The Local Discrete Wave-number Method to Simulate Wave Propagation in Irregular Layer

Hong Zhou[1] Xiaofei Chen[2]

[1] *Associate Professor, Dept. of Structural Engineering , Institute of Geophysics, China Earthquake Administration, Beijing, China*

1. [2] *Professor, School of Earth and Space Science, University of Science and Technology, Hefei 230026, China*
Email: zhouhong@pku.edu.cn, xfchen@pku.edu.cn

ABSTRACT :

In the paper, we present a new method of synthesizing seismograms for irregular multi-layered problems. It is an extension of the *lo*BIE-DWM topography problem. Its computation efficiency increases dramatically, particularly for the problem in which the corrugated part of the layer is highly localized. We first provide the formulation of Bouchon and Campillo's BIE-DWM for the multi-layered problem. Then validate it by several models and finally show its application.

KEYWORDS: local discrete wave-number method, the multi-layered problem, Bouchon and Campillo's BIE-DWM

1. INTRODUCTION

The underground geological structure is so complex and manifold that it can not always be simplified as horizontal-layered media or half-space structure. In order to accurately simulate wave propagation in geological structures it is important to study different methods. Among these methods, Aki-Larner's (1970) method (AL) is a pioneer study. Bard & Bouchon (1980) applied it to study of P-SV waves of topography. Horike et al.(1990) and Uebayashi et al.(1992) extended this method into a three dimensional problem. Implementing of a fast Fourier transform makes it highly efficient, but it is only suitable to slow the interface layer for the assumption of Rayleigh Asanta. Inspired by AL, a global generalized reflection was proposed – the transmission matrices method (GGRTM) (Chen 1990, 1995, 1999), and it works very well for arbitrarily irregular – layered media.

The boundary integral equation method (BIEM) is another major boundary method. It can sorted as the BEM (Sanchez-Sesma and Campillo, 1991, 1993) and BIE-DWN. The BIE-DWN is a combination of Green's function in the wave-number domain and boundary integral equation (Bouchon, 1985, 1989; Campillo and Bouchon, 1985, Campillo, 1987; Kawase and Aki, 1988, 1989). The analytical evaluation of Green's function is one of its advantages. Other merits are its stability and high accuracy. However, the scheme of discretation with equal intervals involves considerable sampling points for high frequency or large-scale layer problems, which results in a large computation and low efficiency. Therefore, any technique to improve efficiency is welcome. Two techniques have been proposed; one is to reduce the unknown number by using a special Green's function. For example, Campillo (1987) applied layer Green's function to study SH waves in irregular-layered media. Gaffet & Bouchon (1991) and Kawase (1990) simulate the P-SV response of a sedimentary basin using half-space Green function. The unknown in these studies is limited to the irregular part of each interface. However, the layer Green's function has no analytical form and must be obtained by numerical computations, while the half-space Green function is not suitable for media with mountainous topography. Bouchon et al. (1996) proposed another approximation technique to reduce the computation. However this approximate technique affects the accuracy, and needs careful analysis.

Zhou and Chen (2006a, 2006b,2008), propose a local discrete wave-number method to simulate the scattering PSV&SH waves of the topography problem. This method is inspired by BIE-DWN (see Bouchon1985, Compillo 1985). However, it is several tens to hundreds of times faster than the BIE-DWM for limiting the unknowns to the irregular part, while still using the simple and analytic full-space Green's function. In this paper we extended it to solve a wave propagation problem in a multi-irregular layer. What follows is the formulation for such problem.

2. THE LOCAL DISCRETE WAVE-NUMBER FOR MULTI-LAYERED PROBLEMS

The local discrete wave-number (*lo*BIE-DWM) is derived from BC. Its theory for SH and P-SV waves of the topography problem has been presented in Zhou (2006a,2006b,2008). But the irregular-layer geological formation is more universal. As extra, the BC's simple formula is given here.

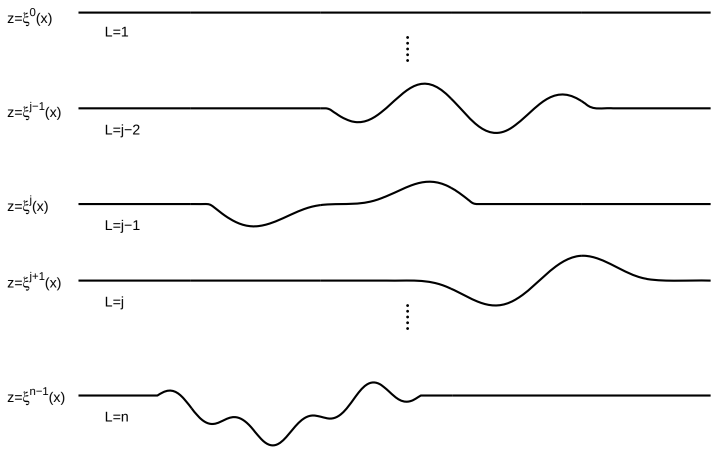


Figure 1 The problem considered in the present paper

The considered problem is illustrated in Fig. 1. There are n layers over a half-space and each layer is bounded by two interfaces $z=\xi^{(j-1)}(x)$ and $z=\xi^{j}(x)$. The uppermost interface $z=\xi^{0}(x)$ is a free surface. According to Huygens ' Principle, the wave-field in layer *l* can be taken as the sum of the incident wave due to the source and the scattering wave induced by the force distributed on the top and bottom of layer *l*, i.e.:

$$\mathbf{u}(l)=\int ds\mathbf{a}^{t}(l)\mathbf{q}^{t}(x,l)+\int ds\mathbf{a}^{b}(l)\mathbf{q}^{b}(x,l)+\delta(l)\mathbf{S}(l), \qquad (2.1)$$

where $\mathbf{u}(l)$ denotes the displacement or traction in layer *l*, $\mathbf{a}^{t}(l)$ and $\mathbf{a}^{b}(l)$ denote the displacement of Green's function or traction of Green's function which are scalar for SH waves and tensor vector for P-SV, $\mathbf{q}^{t}(x,l)$ and $\mathbf{q}^{b}(x,l)$ denote unknown forces distributed on the top and bottom of layer *l*, $\delta(l)\mathbf{S}(l)$ is the displacement or traction produced by a source directly in layer *l*, subscript *t* and *b* mean the top and bottom.
The continuity of traction and displacement on interface provides eq.(2.2)

$$\underline{\underline{\mathbf{A}}}\mathbf{Q}=\mathbf{F} \qquad (2.2)$$

which is a large equation. Directly solving the equation to obtain the force vector $\mathbf{Q}$ on all interfaces is the usual procedure of the discrete wave-number method (BC), and will cost considerable computation.

Similar to the topography problem (Zhou 2006a), firstly the forces $\mathbf{q}(l)$ on the arbitrary interface *l* are distinguished into two orthogonal parts: ones on the flat part $\mathbf{q}_F(l)$, others on the irregular parts $\mathbf{q}_C(l)$. They are:

$$\{\mathbf{q}(l)\}=\{\mathbf{q}_F(l)\}\oplus\{\mathbf{q}_C(l)\}. \qquad (2.3)$$

By perform FFT on the continuity equation of traction and displacement on irregular part and flat part interface eq. (2.2) can give **a mathematical relation between** q_F **and** q_C**,** which is

$$\mathbf{Q}_F=\mathbf{S}_F-\underline{\underline{\mathbf{D}}}\cdot\mathbf{Q}_C \qquad (2.4)$$

The continuity of traction and displacement on the irregular part of the interface can be expressed as:

$$\underline{\underline{\mathbf{C}}}_{\mathbf{CF}}\mathbf{Q}_F+\underline{\underline{\mathbf{C}}}_{CC}\mathbf{Q}_C=\mathbf{S}_C \qquad (2.5)$$

Substituting $\mathbf{Q}_F$ into eq.7 yields:

$$\underline{\underline{\mathbf{C}}}_{CF}\mathbf{S}_F+(-\underline{\underline{\mathbf{C}}}_{CF}\mathbf{D}+\underline{\underline{\mathbf{C}}}_{CC})\mathbf{Q}_C=\mathbf{S}_C \qquad (2.6)$$

Thus, a small linear equation about force $\mathbf{Q}_C$ distributed on irregular parts of the interface is constructed, which is only a small portion of eq.(2.2).

3. VALIDATIONS

In order to validate the *lo*BIE-DWM for a 2-D irregular multi-layered problem, we consider several numerical tests which include an SH and P-SV case. For SH waves, we study the response of a two-layer model for vertical incidence of plane SH waves (fig.2). The solution given by Cao (2003), which was based on Chen's method (1990), is presented for comparison. For the P-SV case, the accuracy of our approach is tested using the resolution of Kawase and Aki (1989), which has been based on a direct boundary element with Green's function in discrete wave-number domain and has been verified extensively. We regard their results as trustworthy. Kawase and Aki studied the trapezoidal valley as shown in Fig. 3. Comparisons with their results are provided here for the vertical incidence of plane SV The agreement is excellent(fig.4), not only for the direct waves but also for the scattering Rayleigh waves.

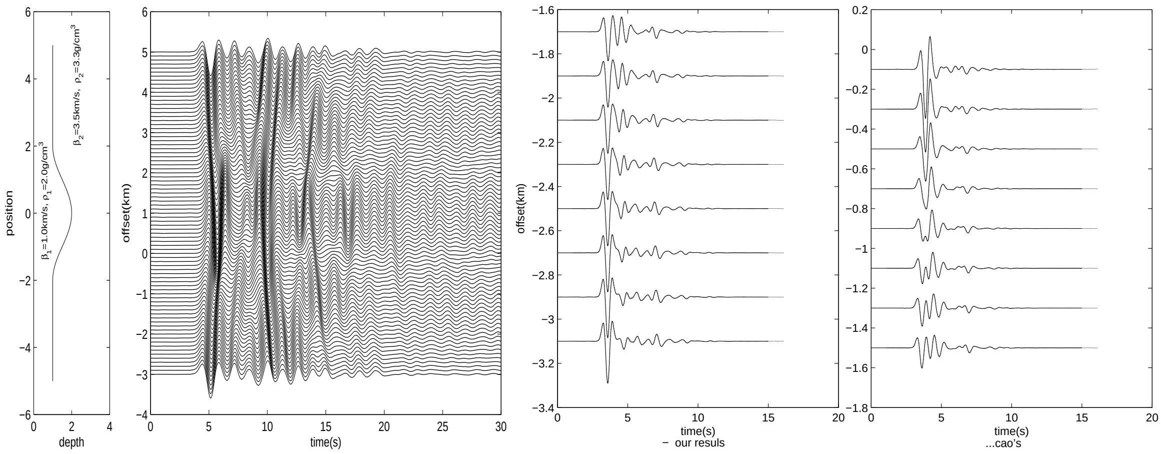


Figure 2(a) The model and the responses in time domain due to vertical incident SH plane waves **(b)** The comparisons of response in time domain between ours and Cao's(2004).Solid line is ours and dot line is Cao's.

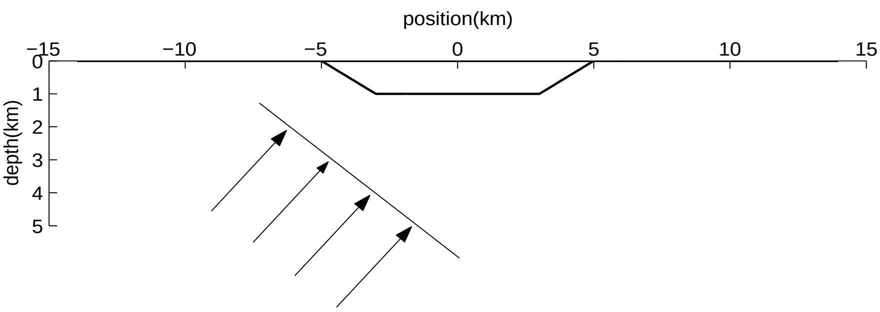
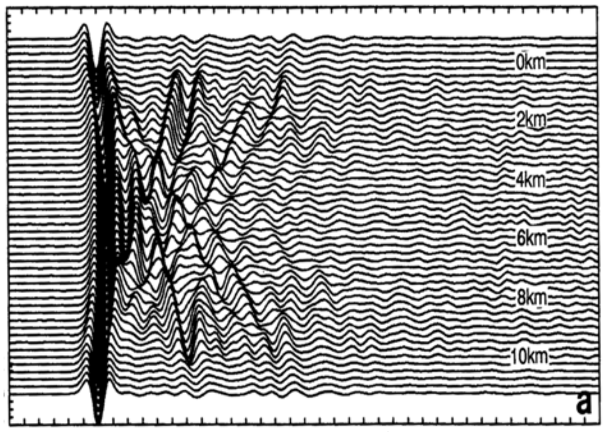


Figure 3 Kawase&Aki basin model

（a）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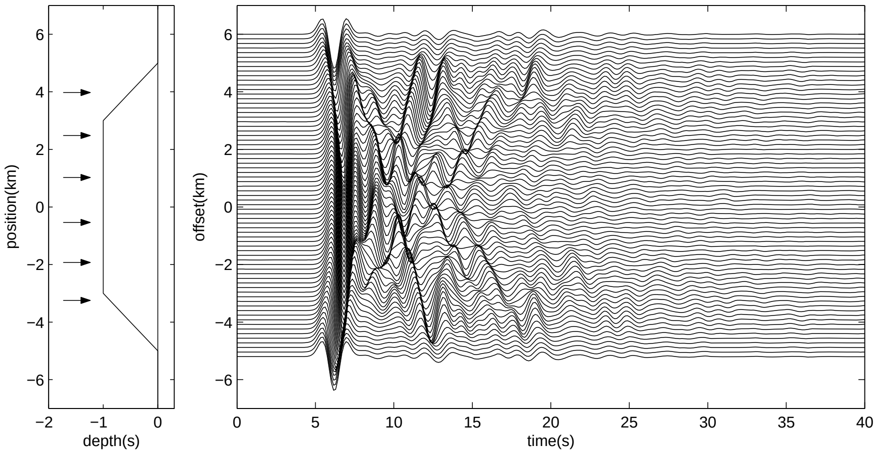
a

（a＇）


position(km)
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25
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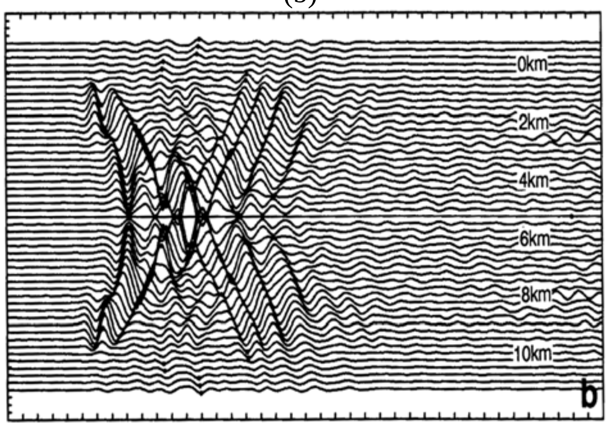
35
40

(b)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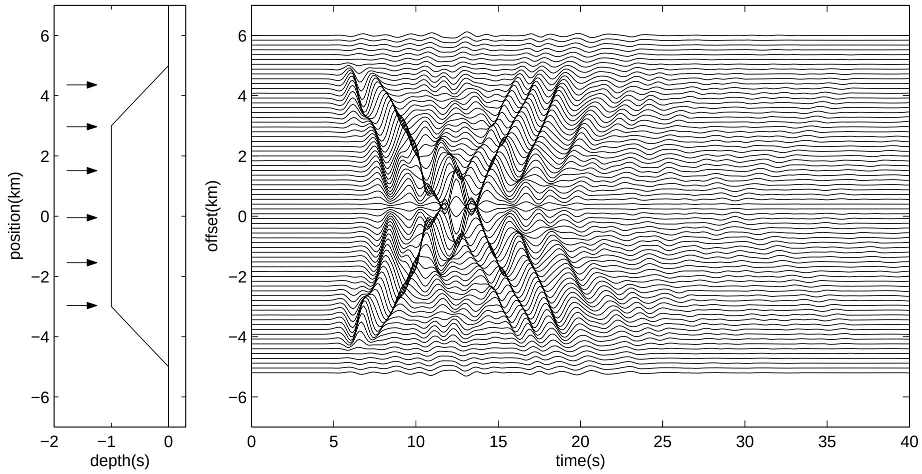

Figure 4 The comparisons of ground responses in time domain from ours and Kawase's due to plane SV incident with 0^0 for Kawase&Aki model. (a) and (b) are horizontal component and vertical component of Kawase's results. （a'），（b'） are horizontal component and vertical component of ours.

4. APPLICATION

In order to illustrate the application of our *lo*BIE-DWM, we present the synthesized responses for selected irregular multi-layered media.

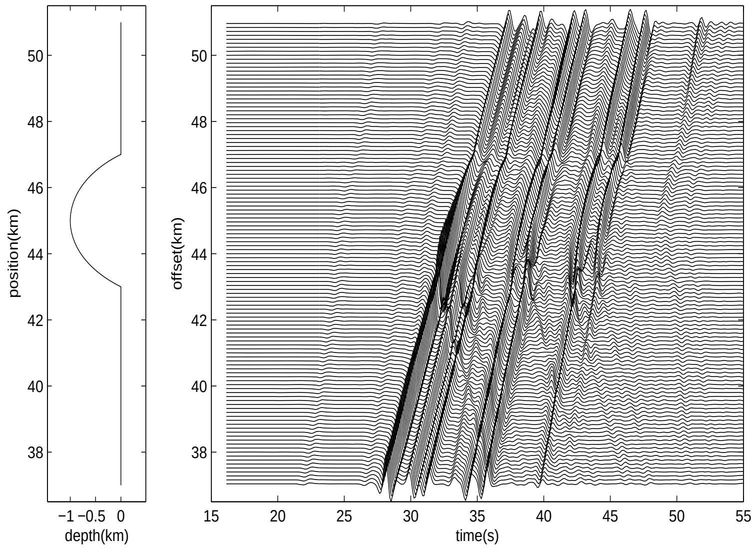

Figure 5 The responses in time domain of two layers model with canyon topography due to exposed source.

For SH case, a two-layered media is shown on the left of Fig.5. In this model, at the free surface there is a valley with a width of 4km and maximum depth of 1km. Shear wave velocity is 2km/s and density is 2.5g/cm^3 for the half-space, while they are 1.5km/s and 2.0g/cm^3 in the first layer, the corresponding thickness is 6km out of the valley area. An explosive source is at a depth of 4.3km. In order to clearly demonstrate wave propagation, the source is located a distance of 45km away from the center of the valley in a horizontal direction.

The right side of Fig. 5 shows the synthetic seismograms in time domain at the free surface between 37km-51km which includes the effect of the valley. In a former part of the time series, the head wave can be seen and has a very weak energy. Reversely, the latter waves have more energy as the Love wave has a stronger energy, multiply reflection wave. Moreover, the scattering waves by the edge of the valley are also clear.

5. CONCLUTIONS

In the present paper we provide a validation and application of the *lo*BIE-DWM for the irregular multi-layered problem. In order to describe it complementally we firstly, summarized its algorithm, then compare our results with those of other methods. The test results demonstrated the *lo*BIE-DWM has the same accuracy as Chen's(1990) for the SH wave and the same as Kawase's (1989) for the P-SV wave. Next, we applied it to simulate the wave propagation in an irregular multi-layered model. In the SH case, the valley on the surface strongly affects near ground wave propagation, but weakly affects near interface waves. The agreement further illustrates our method's application value.

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