# THE ORTHOGONAL HILBERT-HUANG TRANSFORM AND ITS APPLICATION IN EARTHQUAKE MOTION RECORDINGS ANALYSIS

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**ABSTRACT :**
Based on the numerical simulation, the problem that the intrinsic mode functions (IMFs) decomposed by the empirical mode decomposition (EMD) in Hilbert-Huang transform (HHT) are not exactly orthogonal is presented. A new method based on the Gram-Schmidt orthogonalization method referred as the orthogonal empirical mode decomposition (OEMD) is proposed and the complete orthogonal intrinsic mode functions (OIMFs) are attained. The method is validated through the decomposition of a typical time history and demonstrated through the El Centro earthquake recording. The comparison between the Hilbert spectrum, the Hilbert marginal spectrum and the orthogonal Hilbert spectrum, the orthogonal Hilbert marginal spectrum of the El Centro earthquake recording showed that the latter can more faithfully and quantitatively characterize the signal energy distribution at different frequency components.

**KEYWORDS:**
Orthogonality, empirical mode decomposition, intrinsic mode function, Hilbert-Huang transform, Earthquake motion recording analysis

## 1. INTRODUCTION

The Hilbert-Huang transform (HHT), builds on empirical mode decomposition (EMD) and Hilbert spectral analysis (HSA), developed by Huang et al. (Huang et al. 1998, 1999, 2003), can represent nonlinear and non-stationary data such as earthquake recordings. It can decompose any complicated date set via EMD into a finite, often small number of intrinsic mode functions (IMFs) that admit a well-behaved Hilbert transform. Compared with the Fourier decomposition which based on the harmonic functions and wavelet decomposition which based on the 'mother' wavelets, the EMD approach is fitter for analyzing the non-stationary data because it decomposes the signal based on the time scale of the signal itself with adaptive nature. Now the HHT method has been widely applied in many engineering domains such as earthquake engineering, system identification, damage detection and structural health monitoring etc.

Huang et al. (2001) presented a new HHT-based spectral analysis approach and pointed out that it is the only spectral analysis method applicable to non-stationary and nonlinear data. The earthquake record from station TCU 129, at Chi-Chi, Taiwan, collected during the 21 September 1999 earthquake has been used to illustrate its capability. Furthermore, Loh et al. (2001) applied the HHT-based spectral analysis approach to identify near-fault ground-motion characteristics and structural responses. It can detect the time-varying system natural frequency and damping ratio through the seismic response data of structures. Zhang et al. (2003) investigated the rational of HHT for analyzing dynamic and earthquake motion recordings. The research illustrated that HHT is suited for analyzing non-stationary dynamic and earthquake motion recordings, which is better than the conventional Fourier data processing technique in extracting some features of recordings.

Yang et al. (2003a, 2003b) successfully applied the HHT method together with the random decrement technique (RDT) to identify the modal parameters of linear structures. The possibility of using the HHT method for modal parameter identification of linear system with closely spaced modes of vibration was investigated by Chen et al. (2002). The results showed that the HHT method is more capable of identifying modal parameters when the natural frequencies are close to each other than either the FFT-based method or the wavelet transform method. Yang et al. (2004) proposed an EMD-based approach to detect the damage time instants and damage locations by identifying the damage spike due to a sudden change of structural stiffness. Numerical simulation results demonstrate that the proposed method could also identify the damage time instant and damage location using the signal feature of damage spike.

In order to ensure the completeness and orthogonality of this decomposition, all IMFs should reconstruct the original data set and they are orthogonal to each other. However, the EMD method proposed by Huang et al. (1998) is not guaranteed theoretically the orthogonality of IMFs. There is only almost numerically orthogonal among the IMFs. According to the numerical simulation in this study, the extent of orthogonality among the IMFs is actually bad and the energy leakage is severe. It's necessity to improve this problem. In this study, a new method based on the Gram-Schmidt orthogonalization method referred as the orthogonal empirical mode decomposition (OEMD) is proposed to improve the degree of orthogonality among the IMFs and the complete orthogonal intrinsic mode functions (OIMFs) are attained. Then, the method has been validated through the decomposition of a typical time history and demonstrated through the El Centro earthquake recording. The comparison between the Hilbert spectrum, the Hilbert marginal spectrum and the orthogonal Hilbert spectrum, the orthogonal Hilbert marginal spectrum of the El Centro earthquake recording showed that the orthogonal Hilbert spectrum and Hilbert marginal spectrum can more faithfully and quantitatively characterize the signal energy distribution at different frequency components.

## 2. EMPIRICAL MODE DECOMPOSITION

The empirical mode decomposition can decompose any data set into several intrinsic mode functions (IMFs) by a procedure called sifting process. Suppose $X(t)$ is the signal to be decomposed. By EMD, it can be expressed as the sum of n IMF components plus the final residue. More details can be referred Huang et al. (1998).

$$X(t) = \sum_{j=1}^{n} c_j(t) + r_n(t) \tag{2.1}$$

where $c_j(t) =$ $j$th IMF component; and $r_n(t) =$ final residue.

It should be noted that the signals in above equation are all represented in consecutive form. However, the signals collected are generally discrete. Therefore, in order to clearly demonstrate the orthogonality of IMF and the process of the OEMD in next sections, one gives the signal in discrete vector expression, i.e.

$$X(t) \Rightarrow \{X\} = \{X(t_1), X(t_2), \cdots, X(t_i), \cdots X(t_N)\} = \{X_1, X_2, \cdots, X_i, \cdots, X_N\} \tag{2.2}$$

$$c_j(t) \Rightarrow \{c_j\} = \{c_j(t_1), c_j(t_2), \cdots, c_j(t_i), \cdots c_j(t_N)\} = \{c_{j1}, c_{j2}, \cdots, c_{ji}, \cdots c_{jN}\} \tag{2.3}$$

## 3. ORTHOGONALITY OF INTRINSIC MODE FUNCTION

To ensure the rigorousness of EMD, the IMFs from EMD should be completeness which means that the IMFs could reconstruct the original signal. From Eqn. 2.1, we know that the IMFs can theoretically reconstruct the original signal. In order to check the orthogonality of IMFs from EMD, Huang et al. (1998) defined an overall index of orthogonality $IO_{\mathrm{T}}$ and a partial index of orthogonality for any two components $IO_{jk}$, i.e.

$$IO_{\mathrm{T}} = \sum_{j=1}^{n+1}\sum_{\substack{k=1\\k\neq j}}^{n+1}\int_0^T c_j(t)c_k(t)dt \bigg/ \int_0^T X^2(t)dt = \sum_{j=1}^{n+1}\sum_{\substack{k=1\\k\neq j}}^{n+1}\sum_{i=1}^{N} c_{ji}c_{ki} \bigg/ \sum_{i=1}^{N} X_i^2 \quad (3.1)$$

$$IO_{jk} = \int_0^T c_j(t)c_k(t)dt \bigg/ \int_0^T c_j^2(t)dt + \int_0^T c_k^2(t)dt = \sum_{i=1}^{N} c_{ji}c_{ki} \bigg/ \sum_{i=1}^{N}\left(c_{ji}^2 + c_{ki}^2\right) \quad (3.2)$$

Furthermore, we defined an energy index to indicate the orthogonality of IMF components. The energy of original signal $E_x$ and the energy of each IMF component $E_j$ （$j=1,\cdots,n+1$） are given by

$$E_x = \int_0^T X^2(t)\,dt = \sum_{i=1}^{N} X_i^2 \;;\; E_j = \int_0^T c_j^2(t)\,dt = \sum_{i=1}^{N} c_{ji}^2 \quad (j=1,\cdots,n+1) \quad (3.3)\ (3.4)$$

If the IMF components from EMD are exactly orthogonal to each other, the value of $IO_{\mathrm{T}}$ and $IO_{jk}$ should be zeros, the total energy of decomposed signal $E_{tot}$ should be invariable (i.e. $E_{tot} = \sum_{j=1}^{n+1} E_j = E_x$) and the energy leakage between any two IMF components $E_{jk}$ should be zero, i.e. $E_{jk} = \int_0^T c_j(t)c_k(t)\,dt = \sum_{i=1}^{N} c_{ji}c_{ki} = 0 \quad (j,k=1,\cdots,n+1;\ j\neq k)$.

Generally, because the IMFs from EMD aren't theoretically orthogonal, the value of orthogonality index is about from $10^{-2}$ to $10^{-3}$. Therefore, Huang et al. (1998) considered that there is almost orthogonal among IMFs. However, it should be noted that the extent of orthogonality isn't so good for the present computer precision. The numerical simulation in next sections will demonstrate that owing to the minor error in orthogonality that Huang et al. (1998) considered, there is actually severe energy leakage when applied EMD for the decomposition of time histories. In order to ensure the exact orthogonality of IMFs from EMD and no energy leakage due to EMD, a new method based on the Gram-Schmidt orthogonalization method referred as the orthogonal empirical mode decomposition (OEMD) will be proposed to improve the problem in the following.

## 4. THE ORTHOGONAL EMPIRICAL MODE DECOMPOSITION

Through the orthogonal processing for the IMFs from EMD, one obtains the complete orthogonal IMF components, the procedure listed as follow. (1) Using EMD, signal $X(t)$ is expressed as the sum of n IMF components $\bar{c}_j(t)\,(j=1,2,\cdots,n)$ and the final residue $r_n(t)$, i.e. $X(t) = \sum_{j=1}^{n}\bar{c}_j(t) + r_n(t)$. （2） First, $c_1(t)$ is defined as the first orthogonal IMF (OIMF) component of signal $X(t)$, where $c_1(t) = \bar{c}_1(t)$. （3） As we know from the EMD, there isn't theoretically guarantee that $\bar{c}_2(t)$ is orthogonal to $\bar{c}_1(t)$. Therefore, in order to get the second OIMF component of $X(t)$, one may adopt this measure which removes partial $c_1(t)$ from $\bar{c}_2(t)$. Then, $c_2(t)$ is given by

$$c_2(t) = \bar{c}_2(t) - \beta_{21}c_1(t) \quad (4.1)$$

where, $c_2(t)$ is the second OIMF component of $X(t)$ which is orthogonal to $c_1(t)$, $\beta_{21}$ is defined as the orthogonality coefficient between $\bar{c}_2(t)$ and $c_1(t)$. Producing $c_1(t)$ and performing integral transform about time t of both sides of Eqn.4.1 and using the orthogonal characteristic between $c_2(t)$ and $c_1(t)$, it can be shown that $\beta_{21}$ can be deduced as follow.

$$\int_0^T c_1(t)c_2(t)\,dt = \int_0^T \bar{c}_2(t)c_1(t)\,dt - \beta_{21}\int_0^T c_1^2(t)\,dt = 0 \quad (4.2)$$

$$\beta_{21}=\int_0^T \bar{c}_2(t)c_1(t)dt\Big/\int_0^T c_1^2(t)dt \;\; ; \;\; \beta_{21}=\{\bar{c}_2\}^T\{c_1\}/\{c_1\}^T\{c_1\}=\sum_{i=1}^{N}\bar{c}_{2i}c_{1i}\Big/\sum_{i=1}^{N}c_{1i}^2 \qquad (4.3)\ (4.4)$$

（4）Adopting the same measure proposed above, by removing all the former j OIMF components from the (j+1)th IMF component of $X(t)$ from EMD, it can be obtained the (j+1)th OIMF component of $X(t)$. $c_{j+1}(t)(j=2,\cdots,n-1)$ is given by

$$c_{j+1}(t)=\bar{c}_{j+1}(t)-\sum_{i=1}^{j}\beta_{j+1,i}c_i(t) \qquad (4.5)$$

Producing $c_k(t)(k\le j)$ and performing integral transform about time t of both sides of Eqn.4.5 and using the orthogonal characteristic between $c_k(t)$ and $c_i(t)(i\ne k)$, it can be shown that $\beta_{j+1,i}$ can be deduced as follow.

$$\int_0^T c_{j+1}(t)c_k(t)dt=\int_0^T \bar{c}_{j+1}(t)c_k(t)dt-\sum_{i=1}^{j}\beta_{j+1,i}\int_0^T c_k(t)c_i(t)dt=0 \qquad (4.6)$$

$$\beta_{j+1,i}=\int_0^T \bar{c}_{j+1}(t)c_i(t)dt\Big/\int_0^T c_i^2(t)dt \;\; ; \;\; \beta_{j+1,i}=\{\bar{c}_{j+1}\}^T\{c_i\}/\{c_i\}^T\{c_i\}=\sum_{m=1}^{N}\bar{c}_{j+1,m}c_{i,m}\Big/\sum_{m=1}^{N}c_{i,m}^2 \qquad (4.7)\ (4.8)$$

The above orthogonal processing process for IMF components is referred as the orthogonal empirical mode decomposition (OEMD). After performing some algebraic operation, $X(t)$ is expressed as,

$$X(t)=c_1^*(t)+c_2^*(t)+c_3^*(t)+\cdots+c_j^*(t)+\cdots c_{n-1}^*(t)+c_n^*(t)+r_n(t)=\sum_{j=1}^{n}c_j^*(t)+r_n(t)=\sum_{j=1}^{n}a_jc_j(t)+r_n(t) \qquad (4.10)$$

Where $a_j=\sum_{i=j}^{n}\beta_{i,j}(j=1,2,\cdots,n)$, $\beta_{i,j}=1(i=j)$.

It’s obviously shown from the above procedure that the IMF components $c_j(t)(j=1,2,\cdots,n)$ are orthogonal to each other. The algebraic operation for each component $c_j(t)$ will not change its orthogonality among components. Therefore, the components $c_j^*(t)(j=1,2,\cdots,n)$ are also orthogonal to each other. Thus, $X(t)$ is expressed as the sum of n OIMF components $c_j^*(t)(j=1,2,\cdots,n)$ and the final residue $r_n(t)$.

It should be noted that the OEMD method don’t change the extraction process of IMF from EMD, it’s actually an orthogonal processing and regroup process to IMFs in numerical. Furthermore, owing to the almost orthogonality existed among IMFs, the OEMD for extracting OIMF proposed in this study can not only basically guarantee the attribute of intrinsic mode function, but also ensure the exact orthogonal among OIMFs. More details can be referred to Huang (2007). The validity of OEMD will be demonstrated in the following.

## 5. NUMERICAL SIMULATION

### *5.1. Validity of Orthogonality Index*

In order to illustrate the real precision of orthogonality indexes in the present computer level, one takes three sine waves with different frequency into account, i.e., $x_j(t)=\sin(2\pi f_jt)$, $j=1,2,3$. Where, $f_1=1Hz$, $f_2=2Hz$, $f_3=3Hz$, duration $T=5s$ and sampling frequency $f_s=100Hz$. According Eqn.3.1 and Eqn.3.2, the calculated orthogonality indexes among IMFs are

$$IO=\begin{bmatrix} 0.5 & 8.169\times10^{-17} & 5.847\times10^{-17} \\ & 0.5 & 2.430\times10^{-16} \\ \text{symmetry} & & 0.5 \end{bmatrix} \quad \text{and} \quad IO_T = 2.555\times10^{-16}$$

As we know, the above three sine waves are theoretically orthogonal to each other. Therefore, the calculated orthogonality indexes should be zeros. Howerver, because of the present computer precision level, the value of orthogonality indexes are about in the magnitude of $10^{-16}$ which shows that it will be exactly orthogonal when the orthogonality index equals approximately to this magnitude.

Furthermore, for validation the validity of energy index representing the orthogonality , one considers the signal constructed by these above three sine waves, i.e. $x(t)=\sum_{j=1}^{3} x_j(t)$. According Eqn.3.3 and Eqn.3.4, some energy indexes have been calculated, where signal energy $E_x = 750$, sine waves energy $E_j = 250\,(j=1,2,3)$. Based on the results that signal total energy $E_{tot} = \sum_{j=1}^{3} E_j = E_x$ and cross energy among sine waves $E_{jk} = 0 \quad (j,k=1,2,3;\ j\neq k)$, one can conclude that the energy index can also reflect the orthogonality of data.

### *5.2. Orthogonality Validity of IMF from EMD and OEMD for a Typical Time History*

One considers a typical time history, $X(t)=\sum_{j=1}^{3} a_j \sin 2\pi f_j t$, where $f_1 = 1Hz$, $f_2 = 5Hz$, $f_3 = 10Hz$, $a_1 = a_2 = a_3 = 1$, duration $T = 5s$ and sampling frequency $f_s = 100Hz$. The temporal waveform is shown in Figure 1.

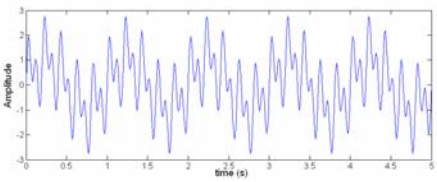
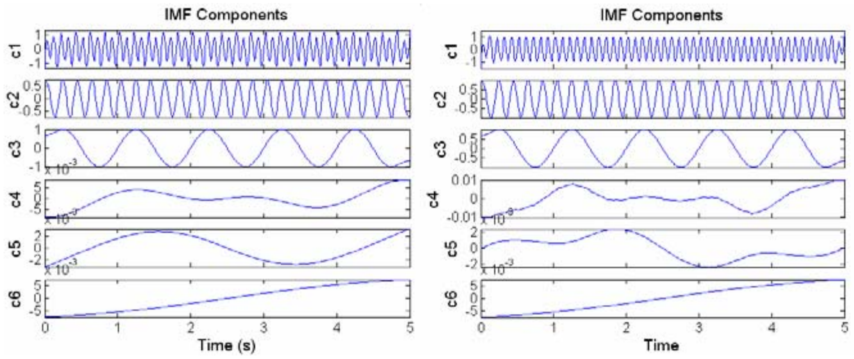


Figure 1 Typical time history




（a）results from EMD （b）results processed by OEMD

Figure 2 IMF components and residue

One gets 5 IMF components c1 to c5 and 1 residue c6 (shown in Figure 2(a)) by EMD to the above time history. Applying OEMD to these five IMF components, we get 5 OIMF components c1 to c5 and 1 residue c6 (shown in Figure 2(b)).

Table 5.1 shows the orthogonality indexes of IMFs from EMD and the orthogonality indexes of OIMFs processed by OEMD in which the numbers in upper triangle blanks represent the orthogonality indexes of IMFs and the numbers in lower triangle blanks represent the orthogonality indexes of OIMFs processed by OEMD. Table 5.1 shows the orthogonality of OIMFs is better than the orthogonality of IMFs from EMD and the magnitude of the orthogonality indexes of OIMFs is less than $10^{-15}$, whereas the magnitude of the orthogonality indexes of IMFs is more than $10^{-5}$. Furthermore, the total orthogonality index $IO_T$ is 0.06633 by EMD and the total orthogonality index $IO_T$ is reduced to 0.0005077 when IMFs has been processed by OEMD. All these results demonstrate the extent of orthogonality of IMFs has been markedly improved and these OIMFs processed by OEMD may be treated as exactly orthogonal to each other.

It should be noted that c6 is the residue in Table 5.1 which isn't orthogonal to all the former IMFs. So, compared with the orthogonality indexes between c6 with the other IMFs, the orthogonality indexes between c6 with the other OIMFs haven't been improved even been declined. However, due to the amplitude of c6 is so small that it has only little impact on the total orthogonality index $IO_T$ .

Table 5.1 Orthogonality index of IMF / OIMF components

| IMF/OIMF | c1 | c2 | c3 | c4 | c5 | c6 |
|---|---|---|---|---|---|---|
| c1 | 0.5 | $1.11\times10^{-1}$ | $3.86\times10^{-3}$ | $8.92\times10^{-5}$ | $3.21\times10^{-5}$ | $7.90\times10^{-5}$ |
| c2 | $8.57\times10^{-17}$ | 0.5 | $4.85\times10^{-2}$ | $2.05\times10^{-4}$ | $8.63\times10^{-5}$ | $1.81\times10^{-4}$ |
| c3 | $4.79\times10^{-18}$ | $4.45\times10^{-18}$ | 0.5 | $1.53\times10^{-3}$ | $5.21\times10^{-4}$ | $1.23\times10^{-3}$ |
| c4 | $3.31\times10^{-19}$ | $1.58\times10^{-19}$ | $1.75\times10^{-18}$ | 0.5 | $3.00\times10^{-1}$ | $1.82\times10^{-3}$ |
| c5 | $3.61\times10^{-20}$ | $3.75\times10^{-20}$ | $8.02\times10^{-20}$ | $1.56\times10^{-16}$ | 0.5 | $6.11\times10^{-2}$ |
| c6 | $1.03\times10^{-4}$ | $1.24\times10^{-4}$ | $1.23\times10^{-3}$ | $1.67\times10^{-1}$ | $1.72\times10^{-1}$ | 0.5 |

Table 5.2 shows the value of signal energy $E_x$, the value of IMF or OIMF component energy (i.e., $E_j$, j=1,2,…,6) and the sum of IMF or OIMF component energy $E_{tot}$ . The error between the signal energy $E_x$ and the sum of IMF component energy $E_{tot}$ is up to 11.69% which illustrates the extent of orthogonality among IMFs is bad. Whereas the error between the signal energy $E_x$ and the sum of OIMF component energy $E_{tot}$ is only 0.09% which further shows the extent of orthogonality among OIMFs is good and the OEMD method have remarkably improved the orthogonality among OIMFs and these OIMFs may be treated as exactly orthogonal.

Table 5.2 Signal energy, IMF component energy and sum of IMF component energy

| IMF/OIMF | $E_x$ | $E_1$ | $E_2$ | $E_3$ | $E_4$ | $E_5$ | $E_6$ | $E_{tot}$ | error/% |
|---|---|---|---|---|---|---|---|---|---|
| EMD | 750 | **265.84** | **136.66** | **259.77** | 0.0076 | 0.0020 | 0.0119 | 662.2914 | -11.69 |
| OEMD | | **251.19** | **240.93** | **258.49** | 0.0134 | 0.0009 | 0.0119 | 750.6410 | 0.09% |

It can be seen that the first three IMF components (i.e., c1 to c3) shown in Figure 2(a) can represent above three sine waves which constructed the typical time history. Table 5.2 shows the energy of the first three IMF components are 251.19, 240.93 and 258.49 respectively which is comparable with the energy value of the ideal sine waves (i.e., 250). Furthermore, the three sine waves and the first three OIMF components processed through OMED are comparably drawn in Figure 3. Figure 3 shows they are almost identical except in end zones. The little error is due to the end effect of EMD and it can be eliminated through the mirror extend to the original signal.

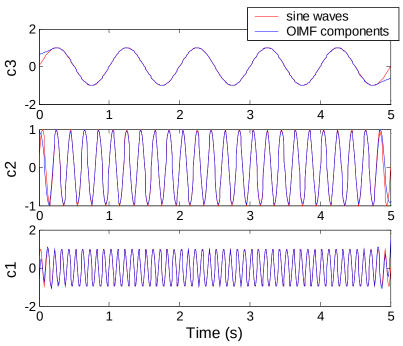



Figure 3 Comparison between the sine waves with OIMF components

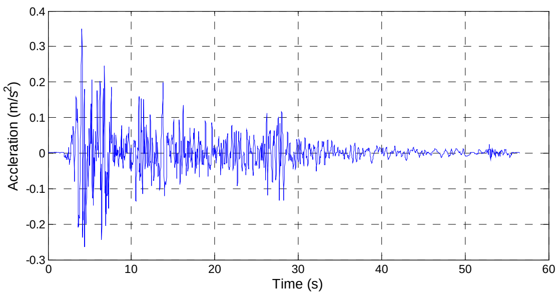
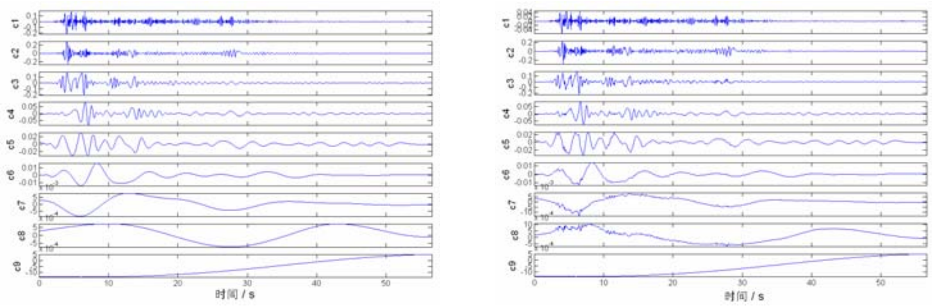



Figure 4 El Centro earthquake recording

## 6. APPLICATION IN EARTHQUAKE MOTION RECORDING

Figure 4 shows the El Centro earthquake recording. One gets 8 IMF components c1 to c8 and 1 residue c9 (shown in Figure 5(a)) by EMD to this recording. Applying OEMD to these eight IMF components, we get 8 OIMF components c1 to c8 and 1 residue c9 (shown in Figure 5(b)). The orthogonality indexes of IMFs from

EMD and the orthogonality indexes of OIMFs processed by OEMD are been calculated. (because of the length of paper, they aren't listed here. It can be referred to Huang (2007)). The value of orthogonality indexes among IMF components is more than $10^{-4}$ and the value of orthogonality indexes among OIMF components is less than $10^{-16}$. Table 6.1 shows the signal energy, each IMF or OIMF component energy and the sum of IMF or OIMF component energy. There is big difference between the signal energy with the sum of IMF component energy, it is up to 43.05%. Whereas the error between the signal energy with the sum of OIMF component energy is minor, it is only 0.18%. Furthermore, the calculated overall orthogonality index of IMF and OIMF components are 0.3992 and 0.001626 respectively. All these results demonstrate the OEMD method can remarkably improve the orthogonality of IMF, get completely orthogonal IMF components and reduce the energy leakage introduced by EMD.



（a）results from EMD　　（b）results processed by OEMD

Figure 5 IMF components and residue

Table 6.1 Signal energy, IMF component energy and sum of IMF component energy

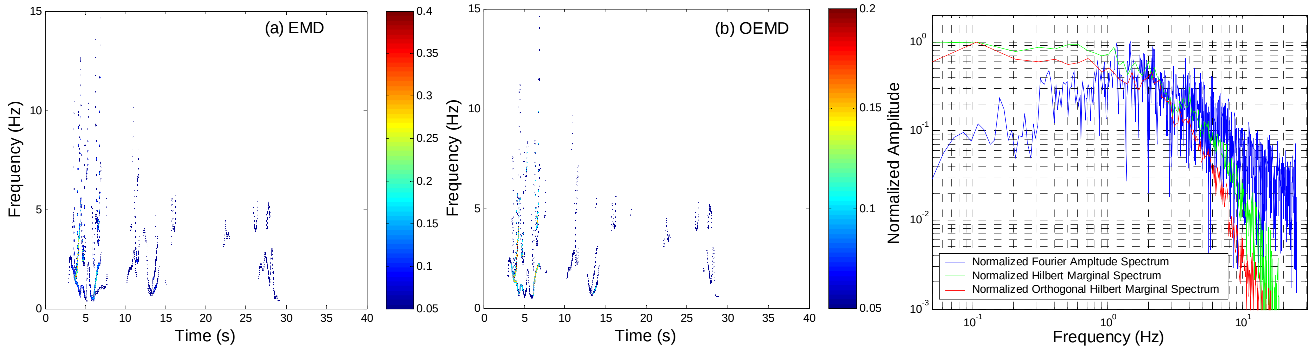
| | $E_x$ | $E_1$ | $E_2$ | $E_3$ | $E_4$ | $E_5$ | $E_6$ | $E_7$ | $E_8$ | $E_9$ | $E_{tot}$ | error /% |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| EMD | 5.7098 | 1.8771 | 2.6763 | 2.8189 | 0.5072 | 0.1943 | 0.054 | 0.0367 | 0.0007 | 0.0025 | 8.1678 | 43.05 |
| OEMD | | 0.1342 | 2.5577 | 2.3026 | 0.4488 | 0.1967 | 0.0388 | 0.0380 | 0.0006 | 0.0025 | 5.7200 | 0.18 |

Figure 6(a) shows the Hilbert spectrum of the El Centro earthquake recording. Figure 6(b) shows the Hilbert spectrum for OIMF components of the El Centro earthquake recording which designated as the orthogonal Hilbert spectrum. Compared with these two figures, we can see that the total energy represented in Figure 6(b) is less than the total energy represented in Figure 6(a) which is consistent with the result listed in Table 6.1.

Figure 6(c) shows the normalized Fourier amplitude spectrum (NFAS), the normalized Hilbert marginal spectrum (NHMS) and the normalized Hilbert marginal spectrum for OIMF which is designated as the normalized orthogonal Hilbert marginal spectrum (NOHMS) of El Centro earthquake recording. Compared the NFAS with the NHMS and the NOHMS, one can conclude that in low-frequency band the energy distribution of NHMS and NOHMS is higher than the energy distribution of NFAS. Compared the NHMS with the NOHMS, one can conclude that the value of the NHMS is larger than the value of the NOHMS whenever in low-frequency band or in high-frequency band. This phenomena shows the sum of IMF components energy is larger than the sum of OIMF components energy which is consistent with the result listed in Table 6.1.

## 7. CONCLUSIONS

In this paper, firstly, the problem that the intrinsic mode functions (IMFs) decomposed by the empirical mode decomposition (EMD) in Hilbert-Huang transform (HHT) are not exactly orthogonal is presented through the numerical simulation. Then, a new method based on the Gram-Schmidt orthogonalization method referred as the orthogonal empirical mode decomposition (OEMD) is proposed and the complete orthogonal intrinsic mode functions (OIMFs) are attained. The validity of the OEMD method is validated through the decomposition of a typical time history. The orthogonality index and the energy index are all used to demonstrate the improvement of the orthogonality of IMF components. The application to the El Centro earthquake recording shows the OEMD method is promising. The comparison between the Hilbert spectrum, the Hilbert marginal spectrum and the orthogonal Hilbert spectrum, the orthogonal Hilbert marginal spectrum of the El Centro earthquake recording showed that the latter can more faithfully and quantitatively characterize the signal energy distribution at different frequency components.




(a) Hilbert Spectrum (b) Orthogonal Hilbert Spectrum (c) Hilbert marginal spectrum

Figure 6 Hilbert spectrum and Hilbert marginal spectrum El Centro earthquake recording

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