Scattering of SH-Wave by a Semi- Cylindrical Hill above a Subsurface Crack in Half-space

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ABSTRACT :

Using the Green's function, the complex function and moving coordinates, an analytical method is developed for scattering of SH -wave by a semi-cylindrical hill above a subsurface crack. Firstly, a suitable Green's function is constructed, which is the fundamental solution of displacement field for an elastic half space with a semi-cylindrical hill under an out-plane harmonic line source loading at an arbitrary point in matrix. Secondly, based on the problem of scattering of SH wave by a semi-cylindrical hill, the Green's function is employed to construct a subsurface crack. Finally, the displacement field of semi-cylindrical hill above a subsurface crack under SH wave is given. The calculating results of the hill motion are plotted to show the influence of crack on the surface displacement, which demonstrates that the presence of the crack has obvious earthquake damping function under certain conditions.

KEYWORDS: Scattering of SH-wave; Crack; Semi-Cylindrical Hill; Green's function

1.INTRODUCTION

It is an important subject in earthquake engineering to study the influence of irregular topography on the ground motion, which provides reasonable parameters for seismic design. In nature, because the difference of geological structure, the cracks often be found near the irregular topography. Therefore, the research on the influence of the cracks in basal body on the irregular topography has an important significance in both theory and practice of engineering. But the references on the problem are seldom found. Most of the former studies focus on the problems that scattering of wave by the interface cracks or the defects containing the cracks and inclusions [1-7]. In 2004, Li Hongliang investigate the scattering of SH wave by a semi-cylindrical canyon and a crack using the Green's function [8]. Compared with the case of canyon, it is more difficult to research the scattering of SH wave by a semi-cylindrical hill and a crack. Obviously, the study on which will provide a more comprehensive theoretical support for the theoretical research on topography.

In this paper, the problem of scattering of SH -wave by a semi-cylindrical hill above a subsurface crack is studied by Green's function. Firstly, a suitable Green's function is constructed by the method of division, which is the solution of displacement field for an elastic half space with a semi-cylindrical hill under an out-plane harmonic line source loading at an arbitrary point in matrix. The whole solution domain is divided into two parts [9,10]. Part I is a circular domain including the boundary of the hill, and all the rest can be considered as part II. If an out-plane harmonic line source loading is applied to an arbitrary point in matrix, the disturbance impacted by the line source loading can be considered as the incident wave, and the corresponding wave fields are generated in two domains respectively. According to the boundary conditions of the common boundary, the Green's function discussed in this paper can be obtained. Secondly, some anti-plane forces with the same magnitude as the stresses results of scattering of SH wave by the semi-cylindrical hill but in the opposite directions are loaded on the region where the crack will appear, therefore the total stresses along this region are zero, and the crack is constructed. Finally, the displacement field of scattering of SH -wave by a semi-cylindrical hill above a subsurface crack is given, and the computational example is discussed.

2. Description of the problem

The model of an elastic half-space containing a semi-cylindrical hill and a subsurface crack is shown in Fig.1. And Fig.2 shows the division of the model under anti-plane harmonic linear source loading. Domain I is a circular domain, including the boundary C and $\overline{C}$. Domain II consists of boundary S and $\overline{S}$. Obviously, $\overline{C}$ and $\overline{S}$ are the common boundary of two parts, which means the displacements and stresses at the common boundary should be continual.

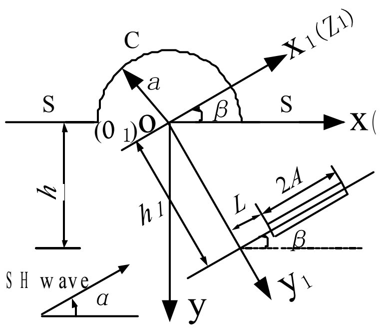

Figure1 The model of a Semi-Cylindrical hill above a subsurface crack

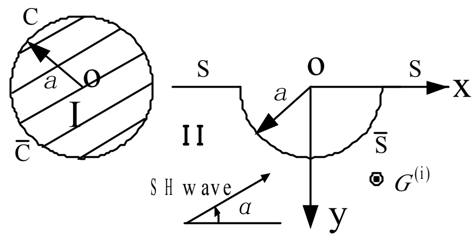

Figure 2 The division of the solution domain

3. GREEN'S FUNCTION

3.1. Governing Equation

The Green's function discussed in this paper is the displacement solution for an elastic half space containing a semi-cylindrical hill impacted by anti-plane harmonic linear source loading at any point in basal body. The dependence relation of G with time factor is $e^{-i\omega t}$ (and will be omitted). In complex plane $(z,\overline{z})$, the displacement function G satisfies the governing equation

$$\frac{\partial^2 G}{\partial x^2}+\frac{\partial^2 G}{\partial y^2}+k^2 G=0 \tag{3.1}$$

where $k=\omega/c_s$, ω is the circular frequency of $W(x,y,t)$; $c_s=\sqrt{\mu/\rho}$ stands for the shear wave velocity of the medium, ρ and μ are mass density and shear modulus of medium respectively.

In the polar coordinate system, the corresponding stresses can be written as

$$\tau_{rz} = \mu\left(\frac{\partial G}{\partial z}\mathrm{e}^{\mathrm{i}\theta} + \frac{\partial G}{\partial \bar{z}}\mathrm{e}^{-\mathrm{i}\theta}\right), \quad \tau_{\theta z} = \mathrm{i}\mu\left(\frac{\partial G}{\partial z}\mathrm{e}^{\mathrm{i}\theta} - \frac{\partial G}{\partial \bar{z}}\mathrm{e}^{-\mathrm{i}\theta}\right) \tag{3.2}$$

3.2. Derivation of Green's function

3.2.1 Standing wave in domain I

As shown in Fig.2, the disturbance impacted by the line source loading on any point in basal body can be considered as the incident wave $G^{(\mathrm{i})}$, so there will be a standing wave in domain I, which should satisfy the conditions that stress free at the hill edge and arbitrary at other point. The standing wave in domain I can be expressed as

$$\tau_{rz,G} = \begin{cases} 0 & z \in C \\ \dfrac{\mu k W_0}{2}\displaystyle\sum_{m=-\infty}^{\infty} C_m\left[J_{m-1}\left(k|z|\right) - J_{m+1}\left(k|z|\right)\right]\left[\frac{z}{|z|}\right]^m & z \in \overline{C} \end{cases} \tag{3.3}$$

the standing wave solution due to Eqn.3.3 can be expressed as[10]

$$G^{(\mathrm{st})} = W_0 \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} C_m \frac{J_{m-1}(ka) - J_{m+1}(ka)}{J_{n-1}(ka) - J_{n+1}(ka)} a_{mn} J_n\left(k|z|\right)\left[\frac{z}{|z|}\right]^n \tag{3.4}$$

where

$$a_{mn} = \begin{cases} \dfrac{1}{2} & m = n \\ \dfrac{\mathrm{e}^{\mathrm{i}(m-n)} - 1}{2\pi \mathrm{i}(m-n)} & m \neq n \end{cases} \tag{3.5}$$

The stress expression from Eqn.3.4 is

$$\tau_{rz,\,G}^{(\mathrm{st})} = \frac{\mu k W_0}{2}\sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty} C_m \frac{J_{m-1}(ka) - J_{m+1}(ka)}{J_{n-1}(ka) - J_{n+1}(ka)} a_{mn}\left[J_{n-1}\left(k|z|\right) - J_{n+1}\left(k|z|\right)\right]\left[\frac{z}{|z|}\right]^n \tag{3.6}$$

3.2.2 Incident wave, reflected wave and scattered wave in domain II

In the whole space, the disturbance impacted by the line source loading can be considered as the incident wave $G^{(\mathrm{i})}$ and expressed in the form

$$G^{(\mathrm{i})} = \frac{\mathrm{i}}{4\mu} H_0^{(1)}\left(k\left|r - r_0\right|\right) \tag{3.7}$$

In complex plane ($z, \bar{z}$), the Eqn.3.7. can be written as

$$G^{(\mathrm{i})} = \frac{\mathrm{i}}{4\mu} H_0^{(1)}\left(k\left|z - z_0\right|\right) \tag{3.8}$$

In the absence of the horizontal boundary, a refected wave $G^{(\mathrm{r})}$ is constructed, and the sum of $G^{(\mathrm{i})}$ and $G^{(\mathrm{r})}$ should satisfy the stress free at the horizontal boundary. $G^{(\mathrm{r})}$ takes the form

$$G^{(\mathrm{r})} = \frac{\mathrm{i}}{4\mu} H_0^{(1)}\left(k\left|z - \overline{z}_0\right|\right) \tag{3.9}$$

The stresses due to $G^{(\mathrm{i})}$ and $G^{(\mathrm{r})}$ can be expressed as

$$\tau_{rz,G}^{(\mathrm{i})} = -\frac{\mathrm{i}k}{8} H_1^{(1)}\left(k\left|z - z_0\right|\right)\left(\frac{\left|z - z_0\right|}{z - z_0}\mathrm{e}^{\mathrm{i}\theta} + \frac{z - z_0}{\left|z - z_0\right|}\mathrm{e}^{-\mathrm{i}\theta}\right) \tag{3.10}$$

$$\tau_{\theta z,G}^{(\mathrm{i})} = \frac{k}{8} H_1^{(1)}\left(k\left|z - z_0\right|\right)\left(\frac{\left|z - z_0\right|}{z - z_0}\mathrm{e}^{\mathrm{i}\theta} - \frac{z - z_0}{\left|z - z_0\right|}\mathrm{e}^{-\mathrm{i}\theta}\right) \tag{3.11}$$

$$\tau_{rz,G}^{(\mathrm{r})} = -\frac{\mathrm{i}k}{8} H_1^{(1)}\left(k\left|z - \overline{z}_0\right|\right)\left(\frac{\left|z - \overline{z}_0\right|}{z - \overline{z}_0}\mathrm{e}^{\mathrm{i}\theta} + \frac{z - \overline{z}_0}{\left|z - \overline{z}_0\right|}\mathrm{e}^{-\mathrm{i}\theta}\right) \tag{3.12}$$

$$\tau_{\theta z,G}^{(\mathrm{r})} = \frac{k}{8} H_1^{(1)}\left(k\left|z - \overline{z}_0\right|\right)\left(\frac{\left|z - \overline{z}_0\right|}{z - \overline{z}_0}\mathrm{e}^{\mathrm{i}\theta} - \frac{z - \overline{z}_0}{\left|z - \overline{z}_0\right|}\mathrm{e}^{-\mathrm{i}\theta}\right) \tag{3.13}$$

In domain II, the scattered wave $G_s^{(\mathrm{s})}$ from the canyon $\overline{\mathrm{S}}$ is constructed to satisfy the traction free at horizontal surface S, $G_s^{(\mathrm{s})}$ takes the form

$$G_s^{(\mathrm{s})} = W_0 \sum_{m=0}^{\infty} A_m H_m^{(1)}\left(k\left|z\right|\right)\left[\left(\frac{z}{\left|z\right|}\right)^m + \left(\frac{z}{\left|z\right|}\right)^{-m}\right] \tag{3.14}$$

where A_m are unknown coefficients.

The corresponding stresses are

$$\tau_{rz,G}^{(\mathrm{s})} = \frac{\mu k W_0}{2} \sum_{m=0}^{\infty} A_m \left[H_{m-1}^{(1)}\left(k\left|z\right|\right) - H_{m+1}^{(1)}\left(k\left|z\right|\right)\right]\left\{\left[\frac{z}{\left|z\right|}\right]^m + \left[\frac{z}{\left|z\right|}\right]^{-m}\right\} \tag{3.15}$$

$$\tau_{\theta z,G}^{(\mathrm{s})} = \frac{\mathrm{i}\mu k W_0}{2} \sum_{m=0}^{\infty} A_m \left[H_{m-1}^{(1)}\left(k\left|z\right|\right) + H_{m+1}^{(1)}\left(k\left|z\right|\right)\right]\left\{\left[\frac{z}{\left|z\right|}\right]^m - \left[\frac{z}{\left|z\right|}\right]^{-m}\right\} \tag{3.16}$$

3.2.3 Boundary conditions and derivation of Green's function

In complex plane ($z,\bar{z}$), domain I and domain II are assembled together, which means that the displacements and stresses at the common boundary should be continual. The conditions are

$$\begin{cases} G^{(st)} = G^{(i)} + G^{(r)} + G_s^{(s)} & on\ \bar{S} \\ \tau_{rz,G}^{(st)} = \tau_{rz,G}^{(i)} + \tau_{rz,G}^{(r)} + \tau_{rz,G}^{(s)} & on\ \bar{S} \end{cases} \tag{3.17}$$

Substituting the expressions of displacements and stresses into Eqn.3.17, and multiplying both sides of equations by $e^{-in\theta}$ and integrating over the interval $(-\pi,\pi)$, so a series infinite algebraic equations solving the unknown coefficients C_m , A_m can be obtained.

So the Green's function discussed in this paper can be given by

$$G_{\mathrm{I}} = G^{(st)} \qquad \text{in domain I} \tag{3.18}$$

$$G_{\mathrm{II}} = G^{(i)} + G^{(r)} + G_s^{(s)} \qquad \text{in domain II} \tag{3.19}$$

4. SCATTERING OF SH-WAVE BY A SEMI-CYLINDRICAL HILL ABOVE A SUBSURFACE CRACK

4.1 Scattering of SH-Wave by a Semi- Cylindrical Hill

4.1.1 Standing wave in domain I

As shown in Fig.2, the solution domain is divided into two parts during the solution of scattering of SH wave by a Semi- Cylindrical Hill. Then in circular domain I , the disturbance impacted by SH wave can be described by a standing wave, which satisfies the conditions that stress free at the edge of the hill and arbitrary at other part.

The standing wave can be written as

$$W^{(st)} = W_0 \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} B_m \frac{J_{m-1}(ka) - J_{m+1}(ka)}{J_{n-1}(ka) - J_{n+1}(ka)} a_{mn} J_n(k|z|) \left[\frac{z}{|z|}\right]^n \tag{4.1}$$

The stress due to Eqn.4.1 is

$$\tau_{rz}^{(st)} = \frac{\mu k W_0}{2} \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} B_m \frac{J_{m-1}(ka) - J_{m+1}(ka)}{J_{n-1}(ka) - J_{n+1}(ka)} a_{mn} \left[J_{n-1}(k|z|) - J_{n+1}(k|z|)\right] \left[\frac{z}{|z|}\right]^n \tag{4.2}$$

where a_{mn} takes the same form with Eqn.3.5.

4.1.2 Wave function in domain II under SH wave

In complex plane $(z,\bar{z})$, incident wave $W^{(i)}$ and reflected wave $W^{(r)}$ can be given by

$$W^{(\mathrm{i})} = W_0 e^{\frac{\mathrm{i}k}{2}\left[\bar{z}e^{\mathrm{i}\alpha} + \bar{z}e^{-\mathrm{i}\alpha}\right]} = W_0 \sum_{n=-\infty}^{\infty} \mathrm{i}^n e^{\mathrm{i}n\alpha} J_n\left(k|z|\right)\left[\frac{z}{|z|}\right]^n \tag{4.3}$$

$$W^{(\mathrm{r})} = W_0 e^{\frac{\mathrm{i}k}{2}\left[\bar{z}e^{-\mathrm{i}\alpha} + \bar{z}e^{\mathrm{i}\alpha}\right]} = W_0 \sum_{n=-\infty}^{\infty} \mathrm{i}^n e^{-\mathrm{i}n\alpha} J_n\left(k|z|\right)\left[\frac{z}{|z|}\right]^n \tag{4.4}$$

The stresses due to $W^{(i)}$ and $W^{(r)}$ can be expressed as

$$\tau_{rz}^{(\mathrm{i})} = \mathrm{i}\mu k W_0 \cos(\theta+\alpha) e^{\mathrm{i}k|z|\cos(\theta+\alpha)} \tag{4.5}$$

$$\tau_{rz}^{(\mathrm{r})} = \mathrm{i}\mu k W_0 \cos(\theta-\alpha) e^{\mathrm{i}k|z|\cos(\theta-\alpha)} \tag{4.6}$$

$$\tau_{\theta z}^{(\mathrm{i})} = -\mathrm{i}\mu k W_0 \sin(\theta+\alpha) e^{\mathrm{i}k|z|\cos(\theta+\alpha)} \tag{4.7}$$

$$\tau_{\theta z}^{(\mathrm{r})} = -\mathrm{i}\mu k W_0 \sin(\theta-\alpha) e^{\mathrm{i}k|z|\cos(\theta-\alpha)} \tag{4.8}$$

The scattered wave from the semi-cylindrical canyon can be expressed as Eqn.4.9, which satisfies the stress free condition on the horizontal surface.

$$W^{(\mathrm{s})} = W_0 \sum_{m=0}^{\infty} F_m H_m^{(1)}\left(k|z|\right)\left[\left(\frac{z}{|z|}\right)^m + \left(\frac{z}{|z|}\right)^{-m}\right] \tag{4.9}$$

where F_m are unknown coefficients.

The corresponding stresses are given by

$$\tau_{rz}^{(\mathrm{s})} = \frac{\mu k W_0}{2} \sum_{m=0}^{\infty} F_m \left[H_{m-1}^{(1)}\left(k|z|\right) - H_{m+1}^{(1)}\left(k|z|\right)\right]\left\{\left[\frac{z}{|z|}\right]^m + \left[\frac{z}{|z|}\right]^{-m}\right\} \tag{4.10}$$

$$\tau_{\theta z}^{(\mathrm{s})} = \frac{\mathrm{i}\mu k W_0}{2} \sum_{m=0}^{\infty} F_m \left[H_{m-1}^{(1)}\left(k|z|\right) + H_{m+1}^{(1)}\left(k|z|\right)\right]\left\{\left[\frac{z}{|z|}\right]^m - \left[\frac{z}{|z|}\right]^{-m}\right\} \tag{4.11}$$

4.1.3 Boundary conditions and determined equations

At the common boundary, the displacements and stresses should be continual, namely

$$\begin{cases} W^{(\mathrm{st})} = W^{(\mathrm{i})} + W^{(\mathrm{r})} + W_s^{(\mathrm{s})} & \text{on } \bar{S} \\ \tau_{rz}^{(\mathrm{st})} = \tau_{rz}^{(\mathrm{i})} + \tau_{rz}^{(\mathrm{r})} + \tau_{rz}^{(\mathrm{s})} & \text{on } \bar{S} \end{cases} \tag{4.12}$$

Substituting the expressions of displacements and stresses into Eqn.4.12, and multiplying both sides of equations by $e^{-in\theta}$ and integrating over the interval $(-\pi, \pi)$, a series infinite algebraic equations solving the unknown coefficients F_m, B_m can be obtained.

Under SH wave, the displacement and stress field in two domains can be expressed respectively as

$$\begin{cases} W_{\mathrm{I}}^{(\mathrm{z})} = W^{(\mathrm{st})} & \text{in domain I} \\ W_{\mathrm{II}}^{(\mathrm{z})} = W^{(\mathrm{i})} + W^{(\mathrm{r})} + W^{(\mathrm{s})} & \text{in domain II} \end{cases} \tag{4.13}$$

$$\begin{cases} \tau_{\theta z,\mathrm{I}}^{(\mathrm{z})} = \tau_{\theta z}^{(\mathrm{st})} & \text{in domain I} \\ \tau_{\theta z,\mathrm{II}}^{(\mathrm{z})} = \tau_{\theta z}^{(\mathrm{i})} + \tau_{\theta z}^{(\mathrm{r})} + \tau_{\theta z}^{(\mathrm{s})} & \text{in domain II} \end{cases} \tag{4.14}$$

4.2 Displacement Function with the Existence of Crack

The displacement field and stress field of scattering of SH wave by the semi-cylindrical hill can be obtained, namely the stress of an arbitrary point in basal body also can be solved. Then a pair of forces with the same value and opposite direction are applied to the region where the crack will appear, therefore the total stresses of this region is zero, which can be thought as a crack. Using the Green's function solved from above discussions, which is the displacement solution for an elastic half space containing a semi-cylindrical hill impacted by anti-plane harmonic linear source force at any point in basal body, the final displacement fields with the coexistence of semi- cylindrical hill and crack take following forms in two domains respectively

$$\begin{cases} W_{\mathrm{I}}^{(\mathrm{t})} = W_{\mathrm{I}}^{(\mathrm{z})} - \int_{(L,h_1)}^{(L+2A,h_1)} \left(\tau_{\theta z,\mathrm{II}}^{(\mathrm{z})} \times G_{\mathrm{I}}\right) dz_1 & \text{in domain I} \\ W_{\mathrm{II}}^{(\mathrm{t})} = W_{\mathrm{II}}^{(\mathrm{z})} - \int_{(L,h_1)}^{(L+2A,h_1)} \left(\tau_{\theta z,\mathrm{II}}^{(\mathrm{z})} \times G_{\mathrm{II}}\right) dz_1 & \text{in domain II} \end{cases} \tag{4.15}$$

5. SUMMARY

Calculation model is shown in Fig.3.

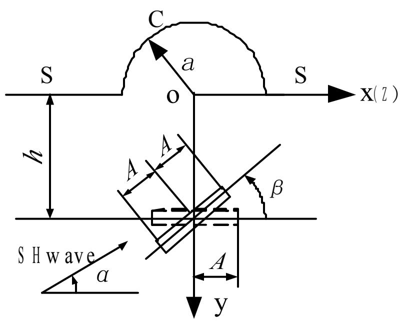

Figure 3 The model of calculation

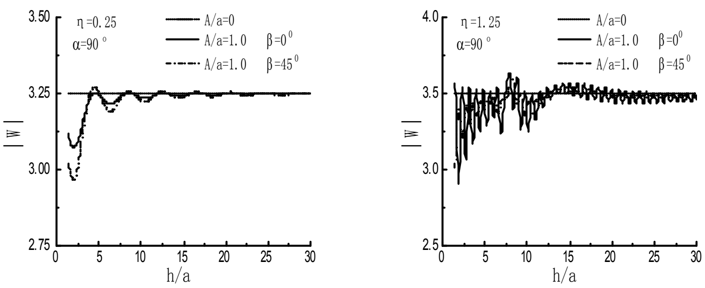

Figure 4 Variation of displacement amplitudes of the hill peak with h/a

(1) The displacement amplitude of the hill peak shows periodical decrease with the increase of h/a. When $h/a \geq 30$, the influence of creak tends to stabilization, as shown in Fig.4.

(2)Form Fig.5, it can be seen that the subsurface crack has notable effects on the displacement of the hill surface. Under the incident SH wave vertically, the level crack illustrates some damping effects, and the larger value of the length of crack is, the more obvious earthquake damping effect is.

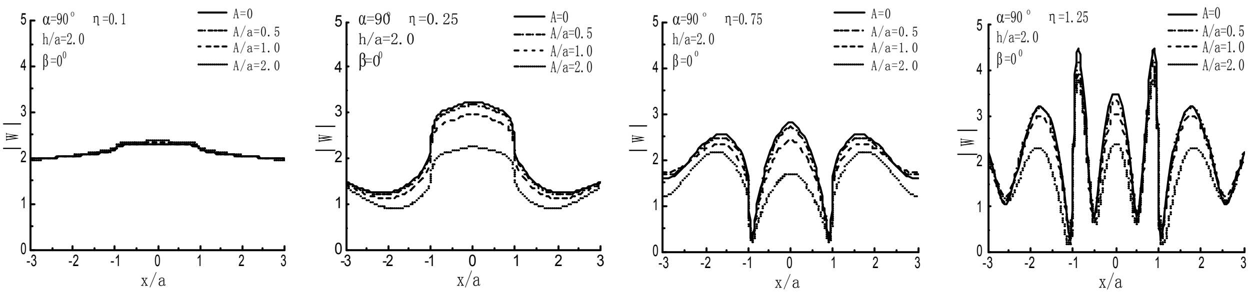

Figure 5 Variation of surface displacement amplitudes with x/a when $\beta = 0^0$

（3）The analysis method presented in this paper is just used to the case that the crack outside of domain I. In the dividing of solution domain, when the common boundary and the crack contact or intersect with each other, a special analysis should be applied.

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