STRUCTURAL SYSTEM IDENTIFICATION BASED ON SUBSYSTEM ANALYSES

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SUMMARY

In estimating the motion characteristics of a building, the modal decomposition technique is frequently employed. However, it doesn't fit to the estimation of a substructure. This paper focuses on the damping estimation of substructure. By means of the transfer matrix method, a structure is divided into a number of substructures, which then are to be identified. A standing wave arisen inside a structure is decomposed into forward and backward moving waves in virtue of the wave propagation theory. Then, a spatial damping factor is estimated from the forward moving wave. This damping factor is regarding the spatial information, which the modal analysis cannot deal with. To demonstrate the validity the proposed method, numerical simulations are conducted with respect to a four-story building models subjected to sinusoidal and white noise excitations induced by AMD.

INTRODUCTION

Along with the remarkable development of sensing, measurement and computer technologies, system identification of structures has recently extended its role in civil engineering field. In addition to the usual case, system identification is conducted for the case of getting more precise information regarding the motion characteristics of a controlled building [Nishitani and Yamada, 1999] and for the case of structural health monitoring systems. In particular, the recent increasing demand for health monitoring technology will expectedly accelerate more practical development of system identification. Despite that, certain difficulties arise in conducting system identification practice of a large structure. They are, for instance, the increase of computation effort and time due to the handling of a large structure.

From the above reason, it would be advantageous to divide an entire structure into a number of substructures or subsystems and then to conduct system identification of each substructure. The idea of substructures is dealt with the transfer matrix method [Fukuwa et al, 1991; Pestel and Leckie, 1963; Yamakawa and Ohnishi, 1982] in conjunction with the employment of wave propagation theory [Doyle, 1997; Flotow, 1986; Fukuwa et al, 1991; Fukuwa et al, 1992; Mead, 1986; Tanaka and Kikushima 1990; Tokuoka, 1985]. This paper focuses on the damping of each substructure in a building instead of dealing with a modal damping ratio. The damping of each substructure is to be estimated in terms of spatial damping factor, which represents the transition of the state from one substructure to other substructure. The spatial damping factor is regarding the spatial information, which the modal analysis cannot deal with. Such a damping factor tells us how many degree of damping any substructure has. This information about the damping of each substructure is used for structural health monitoring systems. To demonstrate how effective this technique is, multi-story building model is employed, which is excited by AMD installed on the top floor.

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EVALUATION OF SPATIAL DAMPING FACTORS

2.1 Transfer Matrix

Consider a shear structure model representing a multi-story building shown in Figure 1. Suppose the model oscillates with frequency ω [rad/s]. The state vector $\boldsymbol{Z}_i$ for ≤th story consists of the displacement x_i relative to the base and the shear force Q_i. The substructure from the upper point of ($i-1$)th mass to the upper point of i th mass is defined as i th story and is utilized as a substructure.

Two sets of states with respect to m_i are dealt with: one is $\boldsymbol{Z}_i^U$ on the upper point of the mass and the other $\boldsymbol{Z}_i^L$ on the lower point of the mass (Figure 2). x_i^U and Q_i^U represent the state components of $\boldsymbol{Z}_i^U$ and the state x_i^L and N_i^L represent the state components of $\boldsymbol{Z}_i^L$. The relationship between these four components are given by the following equation:

$$\begin{bmatrix} x_i^U \\ Q_i^U \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -m_i\omega^2 & 1 \end{bmatrix} \begin{bmatrix} x_i^L \\ Q_i^L \end{bmatrix} \tag{1}$$

The coefficient matrix of $\boldsymbol{Z}_i^L$ in the right-hand side of the above equation is called the point matrix.

The stiffness k_i and damping c_i of i th story are shown in Figure 3. Assuming the Kelvin model that consists of k_i and c_i in the parallel position, one gets

$$\begin{bmatrix} x_i^L \\ Q_i^L \end{bmatrix} = \begin{bmatrix} 1 & \dfrac{1}{k_i + jc_i\omega} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_{i-1}^U \\ Q_{i-1}^U \end{bmatrix} \tag{2}$$

The coefficient matrix of $\boldsymbol{Z}_{i-1}^U$ in the right-hand side of the above equation is called the filed matrix.

Eqs. (1) and (2) yield the following relationship between $\boldsymbol{Z}_i^U$ and $\boldsymbol{Z}_{i-1}^U$.

$$\begin{bmatrix} x_i^U \\ Q_i^U \end{bmatrix} = \begin{bmatrix} 1 & \dfrac{1}{k_i + jc_i\omega} \\ -m_i\omega^2 & 1 - \dfrac{m_i\omega^2}{k_i + jc_i\omega} \end{bmatrix} \begin{bmatrix} x_{i-1}^U \\ Q_{i-1}^U \end{bmatrix} \tag{3}$$

in which, the coefficient matrix of $\boldsymbol{Z}_{i-1}^U$ represents how $\boldsymbol{Z}_i^U$ is effected by $\boldsymbol{Z}_{i-1}^U$ and is referred to as the transfer matrix of i th story from the upper point of m_{i-1} to the upper point of m_i, denoted as $\boldsymbol{U}_i$.

Eq. (3) is the transfer matrix for the case of no external excitation. Since the structure is excited by AMD, only the top story has an external input force. When excited by the movement of AMD f_n applied to m_n, Eq. (3) will be

$$\begin{bmatrix} x_n^U \\ Q_n^U \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & \dfrac{1}{k_n + jc_n\omega} & 0 \\ -m_n\omega^2 & 1 - \dfrac{m_n\omega^2}{k_n + jc_n\omega} & -f_n \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{n-1}^U \\ Q_{n-1}^U \\ 1 \end{bmatrix} \tag{4}$$

If the transfer matrices of the model are known, all the states can be calculated the above formulations with boundary conditions. If the states with respect to all the stories are obtained by the actual measurement, all the story transfer matrices can be estimated. These relationships (Eqs.(3) and (4)) are utilized to get the relationships between the waves from story to story.

2.2 Wave Propagation

Consider the structure subjected to certain sinusoidal excitation due to AMD. The excitation wave propagates from the top floor to the boundary of each story, and the wave partially transmits and partially reflects. The transmitting and reflecting waves are called forward and backward moving waves. There exist both forward and backward moving waves inside the structure in the steady-state sinusoidal oscillation.

The wave vector $\boldsymbol{W}_i$ is defined herein as certain kind of state vector consisting of the complex amplitudes w_i^+ and w_i^- of forward and backward moving waves with respect to i th story. In cooperation with the eigenvalues λ_i and λ_i' of $\boldsymbol{U}_i$, the transition of the wave vector is given by

$$\begin{bmatrix} w_i^+ \\ w_i^- \end{bmatrix} = \begin{bmatrix} e^{(-\alpha - j\beta)} & 0 \\ 0 & e^{(\alpha + j\beta)} \end{bmatrix} \begin{bmatrix} w_{i-1}^+ \\ w_{i-1}^- \end{bmatrix} \quad (5)$$

in which

$$\alpha = \mathrm{Re}(\ln \lambda_i) \text{ and } \beta = \mathrm{Im}(\ln \lambda_i) \quad (6)$$

Eq. (5) illustrates how the wave propagates to the vertical direction. This paper considers about a forward moving wave in the following. In the Eq. (5), as w_i^+ and w_i^- is complex amplitude, the expression of the wave as a function of the time t is represented by multiplying $e^{j\omega t}$. The forward moving wave is extracted.

$$w_i^+ e^{j\omega t} = e^{(-\alpha - j\beta)} w_{i-1}^+ e^{j\omega t} \quad (7)$$

The relationship between $w_i^+ e^{j\omega t}$ and $w_{i-1}^+ e^{j\omega t}$ yields the energy loss factor. Then spatial damping factor will be estimated in the next subsection.

2.3 Spatial Damping Factors Estimated Based on Energy Loss

In this study, a spatial damping factor between stories is dealt with. Such a spatial damping is estimated by comparing the energy dissipated by a forward moving wave as it progresses downwards with the amount of energy dissipated by the damped free vibration of a single-degree-of-freedom oscillatory system.

By means of the equation representing the damped free vibration of the SDOF system induced by an initial displacement, the energy loss factor ψ_f of the vibration can be written as

$$\psi_f = 1 - \exp\left(\frac{-4\pi h_m}{\sqrt{1 - h_m^2}} \right) \quad (8)$$

in which h_m is the viscous damping ratio of this system, describing how the free vibration is diminished as time passes. On the other hand, from the equation for the forward moving wave, Eq. (7), the energy loss factor ψ_w of the forward moving wave is represented by

$$\psi_w = 1 - \exp\left(-4\pi \frac{\alpha}{\beta} \right) \quad (9)$$

Equalizing the two kinds of energy loss factors ψ_f and ψ_w given by Eqs. (8) and (9), the spatial damping factor h_p of the forward moving wave leads to

$$h_p = \frac{\alpha / \beta}{\sqrt{1 + (\alpha / \beta)}} \quad \cdot \quad (10)$$

with α and β given in Eq. (6).

NUMERICAL EXAMPLES

3.1 Models for Numerical Examples

To demonstrate the validity of the proposed methodology, numerical examples are conducted for four-story building models. The data of models are tabulated in Table 1. The specific procedure is: (i) to measure the response displacement x_i; (ii) to calculate Q_i; (iii) to determine $\boldsymbol{U}_i$ with x_i and Q_i from Eq. (3) or (4); (iv) to obtain α and β from the eigenvalues of $\boldsymbol{U}_i$ from Eq. (6); and (v) to evaluate the spatial damping factors through Eq. (10). Repeating the above procedure at the various frequencies, the spatial damping factors as a function of the frequency are estimated.

Table 1: Parameter of building models for numerical examples

	Story	Model				
		A	K1	K2	C1	C2
m [kg]	All	2.6	2.6	2.6	2.6	2.6
k [N/m]	4	9000	8100(-10%)	9000	9000	9000
	3	9000	9000	9000	9000	9000
	2	9000	8100(-10%)	8100(-10%)	9000	9000
	1	9000	9000	9000	9000	9000
c [N/m/s]	4	13.0	13.0	13.0	13.65(+5%)	14.3(+10%)
	3	28.0	28.0	28.0	28.0	28.0
	2	43.0	43.0	43.0	45.15(+5%)	47.3(+10%)
	1	58.0	58.0	58.0	58.0	58.0

3.2 Sinusoidal Excitation Input

Sinusoidal excitation is applied to Model A in the first place. Sinusoidal excitation inputs of amplitude 1 [cm] are given by AMD on the top floor during the frequency range 0 [Hz] to 12 [Hz] with the increment of 0.4 [Hz]. The displacement responses of all the stories are measured as the outputs with the sampling time Δt equal to 0.001 [s] and 0.0001 [s].

The estimated spatial damping factors and actual values obtained from *m*, *c* and *k* given in Table 1. are plotted in Figures 4 and 5.•It is recognized that shorter sampling time is as needed for higher frequency. Although shorter sampling time would be better, the estimation is not always satisfactorily accurate with 0.0001 [s]. Moreover, the accuracy of estimation would be worse with small damping. Therefore, short sampling time is needed to improve the estimation accuracy. However, short sampling time is not realistic in the application of the sinusoidal excitation based methodology to an actual structure.

3.3 Random Input

Accounting for the above discussion, white noise input is used instead of sinusoidal excitation. However, the building response to white noise excitation involves a huge number of frequencies. The direct employment of the presented method for this case is not possible. In estimating x_i for every frequency in Procedure (i), the input/output data are formulated by the linear regression model and the compliance $\boldsymbol{G}(j\omega)$ is estimated by the least squares method.

$$\boldsymbol{G}(j\omega) = \frac{\boldsymbol{X}(j\omega)}{F(j\omega)} \tag{11}$$

in which, $\boldsymbol{X}(j\omega)$: displacement vector and $F(j\omega)$: excitation force due to AMD (scalar).

If $F(j\omega)$ is known, the complex amplitude vector of each story is calculated from $\boldsymbol{G}(j\omega)$. Eq. (11) leads to the estimation of x_i for every frequency. By conducting Procedures (ii), (iii), (iv) and (v), spatial damping factors are estimated. In conducting the analysis, it is assumed that the acceleration responses of all the stories are measured as outputs. The sampling time is set to be 0.01 [s], the number of measured data is 512 with the degree of the linear regression model equal to 10. The estimated spatial damping factors and their actual values are shown in Figure 6. This figure indicates that the estimated results agree with the actual values.

In the following stage, this methodology is applied to structural health monitoring systems. Model A is supposed to be in a good condition, four different kinds of building models are inspected by the methodology: they are Models K1, K2, C1 and C2. The data are shown in Table 1. Models K1 and K2 are those models, which have smaller stiffness than Model A, which, may result from certain damages. On the other hand, Models C1 and C2 are those models with larger damping.

The comparison of estimated spatial damping factors of Model K1, K2, C1 and C2 with those of Model A are shown in Figures 7 through 10. The spatial damping factors of those stories with certain changes in characteristics are different from Model A. The methodology successfully indicates how and where the changes in stiffness and damping occur.

Table 2: Natural frequencies and damping ratios of building models

		Model				
	Mode	A	K1	K2	C1	C2
Natural frequency [Hz]	1	3.25	3.18	3.19	3.25	3.25
	2	9.36	9.19	9.37	9.36	9.36
	3	14.4	13.9	14.2	14.5	14.5
	4	16.8	16.4	16.5	16.8	16.7
Damping ratio	1	0.0513	0.0519	0.0522	0.0522	0.0530
	2	0.108	0.104	0.108	0.109	0.109
	3	0.159	0.174	0.179	0.163	0.168
	4	0.205	0.207	0.194	0.211	0.218

The natural frequencies and damping ratios obtained by use of the modal analysis are shown in Table 2. Each model has different values of both natural frequencies and damping ratios from Model A. Nevertheless, it is hard to decide how and where the local characteristics changes have occurred.

CONCLUSIONS

The spatial damping factor of the subsystems of a structure has been discussed. The idea of subsystems is dealt with the transfer matrix method in conjunction with the employment of wave propagation theory. The presented methodology is also applied to structural health monitoring issue. It is demonstrated that: 1. The damping of each story of a structure can be identified by means of presented methodology: 2. When applied to a structural health monitoring issue, the methodology satisfactorily identifies where in the structure and how the stiffness and damping changes from their original or desired values.

ACKNOWLEDGEMENTS

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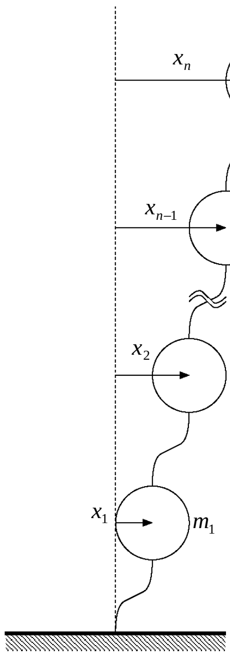

Figure 1: Shear structure model of a multi-story building

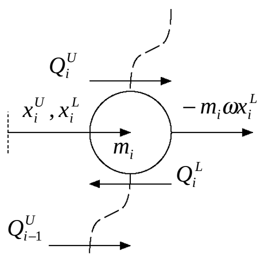

Figure 2: Diagram of state passing through m_i

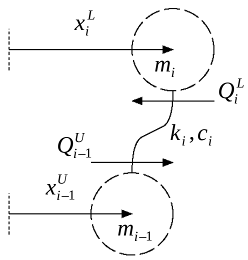

Figure 3: Relationship between two masses

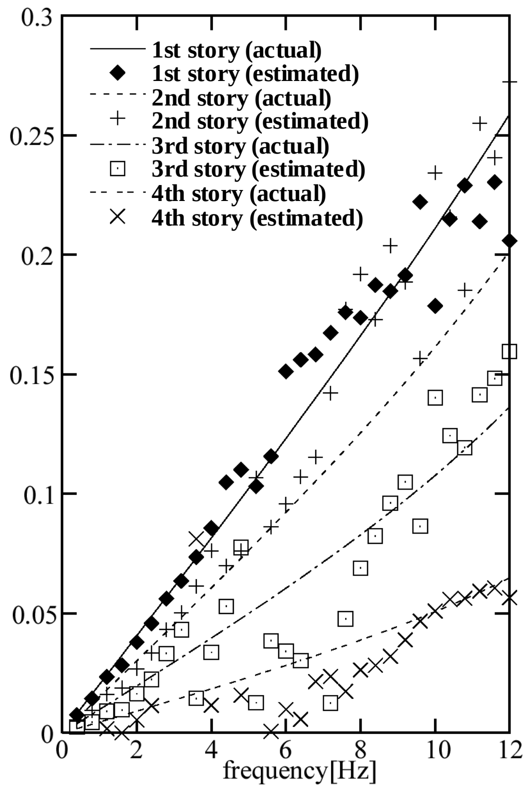

Figure 4: Spatial damping factors from sinusoidal input with sampling time 0.001s for Model A

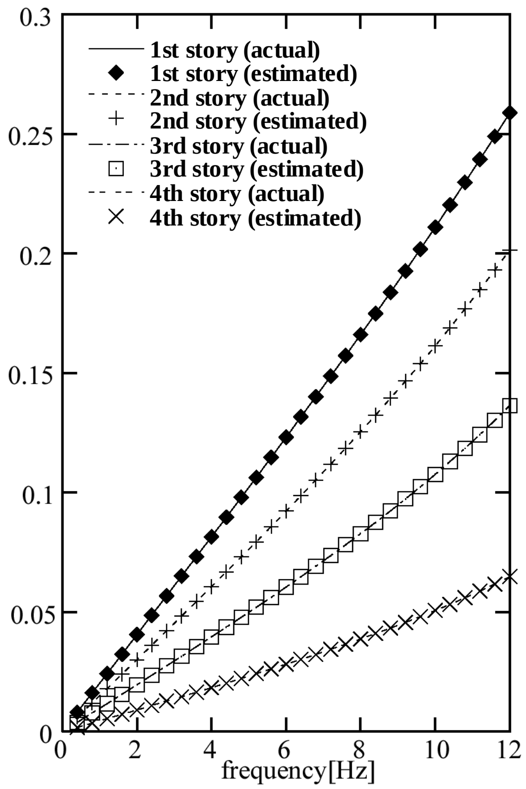

Figure 6: Spatial damping factors from random input with sampling time 0.01s for Model A

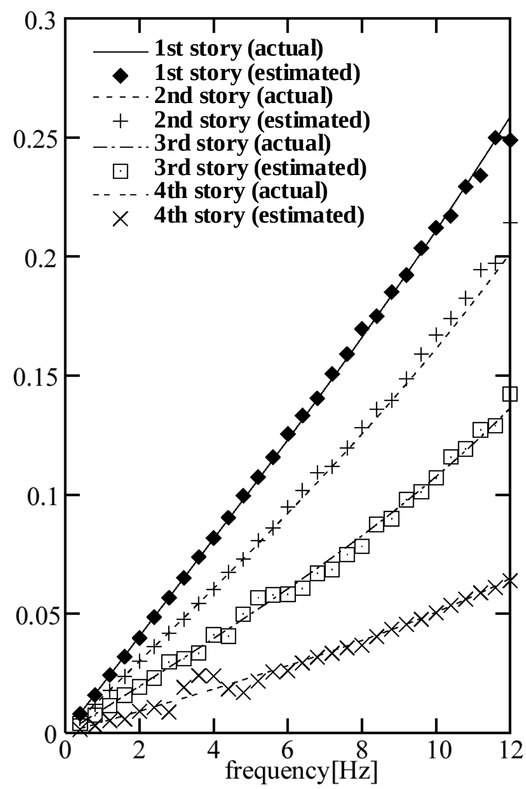

Figure 5: Spatial damping factors from sinusoidal input with sampling time 0.0001s for Model A

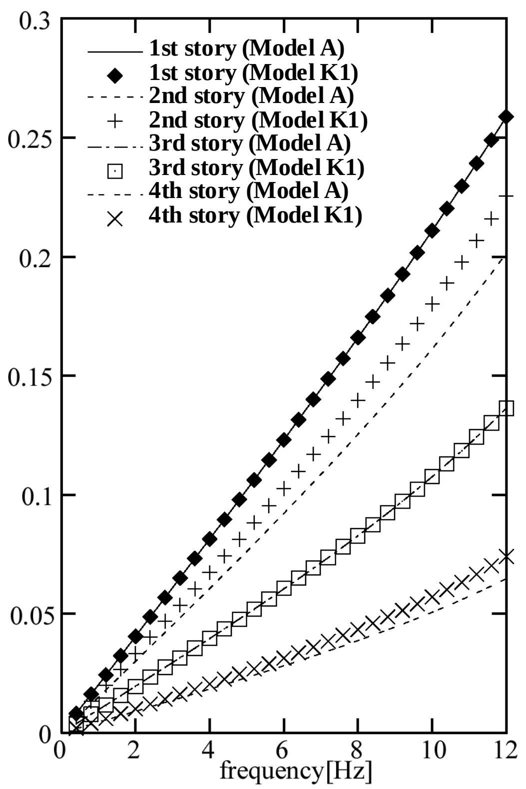

Figure 7: Comparison of Spatial damping factors between Model A and Model K1

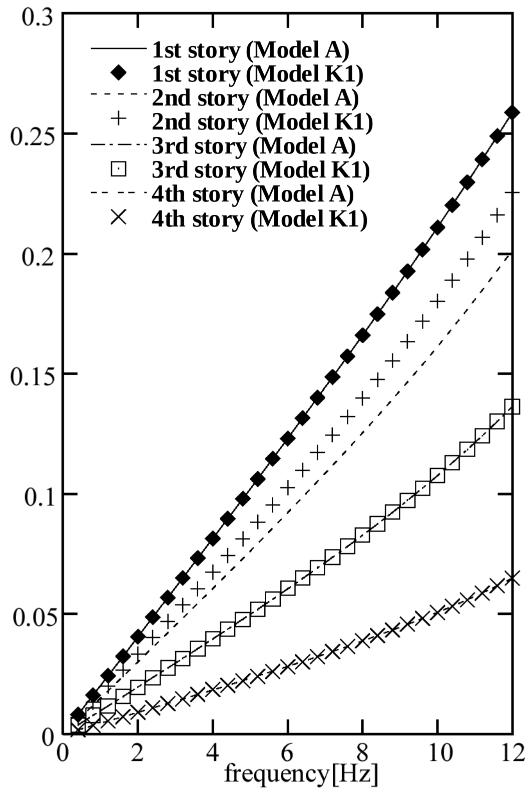

Figure 8: Comparison of Spatial damping factors between Model A and Model K2

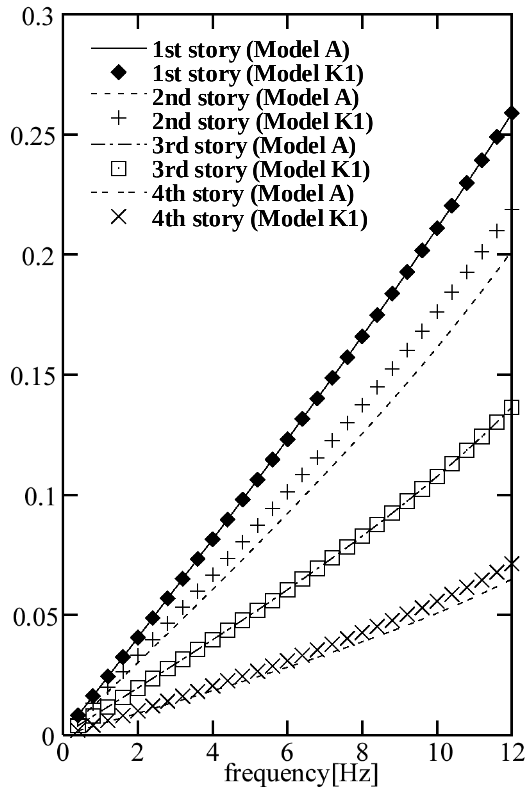

Figure 10: Comparison of Spatial damping factors between Model A and Model C2

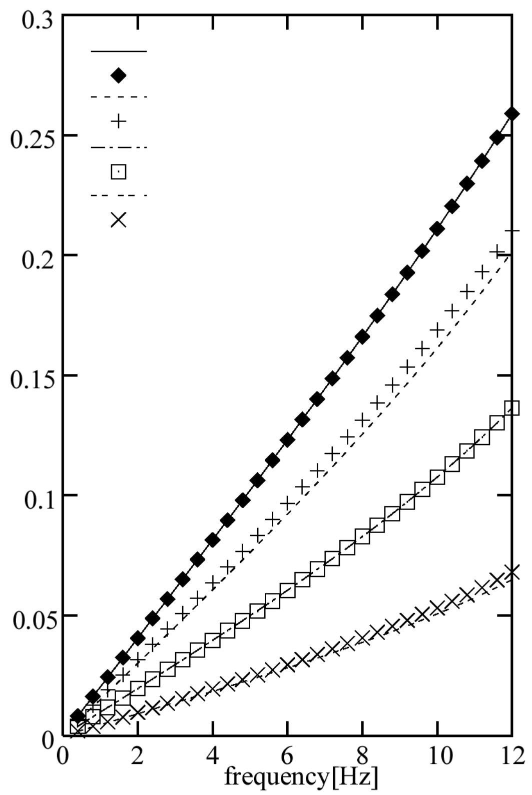

Figure 9: Comparison of Spatial damping factors between Model A and Model C1