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# AN ATTEMPT TO PREDICTION TORSIONAL MOTION FROM TRANSLATIONAL ONES 

G.R. Nouri ${ }^{1}$ and M.R. Ghayamghamian ${ }^{2}$<br>${ }^{1}$ University of Mohaghegh Ardabili, Ardabil, Iran<br>${ }^{2}$ M.R. Ghayamghamian, IIEES, Tehran, Iran<br>Email: r.nouri@iiees.ac.ir


#### Abstract

: The importance of torsional excitation on structural response has been indicated by several studies. However, due to the instrumental limitations to record torsional ground motions, it is important task to investigate on the possibility of torsional motion estimation from translational ones. The attempts were made to estimate the torsional motions using coherency function models based on the assumption that torsional motion induced due to spatial variation of ground motions. Spatial variation of ground motion can be defined by adequate coherency function models that some of them suggested by previous studies. While these coherency functions can be modeled the spatial variations of translational motions, their direct application in torsional ground motion estimation may not be appropriate. In this paper, the relations between power spectra density (PSD) of torsional and translational motions are examined based on Hao's coherency function using Chiba dense array data. Since the coherency function for torsional motion depends on the distance between the two stations, accelerometer pairs in Chiba array are categorized in five groups that varies from close separation distance (<30 m ) to large ones ( $250 \mathrm{~m}-320 \mathrm{~m}$ ). For obtaining the model parameters, $50 \%$ of data are used in each group of separation distance then the PSDs related to each group are averaged at each frequency. Having averaged PSDs for translational and torsional motions at each separation group, the coefficients of the model are evaluated by optimization scheme. In the next step, to check the validity of the results, the identified coefficients are applied to predict the torsional PSDs for the other $50 \%$ of data that are not used in optimization procedure for estimating model parameters. The comparison of the predicted and the calculated torsional motions reveals a weak estimation for close separation distances ( $<30 \mathrm{~m}$ ) and satisfactory predictions in other distances


KEYWORDS: dense array, torsional motion, coherency function

## 1. INTRODUCTION

Rotational motions (torsional and rocking) induced by seismic waves have been essentially ignored for a long time, first because rotational effects were thought to be small for manmade structures (Bouchon and Aki 1986), and second because sensitive measuring devices were not available until quite recently. The benefits of the determination of rotational motion in seismology and engineering are still under investigation (e.g. Trifunac and Todorovska 2001, Takeo and Ito 1997). In seismology, rotational motions can provide accurate data for arrival times of SH waves and, in the near-source distance range, rotational motions might provide more detailed information on the rupture processes of earthquakes (Takeo and Ito 1997). Rotational motions could also be used to better estimate the static displacement from seismic recordings, identifying translational signals caused by rotation (Trifunac and Todorovska 2001 ).
In engineering, dynamic response estimation of structures subjected to earthquake induced base excitations is often simplified by ignoring the rotational components. This has been a widely accepted practice in engineering community, mainly caused by the lack of recorded strong motion accelerograms for these motions. Many structural failures and the damage caused by earthquakes can be linked to differential and rotational ground motions. Torsional responses of tall buildings in Los Angeles, during the San Fernando earthquake in 1971, could be ascribed to torsional excitation, while rotational and longitudinal differential motions may have caused
the collapse of bridges during San Fernando (1971), Miyagi-ken-Oki (1978) (Bycroft 1980) and Northridge (1994) (Trifunac and Todorovska 1996) earthquakes. For the first time Newmark (1960) established a simple relationship between translational and torsional components of the ground motion. He presented a deterministic procedure for estimating the increase in displacement of symmetric-plan buildings caused by rotational ground motions at the base due to horizontal propagation of plane waves with a constant velocity and further explored in the other studies (Nathan and MacKenzie 1975, Rutenberg and Heidebrecht 1985). Several studies have shown the importance of torsional components in seismic analysis and design of structures (Newmark 1969, Ghafory-Ashtiany and Singh 1986, Abdel-Ghaffar and Rubin 1984, Geol and Chopra 1994, De La Llera and Chopra 1994, Shakib and Tohidi 2002).
Three main approaches have been developed to incorporate the rotational motions in engineering applications: one is numerical simulation of radiation field from source mechanism. It requires an appropriate model of faulting mechanism, without considering the effects of path and local site condition (Bouchon and Aki 1986). The second approach is based on theoretical formulation of spatial distribution of ground motion. In this method, some information on source, path and local layering are required (Castellani and Boffi 1986, Lee and Trifunac 1985, 1986, Li et al.). Lee and Trifunac $(1985,1986)$ developed torsional and rocking accelerograms compatible with artificial translational ones with a given proportion of surface and body waves. All of these studies are based on assumed models for ground motion and none of them has the benefit of being tested against field measurements. The third approach is the application of recorded strong motion data from seismic arrays (Niazi 1986, Oliveira and Bolt 1989, Spudich et al. 1995, Bodin et al. 1997, Huang 2003, Suryanto 2006). The average rotational motions can be evaluated from difference of two translational records in an array of stations on the ground. It should be noted that the rotational components could not be reliably identified over the wide frequency range if the accelerometers were installed too far from each other. Thus, dense arrays are one of the unique sources of experimental information on rotational motion estimation by the spatial derivatives.
The interest in the observation of rotational ground motions has increased in recent years due to development of sensors and optical instruments such as ring laser gyros. Solid-state rotational velocity sensors were used to observe rotational ground motions close to sources of seismic energy (Takeo and Ito 1997, Nigbor 1994). However, the resolution of these instruments was too low to be applicable in seismology over a broad magnitude and distance range (Suryanto 2006). Therefore, sensor developments in the past years focused on the refinement of optical instruments, in particular, using laser technology (Stedman 1995, Igel 2005). The first comparison of seismic array-derived rotations with direct measurements has studied by Suryanto et al. (2006). They showed that the overall fit between direct and array-derived measurements is surprisingly good. It is noteworthy that they considered a fairly long-period signal in their study.
An attempt have been made to develop a relation between torsional and translational components of ground motion due to the lack of appropriate torsional motion records. Hao (1996) proposed a relation between power spectral density (PSD) function of torsional and translational ground motions using Smart-1 array data. He also studied the effects of coherency and site condition on torsional motion response spectrum.
In this paper, the Hao's equation for predicting torsional ground motion from translational components is analyzed for different separation distances. To this end, the data of Chiba dense array, in which the separation distances among accelerometers vary from 5 to 320 m , are employed. The Chiba array has been operating since 1982 and has accumulated a large amount of data covering a wide range of seismological parameters. This provides a unique opportunity to examine the proper characteristics of rotational ground motions as they occur in place. Measurement of spatial variation of the seismic wave-field for engineering applications requires dense array recordings. Depending on the aim of observation, there are several manners in the arrangement of seismometers and accelerometers. One of the first arrays was the El Centro differential array that recorded the 1979 Imperial Valley earthquake (Niazi 1986). A typical two-dimensional surface array represented by Smart-1, located in Lotung (North of Taiwan) started operation in 1980 (Oliveira and Bolt 1989). A smaller scale 3D array, the LSST array, was constructed in 1985 within the Smart-l.
A unique 3D dense array system was installed in Chiba, experiment station for institute of industrial science, university of Tokyo in 1982 (Katayama et al. 1990). In this array, seismometers and accelerometers are placed very densely (minimum separation distance is 5 m ) both on ground surface and in boreholes. Chiba experiment station is located about 30 km east of Tokyo (Figure 1). The array system is composed of fifteen boreholes (C0C4 and P0-P9) with 44 three-component accelerometers. Eight surface accelerometers are densely arranged, four of them ( $\mathrm{C} 1-\mathrm{C} 4$ ) are only located 5 m from C 0 and the other four ( $\mathrm{P} 1-\mathrm{P} 4$ ) are 15 m from C 0 . There is a
large triangular network (P0-P8-P5), with each of its sides being approximately 300 m , around the borehole C0 (Figure 1). We categorized the pairs of station for three groups ( $5 \mathrm{~m}<\Delta L<30 \mathrm{~m} 66 \mathrm{~m}<\Delta L \leq 150 \mathrm{~m}$ $150 \mathrm{~m}<\Delta L \leq 320 \mathrm{~m})$ in order to rotational motion analyses.

## 2. CHIBA DENSE ARRAY AND EARTHQUAKE DATA

Measurement of spatial variation of the seismic wave-field for engineering applications requires dense array recordings. Depending on the aim of observation, there are several manners in the arrangement of seismometers and accelerometers. One of the first arrays was the El Centro differential array that recorded the 1979 Imperial Valley earthquake (Niazi 1986). A typical two-dimensional surface array represented by Smart-1, located in Lotung (North of Taiwan) started operation in 1980 (Oliveira and Bolt 1989). A smaller scale 3D array, the LSST array, was constructed in 1985 within the Smart-l.
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Figure 1 Chiba array configuration.
The topographical and geological conditions of the site are generally simple with the ground surface being almost flat. In Figure 1, an example of soil profile at borehole C0 that is the deepest borehole in the array was showed. In spite of a slight difference in the depth of boundaries between different layers from one borehole to another, an overall agreement is good indicating a relatively simple soil structure (Katayama et al. 1990).
The noise level in the events is estimated by taking motions prior to the first arriving energy of the event. Noise to signal ratio (NSR) for different events and station pairs were calculated by taking the root-mean-square (rms) of noise to signal Fourier spectral ratio. Seventeen events that are recorded with low NSRs and cover a wide range of magnitudes and PGAs are selected. Specifications of these events together with their NSRs for station C0 are given in Table 2.1. Variations of noise levels against frequency for each station and event were also studied. The results showed that the noise level is considerable mainly in the frequencies lower than 0.4 Hz .

Table 2.1 Specification of selected events

| No. | Even <br> t No. | Focal <br> Depth <br> (Km) | Distance <br> $(\mathrm{Km})$ | PGA (cm/s2) |  | MJM <br> A* $^{*}$ | Noise to <br> Signal Raito <br> (NSR) \% at C0 | Reliable <br> Frequency <br> Range |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  | 104.5 |  | 52.29 | 59.61 |  | 1.4 | $>0.20$ |
| 2 | 36 | 62.4 | 62.4 | 45.47 | 29.39 | 4.9 | 3.0 | $>0.50$ |
| 3 | 37 | 44.7 | 44.7 | 400 | 292.51 | 6.7 | 0.2 | $>0.20$ |
| 4 | 39 | 42.4 | 42.4 | 18.23 | 30.28 | 4.4 | 4.3 | $>0.50$ |
| 5 | 40 | 52.3 | 52.3 | 34.48 | 28.27 | 4.0 | 4.2 | $>0.50$ |
| 6 | 41 | 37.4 | 37.4 | 48.45 | 52.56 | 4.2 | 2.5 | $>0.50$ |
| 7 | 42 | 37.9 | 37.9 | 117.0 | 79.18 | 5.2 | 1.3 | $>0.30$ |
| 8 | 43 | 16.9 | 16.9 | 37.41 | 27.39 | 4.1 | 5.0 | $>0.50$ |
| 9 | 46 | 47.7 | 47.7 | 57.30 | 70.93 | 5.6 | 1.5 | $>0.30$ |
| 10 | 47 | 55.2 | 55.2 | 31.95 | 34.13 | 6.0 | 1.9 | $>0.30$ |
| 11 | 48 | 51.8 | 51.8 | 28.01 | 48.66 | 4.9 | 3.6 | $>0.50$ |
| 12 | 81 | 42.2 | 42.2 | 71.3 | 86.38 | 6.0 | 1.3 | $>0.30$ |
| 13 | 82 | 62.4 | 62.4 | 38.17 | 51.02 | 5.3 | 2.8 | $>0.30$ |
| 14 | 84 | 40.2 | 40.2 | 90.63 | 121.17 | 5.4 | 1.2 | $>0.30$ |
| 15 | 85 | 39.6 | 39.6 | 40.70 | 46.57 | 5.2 | 2.0 | $>0.40$ |
| 16 | 86 | 7.9 | 7.9 | 29.74 | 31.1 | 4.9 | 2.9 | $>0.50$ |
| 17 | 87 | 52.4 | 52.4 | 91.30 | 93.73 | 5.9 | 1.0 | $>0.20$ |

*Japan Metrological Agency

## 3. TORSIONAL GROUND MOTION

Let $\ddot{u}_{j}(t), \ddot{v}_{j}(t)$ to be the translational accelerations along the $X, Y$ axes. The torsional component of surface motion related to a station pair ( $j=1,2$ ) is computed from the acceleration time-histories according to the following finite difference expressions (Нао 1996):

$$
\begin{equation*}
\psi_{Z}=\frac{1}{2}\left(\frac{\Delta \ddot{u}}{\Delta y}-\frac{\Delta \ddot{v}}{\Delta x}\right)=\frac{1}{2}\left(\frac{\ddot{u}_{2}(t)-\ddot{u}_{1}(t)}{\Delta y}-\frac{\ddot{v}_{2}(t)-\ddot{v}_{1}(t)}{\Delta x}\right) \tag{3.1}
\end{equation*}
$$

The quantity dealt with here IS the torsional acceleration $\psi_{z}$ about the vertical axis. $\ddot{u}_{\mathrm{j}}(t)$, $\ddot{v}_{\mathrm{j}}(t)$ are the recorded acceleration in the NS, EW directions, respectively.
Array-derived rotations are subjected to a limitation. The instruments should be closely spaced enough that the finite difference approximations could be near to true gradients. Specifically, to obtain array gradient estimates accurate to $\sim 90 \%$ of true gradients, the array dimensions must be less than approximately one quarterwavelength of the dominant energy in the wave train (Bodin et al. 1997). Regarding estimated large wave velocity (Yamazaki and Turker 1992) and very closely spaced instruments in Chiba site, the rotational motions could be evaluated within acceptable accuracy range using above finite difference equations.

## 4. ESTIMATION OF TORSIOANL MOTION USING TRANSLATIONAL ONES

Since it is difficult to directly record torsional ground motions, attempts are made to predict the torsional motion from translational ones (Hao 1996). Spatial variation of ground motion can be defined by appropriate coherency function models. Several empirical coherency functions have been suggested (Loh and Yeh 1988). Although these coherency functions model the spatial variations of translational motions well, they cannot be directly used to model the torsional ground motions. Hao (1996) proposed a new form of coherency function as:

$$
\begin{equation*}
\gamma_{k l}(x, y, i \omega)=\left|\gamma_{k l}\right| \exp \left(-\frac{i \omega x}{v_{a}}\right)=\exp \left[-\left(\alpha_{1}(\omega) x^{2}+\alpha_{2}(\omega) y^{2}\right) \omega\right] \exp \left(-\frac{i \omega x}{v_{a}}\right) \tag{4.1}
\end{equation*}
$$

Where $\left|\gamma_{k l}\right|$ is the coherency loss function and $\exp \left(\frac{-i \omega x}{v_{a}}\right)$ is the phase shift between the motions at points $k$ and $l, \omega$ is the circular frequency, $v_{a}$ is the apparent velocity of ground motion, $x$ and $y$ are the separation distances between points $k$ and $l$ in the ground motion propagation direction and its transverse one, respectively. Using this coherency function, the relation between power spectral density function of torsional and translational ground motions can be given as:

$$
\begin{equation*}
S_{\ddot{\phi}}=S_{g}(\omega) \omega\left[\alpha_{1}(\omega)+\alpha_{2}(\omega)+\omega / 2 v_{a}{ }^{2}\right] \tag{4.2}
\end{equation*}
$$

Where $S_{g}(\omega)$ is the PSD of translational accelerations and is obtained by averaging the recorded translational PSDs at station pair. $\alpha_{1}(\omega)$ and $\alpha_{2}(\omega)$ are the parameters with the form:

$$
\begin{equation*}
\alpha_{j}(\omega)=\frac{a_{j}}{\ln (\omega)+b_{j}} \quad \omega \geq 0.314(\mathrm{rad} / \mathrm{s}) \quad j=1,2 \tag{4.3}
\end{equation*}
$$

Where $a_{j}$ and $b_{j}$ are determined by regression method from the recorded motions and $b_{j} \succ|\ln (0.314)=1.1584|$. Furthermore, the apparent velocity $\left(v_{a}\right)$ can be obtained by calculating the crosscorrelation functions between motions of each station pair.
The obtained apparent velocity values ranges between $4 \mathrm{~km} / \mathrm{s}$ to infinity. Yamazaki and Turker (1992) analyzed the apparent velocity at Chiba site using F-K method and found the apparent velocity is $5 \mathrm{~km} / \mathrm{s}$ in the average for a wide frequency range ( $0.5-8 \mathrm{~Hz}$ ). They and Katayama et al. (1990) concluded that the wave propagate almost vertically and the incident angle should be close to normal in Chiba site.
By employing Chiba array data and based on Equation 4.2, the relations between PSD of torsional and translational motions are examined. Since the coherency function depends on the distance between the two stations, accelerometer pairs in Chiba array are categorized in some groups. For obtaining the parameters $a_{j}$
and $b_{j}$ using optimizing scheme, $50 \%$ of data are used in each separation distance group. To this end, the PSDs related to each group are averaged for each frequency. Having an averaged PSDs for translational and torsional motions ( $S_{g}, S_{\ddot{\phi}}$ ) at each separation group, the coefficients $a_{j}$ and $b_{j}$ are evaluated by optimization scheme.
Figure 2 shows the averaged torsional PSD and the predicted one for two events (events 81 and 82) and three groups of distances.
In the next step, to check the validity of the results, the identified coefficients are applied to predict the torsional PSDs for the other $50 \%$ of data in each distance group that are not used in optimization procedure for estimating $a_{j}$ and $b_{j}$. Figure 2 shows the comparison between simulated and actual torsional PSDs for the data that are not used in the optimization (the other 50\% of torsional PSDs). As illustrated in Figure 3, the predicted torsional PSDs are not consistent with the observed ones in small separation distances. In the contrary, a good agreement between predicted and observed torsional PSDs can be seen for intermediate and large distances. This trend can be seen for almost all of the events at the site. Thus, the torsional motion should be calculated with more care using Equation 4.2 in small distances. As shown by Ghayamghamian and Nouri (2007) the values of torsional motion in close distances are very discursive. The scattering of torsional values in close separation distances may also support the idea that some other factors affected the estimation of torsional motion in close distances. However, we do not have a clear explanation for this discrepancy.

## 5. CONCLUSIONS

Because of the difficulties in recording of torsional motion, the evaluation of torsional motion from translational one is focused. The coherency function proposed by Hao (1996) was examined to predict the torsional motion from the translational ones for different separation distances and events at the site. The estimated torsional motion found to be well predicted for intermediate and far distances ( 100 m to 320 m ). However, it seems that

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other factors may need to be considered for reliable estimation of torsional motion in close separation distances.


Figure 2 Comparison of mean of torsional PSDs in each separation distance group with the predicted ones.

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Event 81

(a) $10<\Delta L \leq 30 \mathrm{~m}$



(b) $100<\Delta L \leq 150 \mathrm{~m}$

Stations P4-P5

—Predicted
— Observed

Figure 3 Comparison of the PSDs for the data, which is not employed in optimization, in different separation distances for events 81 and 82.


Figure 3 (Continued)

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