Analysis of Surface Wave Propagation Based on the Thin Layered Element Method

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ABSTRACT :

In this paper, a study is presented to obtain eigenvalues of surface waves, such as Rayleigh and Love waves, based on the two-dimensional thin layered element method. The formulation includes the computation of phase velocities, mode shapes and amplitude response functions (medium responses) of Rayleigh and Love waves propagating in a layered half-space. By comparing the computed values with the theoretical ones, it is concluded that the thin layered element method can be used as a very good alternative to the theoretical solution method. In addition to the eigenvalues, H/V amplitude ratios of the surface waves have been examined by this method, in which a synthesis of higher mode contributions of both Rayleigh and Love waves is formulated. The proposed method was applied to a site where a PS-logging is available. Finally, wave propagation in an irregular ground due to an incident Rayleigh wave is formulated based on the finite element method. It was found that wave propagation is significantly influenced by the irregularity.

KEYWORDS: Surface Wave, Thin Layered Element Method, H/V Spectra, Microtremor, Finite Element Method, Irregular Ground

1. INTRODUCTION

There are a number of methods available that are used for estimating dynamic properties of the ground. Among others, microtremor measurement is one of the simplest and the most inexpensive ways and has been conducted extensively. It is accepted that the peak of its horizontal-to-vertical spectral ratio (H/V spectrum) obtained from microtremor measurement corresponds to the natural frequency of a surface ground. Tokimatsu and Arai (1998) proved that the H/V spectra of microtremor can be explained by the theory of surface wave propagation, including higher modes in addition to the fundamental mode. Although the closed form solution to the eigenvalue problem of surface wave propagation is available, difficulties are encountered when the soil profile is complex. In such a case, a numerical approach can be used. Waas (1972) and other researchers have proposed a finite element approach to the wave propagation problem, in which the soil layer is divided into a number of sublayers and the force-displacement relationship is obtained at the vertical boundary based on the mode superposition method. This technique, known as the thin layered element method, has been used in the two-dimensional finite element analyses as the energy transmitting boundary. However, this method can also be applied to eigenvalue problems of surface wave propagation.

2. THIN LAYERED ELEMENT METHOD

In this study, a finite element technique has been applied to the wave propagation problem due to an excitation force and a surface wave incidence. A lumped mass finite element formulation for a multi-layered system with a rigid base was developed by Lysmer(1970) and was extended by Waas(1972) to include a consistent mass formulation and love waves. This method is briefly reviewed below.

2.1. Eigenvalue Problems

The layered system is treated as a continuum in the horizontal direction but is discretized in the vertical direction by assuming that the displacement is continuous at each interface and varies linearly or curvilinearly within each layer. In this study, quadratic elements are used. Based on the formulation given by Waas(1972), the equation of motion for the layered system may be written as:

$$\left({}_L k^2\left[{}_L A\right]+{}_L\left[G\right]-\omega^2\left[{}_L M\right]\right)\left\{{}_L v\right\}=\left\{0\right\} \tag{2.1}$$
$$\left({}_R k^2\left[{}_R A\right]+{}_R k\left[{}_R B\right]i+\left[{}_R G\right]-\omega^2\left[{}_R M\right]\right)\left\{{}_R v\right\}=\left\{0\right\} \tag{2.2}$$

In which, $\{v\}$ is a vector containing layer interface and mid-point node amplitudes for quadratic elements as shown in Figure 1, $[A]$,$[B]$,$[G]$,and $[M]$ are matrices assembled by the addition of layer submatrices, k is the wave number, subscript R presents Rayleigh wave, and subscript L presents Love wave. Eqs. (2.1) and (2.2) are the quadratic eigenvalue problem. A numerical technique can be applied to find eigenvalues and corresponding eigenvectors. Emphasis can be placed on the fact that among these eigenvalues there exist "physical" modes which correspond to actual Rayleigh and Love waves.

2.2. Line Load Excitation and Corresponding Amplitude Function

The load-displacement relationship, i.e. the dynamic stiffness matrix, at a vertical boundary of a semi-infinite layered region is called a transmitting boundary and is given for the region extending toward right as:

$$\left[{}_L R\right]=i\left[{}_L A\right]\left[{}_L V\right]\left[{}_L K\right]\left[{}_L V\right]^T\left[{}_L A\right] \tag{2.3}$$
$$\left[{}_R R\right]=i\left[{}_R A\right]\left[{}_R V\right]\left[{}_R K\right]\left[{}_R V\right]^{-1}+\left[{}_R D\right] \tag{2.4}$$

where $[V]$ is a matrix which contains mode shapes in its column, $[K]$is a diagonal matrix which contains wave numbers on the diagonal and $[D]$ is related to $[B]$ through $[B]=[D]^T-[D]$. The analysis of a left layered region is analogous to that of a right layered region. The dynamic stiffness matrix, $[L]$, of a left layered region may be computed from $[R]$ by changing only the sign of all the coefficients that relate horizontal forces to vertical displacements or vertical forces to horizontal displacements.

Thus, $[R]+[L]$ is the stiffness matrix at the vertical boundary x=0. Thus, we can calculate the displacement at the vertical boundary, $\{u\}$, when an external force acts at x=0. The mode participation factor,$\{\alpha\}$, can be obtained as follows:

$$\left\{{}_L \alpha\right\}=\left[{}_L V\right]^{-1}\left\{{}_L u\right\} \tag{2.5}$$
$$\left\{{}_R \alpha\right\}=\left[{}_R V\right]^{-1}\left\{{}_R u\right\} \tag{2.6}$$

It is worthy of note that the participation factors of the "physical" mode represent their medium responses (Harkrider(1964)), or amplitude functions.

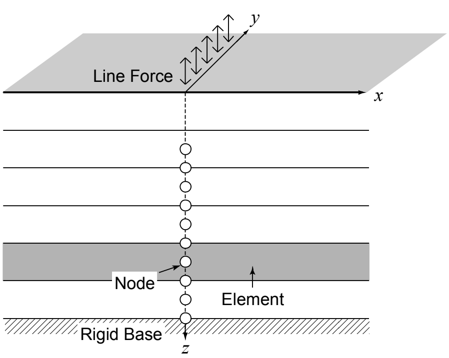

Figure 1 Thin layered element method

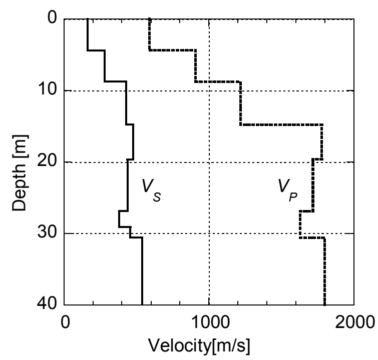

Figure 2 Wave Velocity Profile at site N

2.3. H/V Spectra due to Rayleigh and Love Waves

By consulting the work done by Tokimatsu and Arai(1998), the horizontal-to-vertical displacement ratio at the ground surface, i.e. the H/V spectra, due to Rayleigh and Love waves can be computed from the mode shapes and the participation factor by the following expressions:

$$ {}_R(H/V)=\sqrt{\sum_s \left({}_R\alpha^s\right)^2\left({}_Rv_x^s/{}_Rv_z^s\right)^2\left\{1+\gamma^2\left({}_Rv_x^s/{}_Rv_z^s\right)^2/2\right\}}\Bigg/\sqrt{\sum_s \left({}_R\alpha^s\right)^2\left\{1+\gamma^2\left({}_Rv_x^s/{}_Rv_z^s\right)^2/2\right\}} \tag{2.7}$$

$$ \gamma^2 = 2\sum_s \left({}_R\alpha^s\right)^2\left({}_Rv_x^s/{}_Rv_z^s\right)^2 \Big/ (R/L)^2\sum_s\left({}_L\alpha^s\right)^2-\sum_s \left({}_R\alpha^s\right)^2\left({}_Rv_x^s/{}_Rv_z^s\right)^2 \tag{2.8}$$

$$ (H/V)={}_R(H/V)\sqrt{1+1/(R/L)^2} \tag{2.9}$$

Where, γ is an H/V ratio of excitation forces and R/L is a Rayleigh to Love wave amplitude ratio in horizontal motions. To examine the applicability and the limitations of Eqs.(2.7)-(2.9) to the H/V spectrum of micro-tremors, three component microtremor data observed at a site in Chiba city, say site N, are used in this study. Its wave velocity profile is shown in Figure 2. Mode shapes for the frequency of 8 Hz are shown in Figure 3. Phase velocities, medium responses, and H/V spectra are shown in Figure 4. Here, R/L is set to 0.7, referring to Tokimatsu and Arai(1998). By comparing the computed values with the theoretical ones, it is concluded that the thin layered element method can be used as a very good alternative to the theoretical solution method.

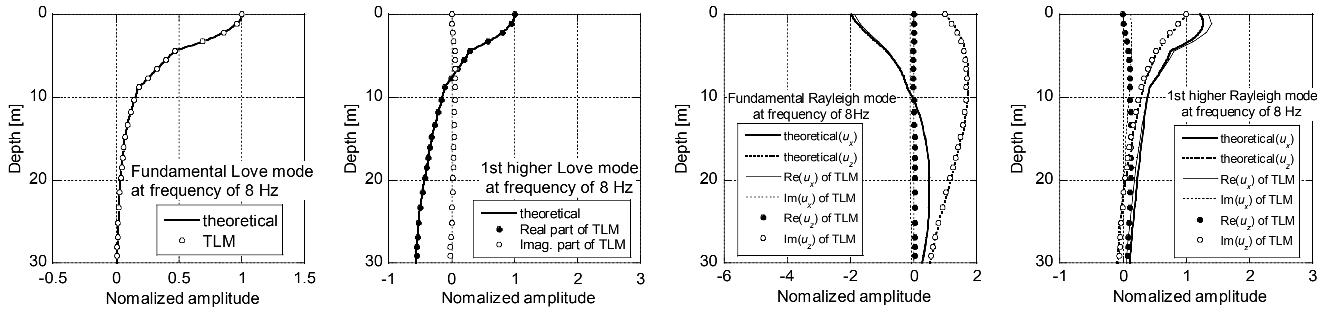

Figure 3 Comparison of mode shapes(x=0) for of 8Hz at site N

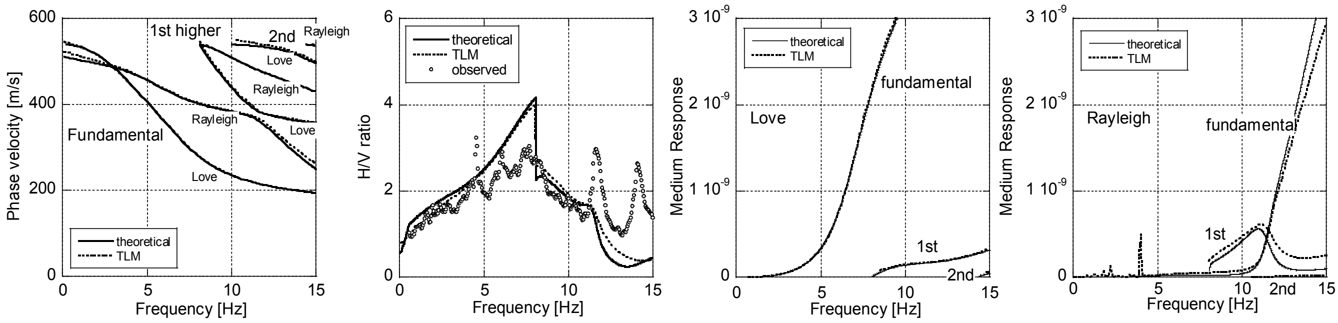

Figure 4 Comparison of phase velocities, medium responses, and H/V spectra at site N

3. INCIDENT RAYLEIGH WAVE IN AN IRREGULAR GROUND

A number of studies have been made on the surface wave propagation based on the hypothesis that the soil medium is horizontally layered. The applicability of H/V spectra of microtremor to estimation of dynamic properties of the ground is usually discussed in this context. However, we often encounter a situation in which it is difficult to assume horizontal layering. Complex landform is a typical example of this. In this study, a two-dimensional finite element approach in conjunction with the thin layered element method described earlier has been used to investigate surface (Rayleigh) wave propagation in an irregular ground. A similar topic was discussed by Drake(1972) and Uebayashi(2006).

3.1. Method of Analysis

The analysis method is based on the so-called substructure approach in elastodynamics. In this approach, an infinite medium is divided into a near field and a far field. In the analysis of the near field, the impedance matrix of the far field is attached to the near field at its boundary and the driving force due to an incident wave from the far field is applied to the boundary. Suppose a separation of a soil model into three parts along the vertical boundaries as shown in Figure 5, the driving force due to an incident Rayleigh wave propagating from left can be expressed as:

$$\{f_c\} = [{}_R L]\sum_s \alpha^s \{{}_R v\}^s + \{{}_R p\} \tag{3.1}$$

In which, $[{}_R L]$ is an impedance matrix of a left layered soil, α^s is a mode participation factor, $\{{}_R v\}^s$ is a mode shape of the incident Rayleigh wave and $\{{}_R p\}$ is a traction due to the incident wave at the boundary of the left layered soil.

3.2. Analysis Model

In this study, a valley shaped irregular ground is considered, as shown in Figure 5. The soil consists of two layers and their boundary is horizontal. The underlying layer is assumed as a half-space by changing its thickness as $H = 4Vs/f$ where Vs is its shear wave velocity and f is a frequency of analysis. The P-wave velocity and the density are the same for both layers; Vp=1500m/s, ρ=1700kg/m^3. The surface layer was partitioned into 5 elements for the vertical direction and the underlying layer was divided into 20. In order to suppress body wave reflection from the bottom, relatively large damping of 30% to 50% is assumed in the lower part of the model. Theoretical values of H/V ratio and the phase velocity of Rayleigh wave at *x*=0 and *x*=600 for the frequency of 10 Hz are shown in Table 1. When a two layered soil is assumed, the Rayleigh wave has three modes for the terrace part and two modes for the valley part, for the frequency of 10 Hz. The phase velocity of a superposed mode is obtained by referring to Tokimatsu et al(1992).

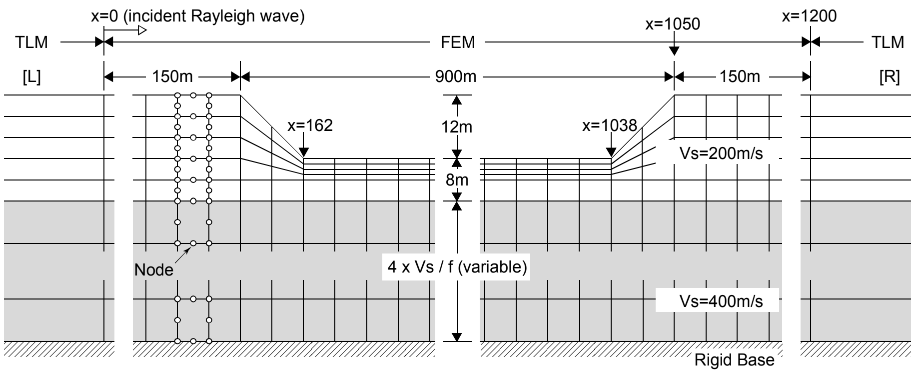

Figure 5 Finite element model

Table 1 Theoretical values of H/V ratio and phase velocity of Rayleigh wave at *x*=0 and *x*=600 for 10Hz

	terrace (*x*=0;Surface layer thickness = 20m)		*valley* (*x*=600;Surface layer thickness = 8m)	
mode	H/V ratio	phase velocity[m/s]	H/V ratio	phase velocity[m/s]
fundamental	0.54	193.1	0.17	315.4
1st higher	0.34	321.0	0.69	393.6
2nd higher	0.87	385.8	-	-
superposed	0.52	205.3	0.31	322.2

3.3. Propagation of an Incident Rayleigh wave

Figure 6 shows the wave propagation in a valley shaped irregular ground due to an incident Rayleigh wave from the left layered soil. The figure represents the results of two cases: the fundamental mode incidence and the first higher mode incidence, respectively. Each case has four different kinds of results: cross sectional distributions of the amplitude and the phase of horizontal and vertical displacements. Left to these distribution diagrams are the displacement distribution of the incident Rayleigh waves along the vertical axis.

There are a number of points that can be made from this figure. First, it is clearly seen from the phase diagrams that body waves are generated as reflected waves from the slope at the left-hand side of the valley. This occurs for each mode incidence. Striped patterns are also clearly seen in the amplitude diagrams, which is considered as a result of the interference of body waves and the incident Rayleigh wave. Next, if we look at the central part of the valley, it is noticed that the phase patterns become vertical as the distance from the left-hand slope increases. This explains that the body waves attenuate rapidly compared to the surface waves because of large damping. In this case, the surface waves dominate in the right side of the valley. Finally, if we look at the right-most part of the valley, it is seen that body waves are again generated from the slope. However, the phase patterns are fairly complex compared to the left-hand side of the valley. This implies that the propagating wave on the right-hand side of the valley is not as simple as a single mode wave propagation.

3.4. Phase Velocity

The effect of irregularity of the ground on the surface wave propagation can be examined in detail from the viewpoint of its phase velocities, by looking into the results described above. Figure 7 shows the phase velocity, as a function of the distance from the left-most boundary of the model. The bottom of the valley spans 876 meters from the distance of 162[m] through 1038[m]. Here, the phase velocity was computed from the phase difference between adjacent two nodes and was given by averaging two components obtained from horizontal and vertical displacements. By reffering to Table 1, it is found that on the terrace in the left-hand side, the phase velocity roughly corresponds to that of a two layered ground with the same soil profile as the terrace, for all cases of wave mode incidence. Variation of the value on the terrace can be attributed to the existence of body waves reflected from the slope. More variation for higher modes may be resulted from discretization errors.

3.5.Contribution of Rayleigh Wave for 10 Hz

Let us look at the lowland. The phase velocity varies a lot near the left-hand slope due to body waves generated from the slope. The variation decreases as the distance from the slope increases. In order to examine this tendency, a contribution ratio of Rayleigh waves to nodal displacements along the ground surface of the lowland (valley) was computed based on Eqs.(2.6). The computation process is as follows: first compute mode participation factors $\{_R\alpha\}$ by substituting nodal displacements $\{u\}$ into Eqs.(2.6), then separate the modal contributions by inverting Eqs.(2.6) as:

$$\{u\} = [_RV]\{_R\alpha\} \tag{3.2}$$

Notice that there exit only two modes of Rayleigh wave for the two layered soil with the same soil profile as the valley part of the model and for the frequency of 10 Hz. Thus, the nodal displacement can be expressed as a sum of the contributions of the fundamental and first higher modes of Rayleigh wave and the body waves. The results are shown in Figure 8. As can be seen from the figure that the surface wave dominates as the distance from the slope increases. The fluctuation of the finite element results is considered to be due to the body waves generated from the left-hand slope. It is also found from the figure that the higher mode prevails over the fundamental mode in the valley, which can be confirmed by the fact that the phase velocity in the right side of the valley is close to that of the first higher mode of a two layered soil as shown in Figure 7.

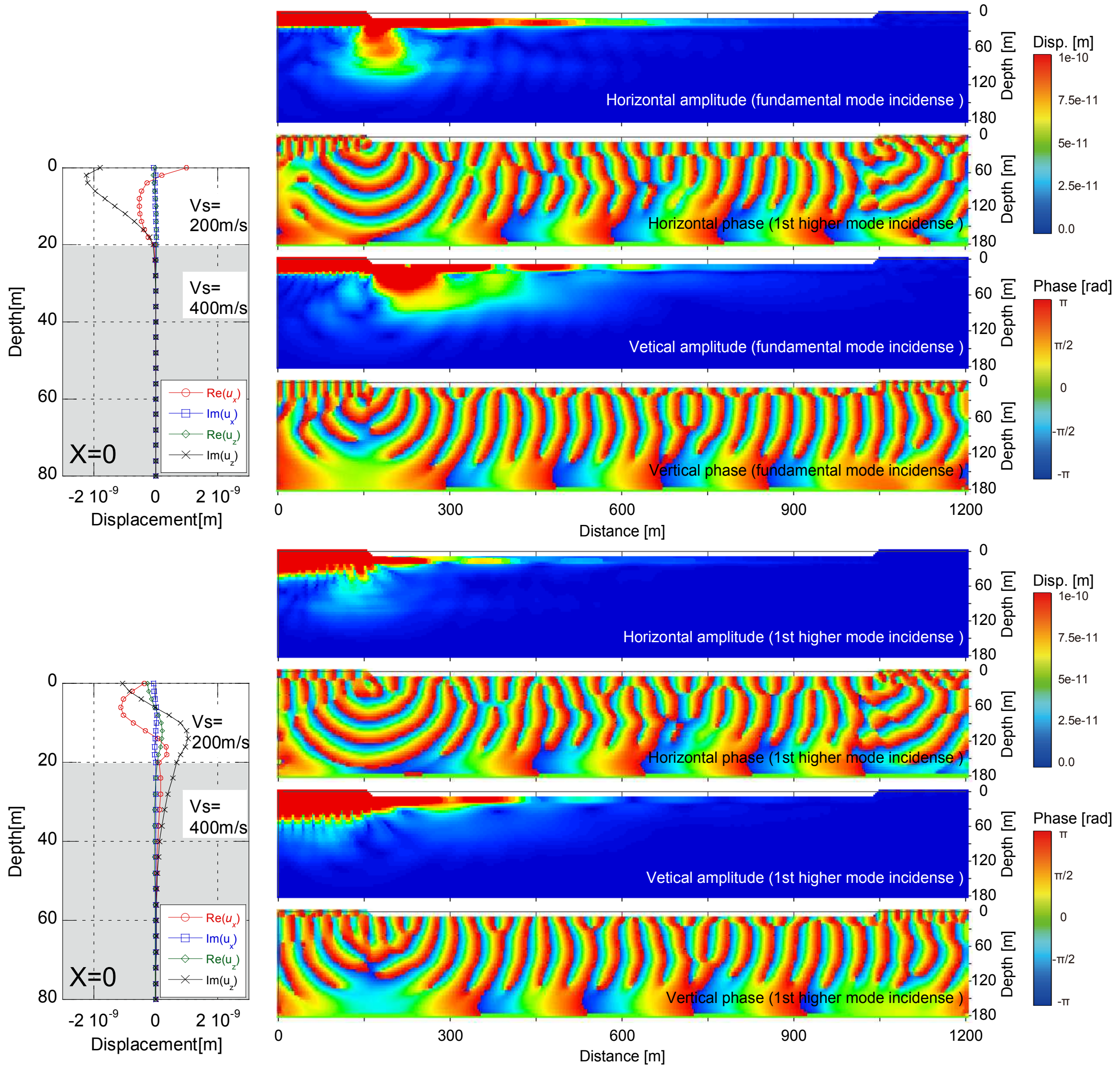

Figure 6 Rayleigh wave propagation in a valley shaped irregular ground for the frequency of 10Hz

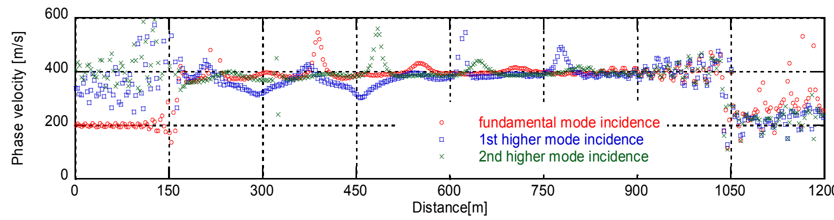

Figure 7 Phase velocity along the ground surface for the frequency of 10Hz

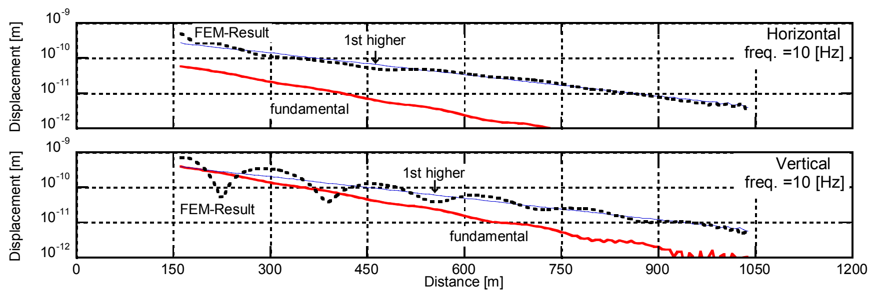

Figure 8 Contribution of the Rayleigh wave for the frequency of 10Hz (Fundamental mode incidence)

3.6.Contribution of Rayleigh Wave : Frequency Characteristics

Next, let us examine how the contribution of Rayleigh wave changes when the frequency changes. Figure 9 shows the frequency dependency of the contribution of each mode to the total displacement for various locations on the ground surface of the lowland (valley). The locations x=162 corresponds to the left-most point of the lowland and at the foot of the left-hand slope. The location x=1038 corresponds to the right-most point of the lowland and at the foot of the right-hand slope. Other locations are situated in-between.

It is clearly seen from Figure 9 that the fundamental mode determines the total displacement (FEM-Result in the figure) in the lower frequency range up to 10 Hz, where the first higher mode appears. It is also clear that, once the first higher mode appears, it dominates over the fundamental mode. This tendency becomes clear as the distance from the slope increases. The contribution of Rayleigh wave modes is relatively small near the slope, especially near the natural frequency of the surface layer (6.25 Hz), which means that the effect of generated body waves is large in this area. This result is compatible with the one out by Tokimatsu and Tamura(1995).

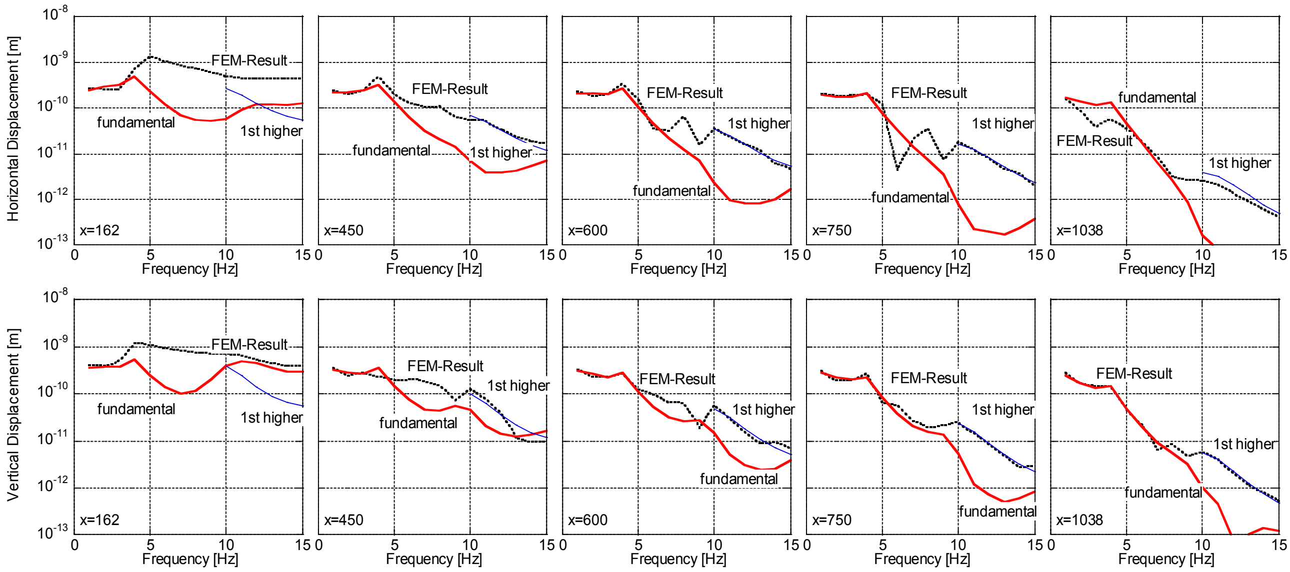

Figure 9 Contribution of the Rayleigh wave (fundamental mode incidence)

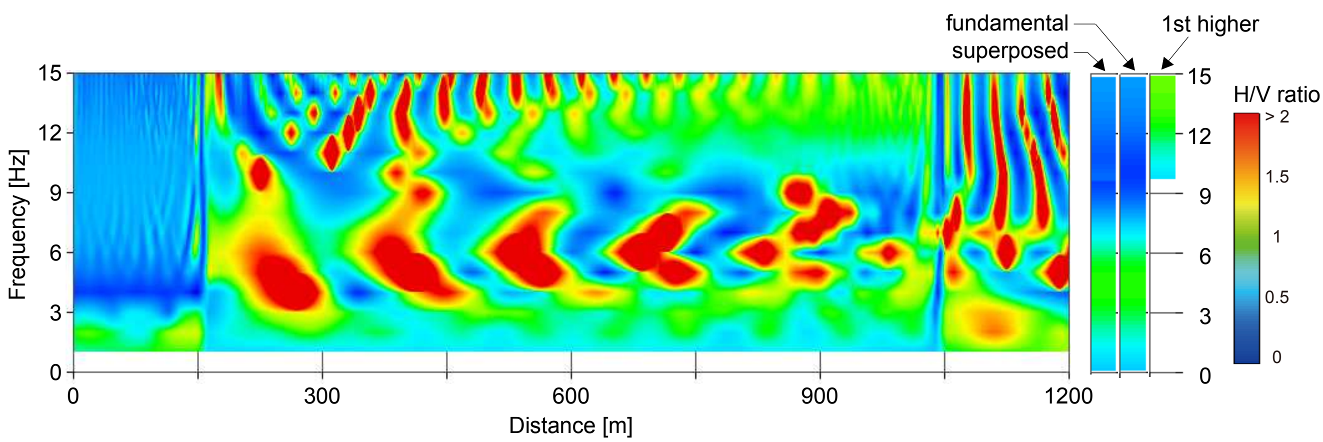

Figure 10 H/V spectrum along the ground surface (fundamental mode incidence)

Figure 10 shows the H/V spectrum distribution along the ground surface of the model. The horizontal axis represents distance, the vertical axis represents frequency, and the color expresses H/V ratios of the displacement. Three color bars on the right-hand side represent the theoretical H/V spectra of the fundamental, first higher and superposed mode of a two-layered soil corresponding to the lowland. It is found from this figure that the peak frequency and the ratio itself of H/V spectrum vary a lot in terms of the location and the frequency. The peak frequency in the low frequency range roughly coincides with that of a fundamental mode of a two-layered soil. However, this tendency becomes unclear as the location moves to the right. In the right-hand side of the lowland, the H/V spectrum in the high frequency range almost coincides with that of the first higher mode of the two-layered soil. This confirms again that the first higher mode dominates in this part. It must be noted that this phenomenon differs from what the theory based on the parallel layer assumption predicts, in which the fundamental mode dominates even in the high frequency range. The effect of irregularity of the ground needs to be further examined from this viewpoint.

4. CONCLUSIONS

From this study, the following conclusions can be made: (1) H/V spectra of a horizontally layered medium as well as the surface wave modes can be obtained based on the thin layered element method. (2) When an incident Rayleigh wave propagates toward an irregular ground, different mode of Rayleigh waves as well as body waves are generated. (3) As the distance from the irregularity increases the surface wave dominates over the body waves. (4) In the high frequency range, higher mode dominates over the fundamental mode, which differs from the theoretical prediction based on a parallel layer assumption.

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