each circular frequency bands

Conduct wavelet transform to the observed earthquake motions  $x^{i}(t)$  using Eqn. 2.1. Then generate time history motions  $x^{i}_{j}(t)$  using Eqn. 2.3.

Step 3 Multiply weight

Generate artificial time history  $y^{i}_{j}(t)$ , which is defined in the form of Eqn. 2.7.

$$y^{i}{}_{j}(t) = W^{i}{}_{j} \cdot x^{i}{}_{j}(t)$$
 (2.7)

Where,



 $W^{i}_{j}$  is weight (refer to 2.3).

### Step 4 Integration

Calculate a response spectrum by time history z(t) which integrates into all of  $y_{ij}^{i}(t)$  as an artificial earthquake motion.

# Step 5 Response spectrum comparison

Collect the weight  $W^{i}_{j}$  and back to step3 until the response spectrum satisfies the certain condition (refer to **2.6**).

# 2.3 Standardize spectrum intensity

In this chapter, The spectrum intensity is standardized in order to prevent from relying on certain motions which has large magnitude characteristics in them. The weight is defined in the form of Eqn. 2.8.

$$W^{i}{}_{j} = C^{i}{}_{j} \cdot W^{i}{}_{Pj} \tag{2.8}$$

Where,

 $C^{i}_{j}$  is the standard coefficient, and

 $W^{i}_{Pj}$  is the pure weight and its initial value is decided at random.

Here,  $C_{j}^{i}$  is the standard coefficient for a target response spectrum  $S_{0}$ .  $C_{j}^{i}$  is decided by the form of Eqn. 2.9.

$$C^{i}{}_{j} = \overline{S}{}_{0} / \overline{S^{i}}{}_{j}$$

$$(2.9)$$

Where,

 $S_0$  is the average of  $S_0$ ,

 $S^{i}_{j}$  is the response spectrum of each observed earthquake motions, and

 $\overline{S_{j}^{i}}$  is the average of  $S_{j}^{i}$ .

# 2.4 Contribution ratio of earthquake motion energy

We define the contribution ratio of *i* th earthquake motion energy  $E_{j}^{i}$  to artificial earthquake motion energy  $E_{j}$  in each of a certain frequency band *j*. It is defined in the form of Eqn. 2.10.

Contribution ratio of 
$$E^{i}_{j}$$
 to  $E_{j} = \left(W^{i}_{j}\right)^{2} \cdot \int x^{i}_{j}(t)^{2} dt \left/ \left\{ \sum_{i} \left(W^{i}_{j}\right)^{2} \cdot \int x^{i}_{j}(t)^{2} dt \right\}$ (2.10)

We also define the whole contribution ratio of *i* th earthquake motion energy  $E^{i}$  to artificial earthquake motion energy *E* in the whole frequency bands. It is form of Eqn. 2.11.

Contribution ratio of 
$$E^{i}$$
 to  $E = \int \left( \sum_{j} W^{i}{}_{j} \cdot x^{i}{}_{j}(t) \right)^{2} dt / \int \left( \sum_{i} \sum_{j} W^{i}{}_{j} \cdot x^{i}{}_{j}(t) \right)^{2} dt$  (2.11)

### 2.5 Databases

In this paper, we use KiK-net and USGS databases. Strong earthquake motion data from October 1997 to

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August 2007 observed in Japan are available from KiK-net. On the other hand, the origin time, magnitudes and epicentral distance are mainly based on USGS. Then magnitudes and epicenters are based on USGS if they show different data compared with USGS and KiK-net. Here, 457842 earthquake motions from 2152 earthquakes are applied to USGS.

### 2.6 Conditions

In this paper, the five conditions are defined as follows to satisfy the artificial earthquake motions.

Condition 1 Minimum spectrum ratio

$$\varepsilon_{\min} = \left\{ \frac{S_a(T_i, h)}{T_a(T_i, h)} \right\}_{\min} \ge 0.85$$
(2.12)

Condition 2 Ratio of spectrum intensity

$$SI_{ratio} = \frac{\int_{1}^{5} S_{pv}(T) dT}{\int_{1}^{5} S_{pv}(T) dT} \ge 1.0$$
(2.13)

Condition 3 Coefficient of variation

$$v = \sqrt{\frac{\sum (\varepsilon_i - 1.0)^2}{N}} \le 0.05$$
 (2.14)

Where, 
$$\varepsilon_i = \frac{S_a(T_i, h)}{{}_T S_a(T_i, h)}.$$
 (2.15)

Condition 4 Error of average value

$$\left|1 - \frac{\sum \varepsilon_i}{N}\right| \le 0.02 \tag{2.16}$$

Condition 5 Number of natural periods of error judgment Pitch is geometric progression.

$$200 \le N \le 300$$
 (2.17)

Where,

 $S_{pv}$  is pseudo-velocity response spectrum,

 $_T S_{pv}$  is target velocity response spectrum,

 $S_a$  is accelerate response spectrum,

 $_T S_a$  is target accelerate response spectrum,

 $\varepsilon_i$  is ratio of  $S_a$  and  $_T S_a$ ,

h is damping constant (h = 0.05) and

T is natural period of error judgment.



# **3. PROBLEM FORMULATION**

### 3.1 Target response spectrum

A target response spectrum is shown in Table 3.1.

# 3.2 Selected range of database

We use underground East-West factor and underground North-South factor from KiK-net in order to generate the artificial earthquake motions in horizontal directions on the engineering base-rock. Here, the parameters of Synthesis Method of Earthquake Motions are magnitude M and the epicentral distance  $\Delta$ . Then ten patterns from different selected ranges are assumed and a hundred artificial earthquake motions in each of the ten patterns are generated. Selected conditions and the number of observed earthquake motions which fulfill them are shown in Table 3.2.

# 3.3 Definition of parameter in the Synthesis Method of Earthquake Motions

Thirty observed earthquake motions are choose in order to make comparison easily (L = 30).

Where,

Circular frequency of data is 100 (Hz.), and Total length of time is 163.84 (sec.). Period bands of circular frequency bands in each period j is shown in Table 3.3.

Table 3.1 Target response spectrum

Natural period of building $T$ (sec.)	Accelerate response spectrum $S_a$ (m/s <sup>2</sup> )	
	An earthquake which rarely occurs	
<i>T</i> < 0.16	0.64 + 6T	
$0.16 \le T < 0.64$	0.16	
$0.64 \le T$	1.04/T	

Table 3.2

Conditions of observed earthquake motions fulfilled our selection

Table 3.3Period bands of frequency bands j

Pattern No.	Magnitude M	Epicentral Distance Δ (km)	Number of observed motions N	Number of observed earthquake motions <i>R</i>
А	5.0 ~ 5.5	$55 \sim 75$	109	966
В		$75 \sim 95$	169	1736
С		55 ~ 95	170	2702
D	5.5~6.5	$55 \sim 75$	46	462
Е		$75 \sim 95$	70	744
F		55~95	70	1206
G	6.5 ~ 7.0	$55 \sim 75$	4	36
Н		$75 \sim 95$	6	50
Ι		55 ~ 95	6	86
J	5.0 ~ 7.0	55~95	226	3748

Frequency bands <i>j</i>	Period bands (sec.)
13	$0.015 \sim 0.060$
12	$0.030 \sim 0.120$
11	$0.060 \sim 0.240$
10	$0.120 \sim 0.480$
9	$0.240 \sim 0.960$
8	0.480 ~ 1.920
7	0.960 ~ 3.840
6	1.920 ~ 7.680
5	3.840 ~ 15.360



# 4. NUMERICAL RESULTS

The whole artificial earthquake motions satisfy condition **2.6**. Number of natural periods of error judgment N is 255, and pitch is defined as geometric progressions. Pattern No. D in Table 3.2 is investigated. We call this case sample motion from now on. Response spectrum of sample motion and target spectrum in the period between  $0.02 \sim 5.00$  (sec.) is shown in Figure 4.1, and the ratio of response spectrum of sample motion and target response spectrum in the period between  $0.02 \sim 5.00$  (sec.) is shown in Figure 4.1, and the ratio of response spectrum of sample motion and target response spectrum in the period between  $0.02 \sim 5.00$  (sec.) is also shown in Figure 4.1.

As a result, the coefficient of variation is 0.049, the minimum spectrum ratio is 0.86, the maximum spectrum ratio is 1.12, and the ratio of spectrum intensity is 1.00. Sample motion is extremely precise to a target response spectrum. Observed earthquake motions and acceleration time history of sample motion are shown in Figure 4.2. Proportion of energy contribution of selected thirty observed earthquake motions  $x^i(t)$  ( $i=1\sim30$ ) in circular frequency bands j ( $j=5\sim13$ ) are shown in Figure 4.3 in order to make sure not being affected by a certain motion's factor excessively. The motions which occupies the largest proportion of energy contribution in each circular frequency bands are put numbers. Any of certain motions are not lopsided all the aspects such as amplitude or energy. Comparison between the  $\mu_{tgr}^{o}$  of observed motion and  $\mu_{tgr}^{s}$  of sample motion is shown in Figure 4.4, where  $\mu_{tgr}$  means the mean of group delay time of motion<sup>1)</sup>. According to Figure 4.4,  $\mu_{tgr}^{s}$  is located in around the middle of the observed earthquake motions. It means that  $\mu_{tgr}^{s}$  shows the average time-frequency characteristics.

### **5. CONCLUSION**

In this paper, we propose the Synthesis Method of Earthquake Motions which meets a target response spectrum by using observed earthquake motions. The Synthesis Method of Earthquake Motions does not assume any parametric model such as regression model. And the method makes use of time-frequency characteristics and amplitude characteristics including observed earthquake motions. Here, we show through the numerical test that the Synthesis Method of Earthquake Motions is able to make the artificial earthquake motions that satisfy target response spectrum by using the 30 observed earthquake motions. Moreover, the energy contribution of the artificial earthquake motions made by this method are unaffected by the energy contribution of particular observed earthquake motion. And  $\mu_{tgr}^{s}$  of the artificial earthquake motions made by the method show the similar characteristic as  $\mu_{tgr}^{o}$  of average of 30 observed earthquake motions. Therefore the artificial earthquake motion made by the Synthesis Method of Earthquake Motions shows the average time-frequency characteristics of plural observed earthquake motions.

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Figure 4.3 Proportion of energy contribution



Figure 4.4 Average of  $\mu_{tgr}^{o}$  and  $\mu_{tgr}^{s}$ 



Figure 4.2 Sample motion and acceleration time histories of observed earthquakes  $x^{i}(t)$