GROUND MOTION OF NON-CIRCULAR ALLUVIAL VALLEY FOR INCIDENT PLANE SH-WAVE

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ABSTRACT :

The seismic ground motions are studied in an infinite half-space with a non-circular alluvial valley under time harmonic incident anti-plane shear waves. Based on the conformal mapping method and Fourier series expansions, the conditions of displacement continuity and stress equilibrium at the interface of alluvial valley are set as semi-circular alluvial valley in conformal plane, then the result is obtained by constructing a set of infinite linear algebraic equations with boundary discretization. The unknown coefficients in the algebraic system can be easily determined. The present method is treated as a semi-analytical solution since error only attributes to the truncation of Fourier series. Earthquake analysis for the site response of alluvial valley or canyon subject to the incident SH-wave is the main concern. Numerical examples for semi-elliptic alluvial valley are given to test our program. The research indicates that great interaction exists between the alluvial valley and the horizontal surface, which will bring on great influence on ground motion. Therefore enough importance must be attached to the existing of subsurface cavity while finishing seismic design.

KEYWORDS: ground motion, SH-waves, non-circular alluvial valley, conformal mapping method

1. INTRODUCTION

One of the major concerns of engineering seismology is to understand and explain vibrational response of the soil excited by earthquakes. The problem of the scattering and diffraction of SH-waves by a two-dimensional arbitrary number and location of cavities and inclusions in full and half space has obtained abundant fruits. In 1971, Trifunac has solved the problem of a single semi-cylindrical alluvial valley. Later, Pao and Mow have published a book on the stress concentration in 1972. In the following years, In 1999, Lee V W and Chen S studied the problem of anti-plane diffraction from canyon above subsurface unlined tunnel. In 2001, Abdul Hayir considered a dike with flexible soil-structure interface to incident SH waves. In 2005, Qiu F Q studied the ground motion of isosceles triangular hill. In order to extend to arbitrary shape inclusion problems, Lee and Manoogian have used the weighted residual method to revisit the problem of scattering and diffraction of SH-wave with respect to an underground cavity of arbitrary shape in a two-dimensional elastic half-plane. According to the literature review, it is observed that exact solutions for boundary value problems are only limited for simple cases. In this paper, the conformal mapping method is utilized to solve the half-plane radiation and scattering problems with non-circular boundaries. Owing to the existence of the non-circular interface, the elastic wave, which produced by alluvial valley, is reflected and scattered many times. Thus the solution of displacement field satisfying the boundary conditions is difficult to present. To overcome this difficulty, using conformal mapping method in here, this problem can be solved as circular boundaries in conformal plane.

2. THE GOVERNING EQUATIONS

The half-space problem with a non-circular alluvial valley to be analyzed is shown in Fig.1. The infinite half-space and alluvial valley are assumed to be elastic, isotropic and homogenous. The mechanical field corresponding to a steady state incident elastic wave can be expressed as follows

$$w^*(x,y,z,t) = w(x,y,z)e^{-i\omega t} \tag{2.1}$$

in which w^* represent the desired field variable. The exponential time harmonic factor $e^{-i\omega t}$ will be suppressed and only time-independent variable $w(x,y,z)$ will be considered for the sake of convenience.

In the absence of body forces and free charges, the governing equation of the anti-plane SH-wave harmonic motion is

$$\frac{\partial^2 w_j}{\partial z \partial \bar{z}} + \frac{k_j^2}{4} w_j = 0 \tag{2.2}$$

where, $k_j = \omega / c_s^j, c_s^j = \sqrt{\mu_j / \rho_j}, (j = 1,2)$, μ_j, ρ_j and ω are the material properties of shear modulus, the density and the frequency, respectively. The anti-plane displacement field is defined as

$$u = v = 0, \quad w = w(x,y) \tag{2.3}$$

where, w is the only non-vanishing component of displacement with respect to the *Cartesian* coordinate which is a function of x and y.
The relative shear stresses in the complex plane can be expressed as

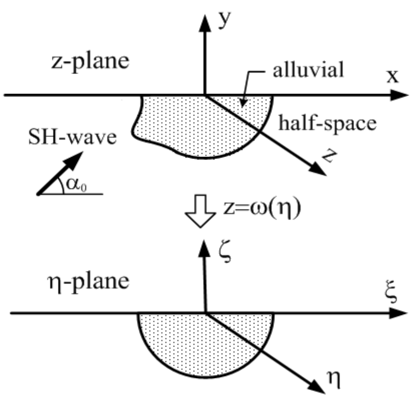
$$\tau_{rz} = \mu_j \left[\partial w_j / \partial z + \partial w_j / \partial \bar{z} \right], \tau_{\theta z} = i\mu_j \left[\partial w_j / \partial z - \partial w_j / \partial \bar{z} \right] \tag{2.4}$$

Complex variable and conformal mapping method can be employed conveniently to solve the boundary value problem for an infinite half-space with a non-circular alluvial valley. Introducing a complex variable $z = \omega(\eta)$, if only $\omega'(\eta) \neq 0$ in the mapping domain, the non-circular alluvial valley in the z-plane can be transformed into a semi-circular alluvial valley in the η-plane with the help of conformal mapping method (see figure 1). In the mapping η, the governing equation (2.1) can be rewritten as

$$\frac{1}{\omega'(\eta)\overline{\omega'(\eta)}} \frac{\partial^2 w_j}{\partial \eta \partial \bar{\eta}} + \frac{k_j^2}{4} w_j = 0 \tag{2.5}$$

The constitution relations (3) can be rewritten η-plane in the as

$$\tau_{rz} = \frac{\mu_j}{|\omega'(\eta)|} \left[\eta \frac{\partial w_j}{\partial \eta} + \bar{\eta} \frac{\partial w_j}{\partial \bar{\eta}} \right], \tau_{\theta z} = \frac{i\mu_j}{|\omega'(\eta)|} \left[\eta \frac{\partial w_j}{\partial \eta} - \bar{\eta} \frac{\partial w_j}{\partial \bar{\eta}} \right] \tag{2.6}$$



ρ_1: density of alluvial vally
ρ_2: density of half-space
μ_1: density of alluvial vally
μ_2: density of half-space
α_0: the angle of incident wave and x-axis

Figure 1 Conformal mapping from z-plane to η-plane

3. BOUNDARY VALUE PROBLEMS

Assuming that the infinite half-space is subjected to an anti-plane harmonic shear wave direct at an incident angle α_0 with the positive x-axis (see figure 1). In the mapping η, the incident elastic displacement $w^{(i)}$ can be generally expressed as follows

$$w^{(i)} = w_0 \exp\left\{\frac{ik_1}{2}\left[\omega(\eta)e^{-i\alpha_0} + \overline{\omega(\eta)}e^{i\alpha_0}\right]\right\} \tag{3.1}$$

in which w_0 is the magnitude of incident displacement wave and time-harmonic factory $e^{-i\omega t}$ has been omitted.

Relatively, the reflected wave produced by the free surface of half space can be expressed as

$$w^{(r)} = w_0 \exp\left\{\frac{ik_1}{2}\left[\omega(\eta)e^{i\alpha_0} + \overline{\omega(\eta)}e^{-i\alpha_0}\right]\right\} \tag{3.2}$$

The scattering elastic displacement $w^{(s)}$ caused by the semi-circular alluvial valley in the η-plane which satisfy the government equation (1) has the following forms

$$w^{(s)} = \sum_{n=0}^{\infty} A_n H_n^{(1)}\left[k_1\left|\omega(\eta)\right|\right]\left\{\left[\frac{\omega(\eta)}{\left|\omega(\eta)\right|}\right]^n + \left[\frac{\omega(\eta)}{\left|\omega(\eta)\right|}\right]^{-n}\right\} \tag{3.3}$$

in which $H_n^{(1)}(*)$ is *Hankel* function of the first kind of n order, A_n are unknown constants.

The standing wave produced by the semic-circular alluvial valley in the η-plane can be expressed as

$$w^{(st)} = \sum_{n=0}^{\infty} B_n J_n\left[k_2\left|\omega(\eta)\right|\right]\cdot\left\{\left[\frac{\omega(\eta)}{\left|\omega(\eta)\right|}\right]^n + \left[\frac{\omega(\eta)}{\left|\omega(\eta)\right|}\right]^{-n}\right\} \tag{3.4}$$

in which J_n is *Bessel* function of the first kind of n order, B_n are unknown constants.

So the relative stress caused by incident wave $w^{(i)}$、reflected wave $w^{(r)}$、scattering wave $w^{(s)}$ and standing wave $w^{(st)}$ can be obtained, they are

$$\begin{aligned}
\tau_{rz}^{(i)} &= \frac{i\mu_1 k_1 w_0}{2}\exp\left\{\frac{ik_1}{2}\left[\omega(\eta)e^{-i\alpha_0} + \overline{\omega(\eta)}e^{i\alpha_0}\right]\right\}\cdot\left[\frac{\eta\omega'(\eta)}{\left|\omega'(\eta)\right|}e^{-i\alpha_0} + \frac{\overline{\eta}\overline{\omega'(\eta)}}{\left|\omega'(\eta)\right|}e^{i\alpha_0}\right] \\
\tau_{\theta z}^{(i)} &= -\frac{\mu_1 k_1 w_0}{2}\exp\left\{\frac{ik_1}{2}\left[\omega(\eta)e^{-i\alpha_0} + \overline{\omega(\eta)}e^{i\alpha_0}\right]\right\}\cdot\left[\frac{\eta\omega'(\eta)}{\left|\omega'(\eta)\right|}e^{-i\alpha_0} + \frac{\overline{\eta}\overline{\omega'(\eta)}}{\left|\omega'(\eta)\right|}e^{i\alpha_0}\right]
\end{aligned} \tag{3.5}$$

$$\begin{aligned}
\tau_{rz}^{(r)} &= \frac{i\mu_1 k_1 w_0}{2}\exp\left\{\frac{ik_1}{2}\left[\omega(\eta)e^{i\alpha_0} + \overline{\omega(\eta)}e^{-i\alpha_0}\right]\right\}\cdot\left[\frac{\eta\omega'(\eta)}{\left|\omega'(\eta)\right|}e^{i\alpha_0} + \frac{\overline{\eta}\overline{\omega'(\eta)}}{\left|\omega'(\eta)\right|}e^{-i\alpha_0}\right] \\
\tau_{\theta z}^{(r)} &= -\frac{\mu_1 k_1 w_0}{2}\exp\left\{\frac{ik_1}{2}\left[\omega(\eta)e^{i\alpha_0} + \overline{\omega(\eta)}e^{-i\alpha_0}\right]\right\}\cdot\left[\frac{\eta\omega'(\eta)}{\left|\omega'(\eta)\right|}e^{i\alpha_0} + \frac{\overline{\eta}\overline{\omega'(\eta)}}{\left|\omega'(\eta)\right|}e^{-i\alpha_0}\right]
\end{aligned} \tag{3.6}$$

$$\tau_{rz}^{(s)}=\frac{\mu_1 k_1}{2}\sum_{n=0}^{\infty}A_n\left\{T_{n-1}\cdot\frac{\eta\omega'(\eta)}{|\omega'(\eta)|}-T_{n+1}\cdot\frac{\overline{\eta\omega'(\eta)}}{|\omega'(\eta)|}\right\}$$
$$\tau_{\theta z}^{(s)}=\frac{i\mu_1 k_1}{2}\sum_{n=0}^{\infty}A_n\left\{T_{n-1}\cdot\frac{\eta\omega'(\eta)}{|\omega'(\eta)|}+T_{n+1}\cdot\frac{\overline{\eta\omega'(\eta)}}{|\omega'(\eta)|}\right\} \tag{3.7}$$

$$\tau_{rz}^{(st)}=\frac{\mu_2 k_2}{2}\sum_{n=0}^{\infty}B_n\left\{S_{n-1}\cdot\frac{\eta\omega'(\eta)}{|\omega'(\eta)|}-S_{n+1}\cdot\frac{\overline{\eta\omega'(\eta)}}{|\omega'(\eta)|}\right\}$$
$$\tau_{\theta z}^{(st)}=\frac{i\mu_2 k_2}{2}\sum_{n=0}^{\infty}B_n\left\{S_{n-1}\cdot\frac{\eta\omega'(\eta)}{|\omega'(\eta)|}+S_{n+1}\cdot\frac{\overline{\eta\omega'(\eta)}}{|\omega'(\eta)|}\right\} \tag{3.8}$$

where

$$T_{n-1}=\left[H_{n-1}^{(1)}\left(k_1|\omega(\eta)|\right)\left[\frac{\omega(\eta)}{|\omega(\eta)|}\right]^{n-1}-H_{n+1}^{(1)}\left(k_1|\omega(\eta)|\right)\left[\frac{\omega(\eta)}{|\omega(\eta)|}\right]^{-(n+1)}\right]$$
$$T_{n+1}=\left[H_{n+1}^{(1)}\left(k_1|\omega(\eta)|\right)\left[\frac{\omega(\eta)}{|\omega(\eta)|}\right]^{n+1}-H_{n-1}^{(1)}\left(k_1|\omega(\eta)|\right)\left[\frac{\omega(\eta)}{|\omega(\eta)|}\right]^{-(n-1)}\right]$$
$$S_{n-1}=\left[J_{n-1}\left(k_2|\omega(\eta)|\right)\left[\frac{\omega(\eta)}{|\omega(\eta)|}\right]^{n-1}-J_{n+1}\left(k_2|\omega(\eta)|\right)\left[\frac{\omega(\eta)}{|\omega(\eta)|}\right]^{-(n+1)}\right]$$
$$S_{n+1}=\left[J_{n+1}\left(k_2|\omega(\eta)|\right)\left[\frac{\omega(\eta)}{|\omega(\eta)|}\right]^{n+1}-J_{n-1}\left(k_2|\omega(\eta)|\right)\left[\frac{\omega(\eta)}{|\omega(\eta)|}\right]^{-(n-1)}\right]$$

The boundary conditions of displacement continuity and stress equilibrium at the interface of alluvial valley are given by

$$w^{(i)}+w^{(r)}+w^{(s)}=w^{(st)}$$
$$\tau_{rz}^{(i)}+\tau_{rz}^{(r)}+\tau_{rz}^{(s)}=\tau_{rz}^{(st)} \tag{3.9}$$

In order to solve the equations (3.9), multiplying $e^{-im\theta}\ (m=0,\pm1\cdots)$ at the two sides of equations and integrating between the interval $(-\pi,0)$, and the limitless algebraic equations can be written into the matrix form. So the equations to determinate the unknown quantities $A_n, B_n\ (n=0,\pm1,\pm2\cdots)$ can be obtained.

4. CALCULATING EXAMPLEMS AND DISCUSSIONS

As examples, the displacements of ground motion are given for the scattering of SH-wave by a semi-elliptic alluvial valley in half space. Because the investigation of present problem in this paper is based on the complex variables and conformal mapping methods, the alluvial valley can be arbitrary shape. Here, we give several kinds of special examples, and analyse the calculating results.

$$w^{(t)}=w^{(i)}+w^{(r)}+w^{(s)} \quad \text{at } -1<x/a<1$$
$$w^{(t)}=w^{(st)} \quad \text{at } 1<x/a \text{ and } x/a<-1 \tag{4.1}$$

or

$$w^{(t)}=|w^{(t)}|e^{i(\omega t-\varphi)} \tag{4.2}$$

in which $|w^{(t)}|$ is the absolute value of ground motion displacement, and φ is the phase of $w^{(t)}$

$$\varphi = \tan^{-1}[\frac{\mathrm{Im}W^{(t)}}{\mathrm{Re}W^{(t)}}] \tag{4.3}$$

The frequency of incidence wave ω and the short axis of semi-elliptic alluvial valley a can compose the incidence wave number, and it can be expressed as

$$k\,a = \omega a / c_s \quad \text{or} \quad k\,a = 2\pi a / \lambda \tag{4.4}$$

in which λ is wave length of incidence wave, it can be written as

$$\eta = 2a / \lambda \tag{4.5}$$

It can been seen from figures 2 that when dismensinless wave number $\eta_1 = 0.25$, that is, the quasi-static case, the existence of semi-elliptic alluvial valley increases the maximum of displacements of ground motion $|W^{(t)}|$ obviously, and the extent increased with shape of semi-elliptic a/b. Figures 2(a) and 2(b) give the cases of SH-wave vertical incidence and horizontal incidence respectively. Figures 3 give the distribution of seismic ground motion $|W^{(t)}|$ while wave number $\eta_1 = 1.25$. It can also been seen that the fluctuating frequency increases while the frequency wave number is great.

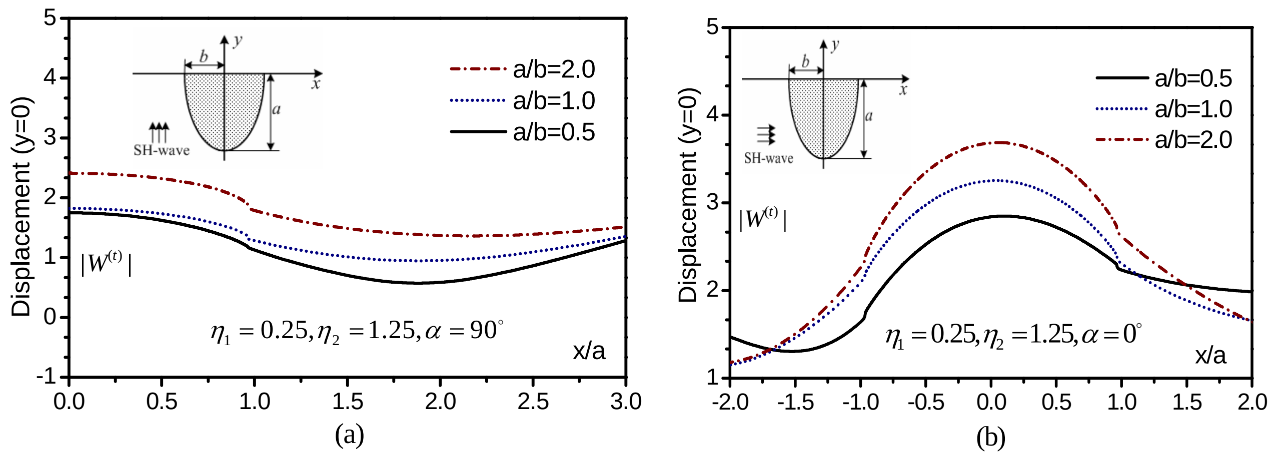


Figure 2 Distribution of seismic ground motion while incidence wave number $\eta_1 = 0.25$

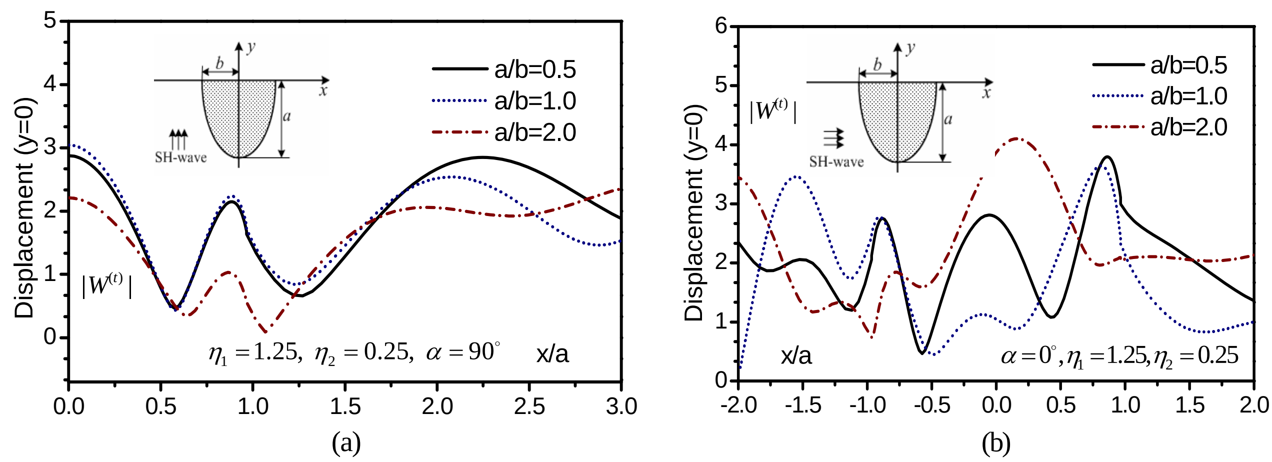


Figure 3 Distribution of seismic ground motion while incidence wave number $\eta_1 = 1.25$

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