A FIXED-BASE MODEL WITH CLASSICAL NORMAL MODES FOR SOIL-STRUCTURE INTERACTION SYSTEMS

Wen-Hwa WU[1]

SUMMARY

To simplify the analysis of soil-structure interaction (SSI) systems, different fixed-base models have been recently developed by the author to efficiently represent the SSI system and shown to have good accuracy. However, the modified mass and damping matrices of these models do not hold the properties of symmetry and orthogonality, which are usually valid or assumed for ordinary structural systems. In the present paper, this problem is further explored to establish a lumped-parameter model possessing classical normal modes. An iteration algorithm is suggested to incorporate the Gram-Schmidt orthogonalization process and the formulation in the modal space for the determination of orthogonal mode shapes, natural frequencies, and modal damping ratios. It is demonstrated with a numerical example that this new fixed-base model has excellent accuracy. Consequently, the complicated SSI systems can be directly analyzed using conventional computer codes for structural dynamics with the fixed-base model developed in this study.

INTRODUCTION

Even though various methods have been proposed by different researchers to simplify the analysis of soil-structure interaction (SSI) systems, the complicated formulation and intensive computation required to obtain the analytical solution for this problem still limits its popular application to engineering practice. Since most of the analysis complexity results from the frequency dependence of the dynamic soil stiffness, many attempts have been made to simplify the SSI analysis by representing the soil with frequency-independent models. Recently, several lumped-parameter models for the soil were developed by minimizing the total square errors between the dynamic stiffness of these models and that for the actual soil [De Barros and Luco 1990, Jean et al. 1990, Wolf 1991]. Other than using frequency-independent models to replace the soil, an alternative way to simplify the procedure in the SSI analysis is to represent the whole structure-foundation-soil system with a lumped-parameter model as that of a fixed-base structure. Using this type of fixed-base model, it is not necessary to introduce any additional degrees of freedom. A methodology using system identification technique has been recently developed by the author to determine an equivalent fixed-base model for the whole SSI system [Wu 1997]. The damping matrix of this fixed-base model was modified from the structural damping matrix to account for the altered resonant peak responses and its mass matrix was also modified from the structural mass matrix to resemble the decrease in the system natural frequencies. In a related study, an effort was further made to establish a uniquely solved fixed-base model in order to avoid the intensive computation due to applying system identification techniques [Wu 1998].

While almost perfect accuracy has been demonstrated for the above fixed-base models, the modified mass and damping matrices of these models do not hold the properties of symmetry and orthogonality, which are usually valid or assumed for ordinary structural systems and can considerably simplify the dynamic analysis. Consequently, these fixed-base models may not be conveniently solved by directly applying conventional computer codes for structural dynamic analysis. In the present paper, this problem is further explored to develop.

[1] *Department of Construction Engineering, National Yunlin University of Science and Technology, Yunlin, Taiwan 640, R.O.C.*

a lumped-parameter model possessing classical normal modes. The resonant frequencies and their corresponding peak responses are firstly obtained using the method suggested in the previous work [Wu 1998] to calibrate the lumped-parameter model. With these resonant responses and a set of initially assumed orthogonal modes, an iteration algorithm incorporating the Gram-Schmidt orthogonalization process and the formulation in the modal space is designed to determine the orthogonal mode shapes, natural frequencies, and modal damping ratios to synthesize a fixed-base model with classical normal modes. An illustrative five-story shear building resting on soft soil is used for demonstrating the effectiveness of this new lumped-parameter model.

REVIEW OF PREVIOUSLY DEVELOPED FIXED-BASE MODELS

Actual SSI System vs. Fixed-base Model

It has been shown that the SSI effects can be conveniently quantified by applying an SSI transfer matrix in the frequency domain to modify the effective load from the free-field ground motion [Gupta and Trifunac 1991, Wu and Smith 1995]. Therefore, the governing equations for the SSI system with multi-degree-of-freedom (MDOF) structural model can be expressed in the frequency domain as

$$-\omega^2\mathbf{MX}(\omega)+\mathrm{i}\omega\mathbf{CX}(\omega)+\mathbf{KX}(\omega)=-\mathbf{S}(\omega)\boldsymbol{\Gamma}\ddot{X}_g(\omega) \quad (1)$$

where $\ddot{X}_g(\omega)$ = Fourier transform of ground acceleration, $\mathbf{X}(\omega)$ = Fourier transform of structural displacement vector, $\mathbf{M}$ = mass matrix of structure, $\mathbf{C}$ = damping matrix of structure, $\mathbf{K}$ = stiffness matrix of structure, $\mathbf{S}(\omega)$ = SSI transfer matrix, $\boldsymbol{\Gamma}$ = $\mathbf{M1}$, and $\mathbf{1}$ = column vector in which each element is unity. If the structural displacement vector $\mathbf{X}(\omega)$ is normalized to the ground acceleration $\ddot{X}_g(\omega)$, eq. (1) can be written as

$$-\omega^2\mathbf{MY}(\omega)+\mathrm{i}\omega\mathbf{CY}(\omega)+\mathbf{KY}(\omega)=-\mathbf{S}(\omega)\boldsymbol{\Gamma} \quad (2)$$

where $\mathbf{Y}(\omega)$ = $\mathbf{X}(\omega)/\ddot{X}_g(\omega)$ = normalized structural displacement vector.

The basic idea of fixed-base model is to account for the SSI effects by modifying the original structural parameters such that the dynamic structural responses of the SSI system can be well represented. It was recently discussed by the author [Wu 1997] that the effectiveness of the fixed-base model depends on the choice of the parameters to modify and the algorithm to determine them. It was also shown that the best choice for the fixed-base model is to modify the structural mass and damping but keep the stiffness unchanged. The structural damping is modified to account for the altered resonant peak responses due to the wave radiation in the soil. On the other hand, the structural mass is modified to resemble the decrease in the system natural frequencies due to soil flexibility. Based on the above studies, the fixed-base model can be formulated as

$$-\omega^2\hat{\mathbf{M}}\mathbf{Y}(\omega)+\mathrm{i}\omega\hat{\mathbf{C}}\mathbf{Y}(\omega)+\mathbf{KY}(\omega)=-\boldsymbol{\Gamma} \quad (3)$$

where $\hat{\mathbf{M}}$ and $\hat{\mathbf{C}}$ are the modified mass and damping matrices for the equivalent model. Since the matrices to be determined are $\hat{\mathbf{M}}$ and $\hat{\mathbf{C}}$, eq. (3) can be reformulated further as

$$-\omega^2\hat{\mathbf{M}}\mathbf{Y}(\omega)+\mathrm{i}\omega\hat{\mathbf{C}}\mathbf{Y}(\omega)=-\mathbf{KY}(\omega)-\boldsymbol{\Gamma} \quad (4)$$

For an N-DOF system, the $2N^2$ entries in $\hat{\mathbf{M}}$ and $\hat{\mathbf{C}}$ can be determined from the formulation of eq. (4) by applying certain algorithms to make the responses of the fixed-base model resemble those of the actual SSI system as close as possible.

Fixed-base Model Using System Identification and Uniquely-Solved Fixed-base Model

For each fixed frequency, the formulation in eq. (4) can provide 2N equations to determine the modified mass and damping matrices by comparing its corresponding N entries in the real and imaginary parts, respectively. As a result, if L exciting frequencies are chosen, $2L\times N$ equations will be available to solve for the $2N^2$ entries in $\hat{\mathbf{M}}$ and $\hat{\mathbf{C}}$. An algorithm using system identification techniques was recently proposed by the author [Wu 1997] where a great number (L>>N) of actual SSI responses covering an extensive frequency range were used to identify $\hat{\mathbf{M}}$ and $\hat{\mathbf{C}}$. The least squares method was adopted to minimize the total error between the responses for the actual SSI system and for the fixed-base model. In addition, to improve the accuracy of the equivalent model, different weightings applied to the response equations at different frequencies were suggested such that the contribution at resonant frequencies to the total error can be emphasized. Even though excellent accuracy has been demonstrated for the above model, its computation cost resulting from the calculation of numerous SSI responses may be very expensive.

The success for the weighted fixed-base model to achieve better accuracy is based on the fact that the frequency response near the resonant frequencies of a system shows peak values and contributes most crucially to the composition of its corresponding time response. Further exploration of this idea would naturally lead to a meaningful limiting case where the weightings introduced on the resonant frequency responses are infinitely large compared to those applied to the other frequency responses. This case is equivalent to simply selecting the responses at the N resonant frequencies for the determination of $\hat{\mathbf{M}}$ and $\hat{\mathbf{C}}$. Since $L = N$ in this case, the modified system matrices can be uniquely solved and the responses for the fixed-base model must be identical to those for the actual SSI system at these N resonant frequencies. Consequently, good accuracy can be expected for this fixed-base model and the computation cost is significantly reduced because only N frequency responses are needed. However, the N resonant frequencies for the SSI system are not known beforehand. For the above fixed-base model to be feasible, an effective algorithm to determine the resonant frequencies of the complicated SSI system is necessary. It is well known that each modal response reaches the peak value at its corresponding resonant frequency. Therefore, if the SSI analysis is performed in the modal space [Wu and Smith 1995], the resonant frequencies can be directly obtained by examining each modal frequency response and then locating the frequency where its maximum amplitude occurs. Many codes are available to solve this optimization problem.

FIXED-BASE MODEL WITH CLASSICAL NORMAL MODES

To establish a fixed-base model whose mass, damping, and stiffness matrices can all be diagonalized through the transformation with a set of orthogonal modes, the most convenient way would be to directly determine the orthogonal mode shapes, natural frequencies, and modal damping ratios. Therefore, the crucial parts for developing of this model are how the dynamic analysis is formulated in the modal space and how the orthogonal bases can be extracted from a given set of vectors. In this section, these two constituents are firstly described, followed by developing an iteration algorithm to integrate them for determining the new fixed-base model.

Formulation for Fixed-base Model in Modal Space

Assuming that the fixed-base model formulated in eq. (3) possesses classical normal modes and $\hat{\mathbf{\Phi}}$ = matrix consisting of orthogonal mode shapes of the fixed-base model, a coordinate transformation can be performed by letting

$$\mathbf{Y}(\omega) = \hat{\mathbf{\Phi}}\overline{\mathbf{Y}}(\omega) \tag{5}$$

where $\overline{\mathbf{Y}}(\omega)$ = normalized modal structural displacement vector. Using eq. (5), eq. (3) can be reformulated into

$$-\omega^2\overline{\hat{\mathbf{M}}}\overline{\mathbf{Y}}(\omega) + \mathrm{i}\omega\overline{\hat{\mathbf{C}}}\overline{\mathbf{Y}}(\omega) + \overline{\mathbf{K}}\,\overline{\mathbf{Y}}(\omega) = -\overline{\mathbf{\Gamma}} \tag{6}$$

using the following orthogonality conditions:

$$\overline{\mathbf{K}} = \hat{\mathbf{\Phi}}^{\mathrm{T}}\mathbf{K}\hat{\mathbf{\Phi}} = \mathbf{I}; \quad \overline{\hat{\mathbf{C}}} = \hat{\mathbf{\Phi}}^{\mathrm{T}}\hat{\mathbf{C}}\hat{\mathbf{\Phi}} = \mathbf{diag}[2\hat{\xi}_j / \hat{\omega}_j]; \quad \overline{\hat{\mathbf{M}}} = \hat{\mathbf{\Phi}}^{\mathrm{T}}\hat{\mathbf{M}}\hat{\mathbf{\Phi}} = \mathbf{diag}[1/\hat{\omega}_j^2] \tag{7}$$

where $\hat{\omega}_j$ = j-th mode natural frequency of fixed-base model, $\hat{\xi}_j$ = j-th mode percentage of critical damping of fixed-base model, $\mathbf{I}$ = $N \times N$ identity matrix, and $\mathbf{diag}[\cdot]$ denotes a diagonal matrix. In eq. (6), $\overline{\mathbf{\Gamma}} = \hat{\mathbf{\Phi}}^{\mathrm{T}}\mathbf{\Gamma}$. Since all the property matrices are diagonalized in the modal space, each modal displacement $\overline{Y}_j(\omega)$ can be independently solved as

$$\overline{Y}_j(\omega) = \frac{-\overline{\Gamma}_j}{[1-(\omega/\hat{\omega}_j)^2 + \mathrm{i}2\hat{\xi}_j(\omega/\hat{\omega}_j)]} \tag{8}$$

In addition, the amplitude of each modal displacement $\left|\overline{Y}_j(\omega)\right|$ can be shown through differentiation to reach its corresponding peak value

$$\left|\overline{Y}_j(\omega)\right|_{\max} = \frac{\left|\Gamma_j\right|}{2\hat{\xi}_j\sqrt{1-\hat{\xi}_j^{\,2}}} \qquad \text{at} \quad \omega = \hat{\omega}_j\sqrt{1-2\hat{\xi}_j^{\,2}} \tag{9}$$

Based on the above analysis, it is obvious that the normalized modal structural displacement vector at any frequency can be efficiently obtained from

$$\overline{\mathbf{Y}}(\omega) = \hat{\mathbf{\Phi}}^{\mathrm{T}}\mathbf{K}\mathbf{Y}(\omega) \tag{10}$$

if the normalized structural displacement vector $\mathbf{Y}(\omega)$ at that frequency is known and the orthogonal modes have been accurately estimated. Moreover, if the responses at the resonant frequencies are given, eq. (10) can be firstly utilized to evaluate the resonant modal responses. Eq. (9) can then be applied to determine the natural frequencies and modal damping ratios under the constraint that the modal displacements will reach their peak amplitudes at these resonant frequencies.

Extraction of Orthonormal Basis through Gram-Schmidt Process

Since all the mode vectors of the fixed-base model are required to be orthogonal to each other with respect to all the property matrices, an orthogonalization process is necessary to preserve the orthogonality of mode vectors. Therefore, the Gram-Schmidt orthogonalization process, which has been established in applied mathematics (e.g., Wylie and Barrett 1995), is briefly reviewed herein for its application in this study. Assuming that $\mathbf{v}_1, \mathbf{v}_2, \ldots\ldots, \mathbf{v}_r$ are r linearly independent vectors, there must exist an orthogonal basis $\{\mathbf{u}_1, \mathbf{u}_2, \ldots\ldots, \mathbf{u}_r\}$ for these vectors. According to the Gram-Schmidt orthogonalization process, this set of basis can be generalized to be orthonormal with respect to any coefficient matrix $\mathbf{B}$ and obtained through the following procedures:

$$\begin{cases} \mathbf{w}_k = \mathbf{v}_k - \sum\limits_{i=1}^{k-1}(\mathbf{u}_i^{\mathrm{T}}\mathbf{B}\mathbf{v}_k)\mathbf{u}_i \\ \mathbf{u}_k = \dfrac{\mathbf{w}_k}{\sqrt{\mathbf{w}_k^{\mathrm{T}}\mathbf{B}\mathbf{w}_k}} \end{cases} \qquad \text{for } k = 1, 2, \ldots\ldots, r \tag{11}$$

where $\mathbf{w}_k$'s are the transition vectors used to calculate the orthonormal basis.

Iteration Algorithm

Let $[\mathbf{Y}(\omega)]_{N\times N}$ denote the $N \times N$ complex matrix consisting of the N resonant frequency responses. With a given orthogonal matrix (with respect to $\mathbf{K}$) $\hat{\mathbf{\Phi}}$, its corresponding matrix $[\overline{\mathbf{Y}}(\omega)]_{N\times N}$ in the modal space can be obtained using eq. (10). Adopting the diagonals in $[\overline{\mathbf{Y}}(\omega)]_{N\times N}$ (i.e., the dominant modal response at each resonant frequency) and applying eq. (9), the natural frequencies and modal damping ratios can then be determined. However, employing these modal parameters in eq. (8) to reconstruct $[\overline{\mathbf{Y}}(\omega)]_{N\times N}$, the previous values may not be recovered unless the initially assumed orthogonal matrix $\hat{\mathbf{\Phi}}$ is the true mode matrix for the fixed-base model. This difficulty results from the fact that the orthogonal mode matrix $\hat{\mathbf{\Phi}}$ and the other modal parameters cannot be decided simultaneously. In other words, the natural frequencies and modal damping ratios are determined following the selection of $\hat{\mathbf{\Phi}}$, or vice versa. Accordingly, an iteration algorithm is necessary for the convergence of $\hat{\mathbf{\Phi}}$, $\hat{\omega}_j$, and $\hat{\xi}_j$ to satisfy eqs. (5) to (8).

Eq. (5) can be reformulated as

$$\begin{bmatrix} \mathbf{R}\{[\overline{\mathbf{Y}}(\omega)]_{N\times N}\}^{\mathrm{T}} \\ \mathbf{I}\{[\overline{\mathbf{Y}}(\omega)]_{N\times N}\}^{\mathrm{T}} \end{bmatrix} \hat{\mathbf{\Phi}}^{\mathrm{T}} = \begin{bmatrix} \mathbf{R}\{[\mathbf{Y}(\omega)]_{N\times N}\}^{\mathrm{T}} \\ \mathbf{I}\{[\mathbf{Y}(\omega)]_{N\times N}\}^{\mathrm{T}} \end{bmatrix} \tag{12}$$

where $\mathbf{R}\{\cdot\}$ and $\mathbf{I}\{\cdot\}$ denote the real and imaginary parts of a complex-valued quantity, respectively. In this study, the regressive solution of eq. (12), which minimizes the total square errors of the equation, is suggested to determine a new set of mode vectors. Since orthogonality may not be preserved during the regression process, the Gram-Schmidt process is consequently applied to obtain a set of orthogonal modes with respect to $\mathbf{K}$. Incorporating the above procedures, an efficient iteration algorithm can be summarized as follows:

(1) Determine the normalized frequency responses $[\mathbf{Y}(\omega)]_{N\times N}$ for the actual SSI system at the N resonant frequencies using the method previously proposed by the author [Wu 1998];

(2) Select an initial assumption for the orthogonal mode shapes $\hat{\mathbf{\Phi}}$ of the fixed-base model (the orthogonal mode shapes $\mathbf{\Phi}$ of the superstructure is usually a reasonable guess);
(3) Use eq. (10) to compute the normalized frequency responses $\left[\overline{\mathbf{Y}}(\omega)\right]_{N\times N}$ in the modal space;
(4) Adopt the diagonals in $\left[\overline{\mathbf{Y}}(\omega)\right]_{N\times N}$ and apply eq. (9) to determine the natural frequencies and modal damping ratios;
(5) Employ the modal parameters obtained from the previous step into eq. (8) to reconstruct a new set of $\left[\overline{\mathbf{Y}}(\omega)\right]_{N\times N}$;
(6) Create a set of raw mode shapes from the regressive solution of eq. (12);
(7) Apply the Gram-Schmidt orthogonalization process with eq. (11) to produce a new set of orthogonal mode shapes with respect to **K**;
(8) Check if this new set of orthogonal mode shapes is convergent to the previous set of mode shapes under a prescribed tolerance? If yes $\Rightarrow$ stop the iteration; Otherwise $\Rightarrow$ go to step (3).

With the convergent orthogonal mode shapes, natural frequencies, and modal damping ratios obtained from the above iteration algorithm, a new fixed-base model with classical normal modes is completely established.

NUMERICAL EXAMPLE

A five-story shear building resting on a homogeneous elastic soil through a rigid square foundation is considered to evaluate the accuracy of the new lumped-parameter model. All the parameters for this system are taken the same as those adopted in a recent study by the author [Wu 1997]. The natural frequencies for the superstructure are listed in Table 1. In addition, a uniform modal damping is assumed to be 2% of the critical damping for each structural mode. A high column stiffness 180,000kN/m and low shear velocity 150m/sec of soil are intentionally selected for the demonstration of significant SSI effects. Two half-side lengths of foundation, 5m and 3m are chosen to illustrate different types of SSI effects.

The normalized modal frequency responses $\overline{\mathbf{Y}}(\omega)$ for the actual SSI system with b = 3m and its corresponding fixed-base model are plotted in Fig. 1 where the response amplitude is normalized with respect to the square of the corresponding natural frequency of superstructure. As expected, each modal response of the actual SSI system reaches its peak value at the corresponding resonant frequency and can be well represented by the fixed-base model in the neighborhoods of these resonant frequencies. In this study, the optimization toolbox of MATLAB is utilized to obtain these resonant frequencies and the results for the two SSI systems are also listed in Table 1. With the 5 resonant frequencies for the actual SSI system determined, the SSI responses at these frequencies are computed and the natural frequencies and modal damping ratios for the fixed-base model can be obtained using the suggested iteration algorithm. The results for both the cases of b = 3m and b = 5m are listed in Table 2.

To evaluate the accuracy of the new proposed model, the responses calculated from the actual SSI system and the new fixed-base model with classical normal modes are compared in the following different perspectives. The normalized frequency responses of the top floor are firstly displayed in Fig. 2 for the systems with b = 3m and b = 5m. The results show that the new fixed-base model can accurately represent the actual SSI system. This approximation is excellent in the low frequency range. In the high frequency range where the contribution to the total structural response is usually insignificant, the accuracy has deteriorated but the approximate responses are still representative of the actual system. To further investigate the effectiveness of the new model, the earthquake responses for the actual SSI system and the fixed-base model are also computed. The 1940 El Centro earthquake and the 1989 Loma Prieta earthquake are chosen as the free-field ground motions. The time histories for the base shear v_b are displayed in Fig. 3 for El Centro earthquake and in Fig. 4 for Loma Prieta earthquake. The almost perfect accuracy of the new model is demonstrated for both SSI systems. Furthermore, to have a thorough evaluation on this new model, the response spectra that present the peak responses for the structures with different fundamental frequencies are displayed in Fig. 5 for the two chosen earthquakes. In preparing these spectra, the structure with a half side-length of 3m is considered and its column stiffness is varied to produce different fundamental natural frequencies. Again, the results show that the new model accurately represents the actual SSI system in all the different cases.

CONCLUSIONS

A new fixed-base model possessing classical normal modes for a general MDOF soil-structure interaction system is developed in this paper applying an iteration algorithm incorporating the Gram-Schmidt orthogonalization process and the formulation in the modal space. The convergent orthogonal mode shapes, natural frequencies, and modal damping ratios can be determined to establish this fixed-base model. It is shown with a numerical example that this new fixed-base model has excellent accuracy. Consequently, the complicated SSI systems can be directly analyzed using conventional computer dynamic codes with the fixed-base model developed in this study.

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Table 1. Natural frequencies of superstructure and SSI systems

Mode		1	2	3	4	5
Natural frequency (Hz)	Superstructure	2.48	7.24	11.42	14.67	16.73
	SSI system (*b*=5m)	2.07	7.02	11.34	14.64	16.72
	SSI system (*b*=3m)	1.50	6.62	11.15	14.67	16.73

Table 2. Natural frequencies and damping ratios of fixed-base model

Mode		1	2	3	4	5
Natural frequency (Hz)	b = 5m	2.07	7.03	11.35	14.65	16.72
	b = 3m	1.50	6.63	11.16	14.67	16.73
Damping ratio (%)	b = 5m	2.35	3.41	2.77	2.03	1.48
	b = 3m	0.81	4.02	3.04	1.92	1.05

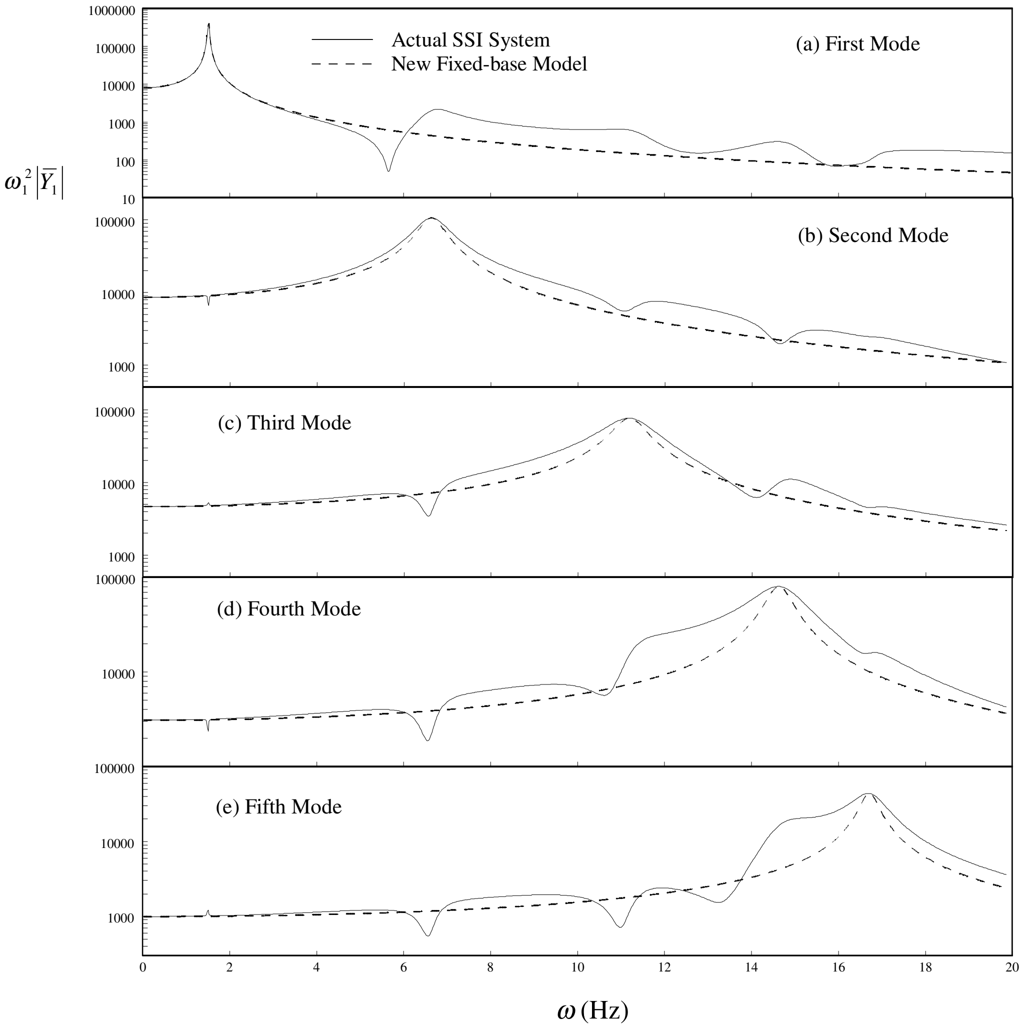

Figure 1. Comparison of modal responses in the frequency domain

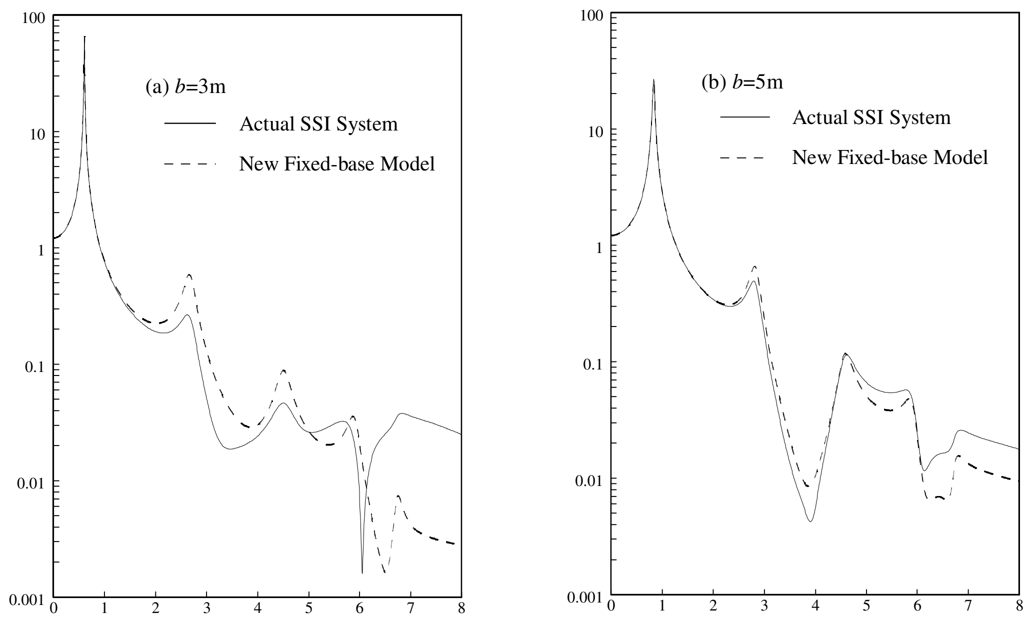

Figure 2. Comparison of roof floor displacement in the frequency domain

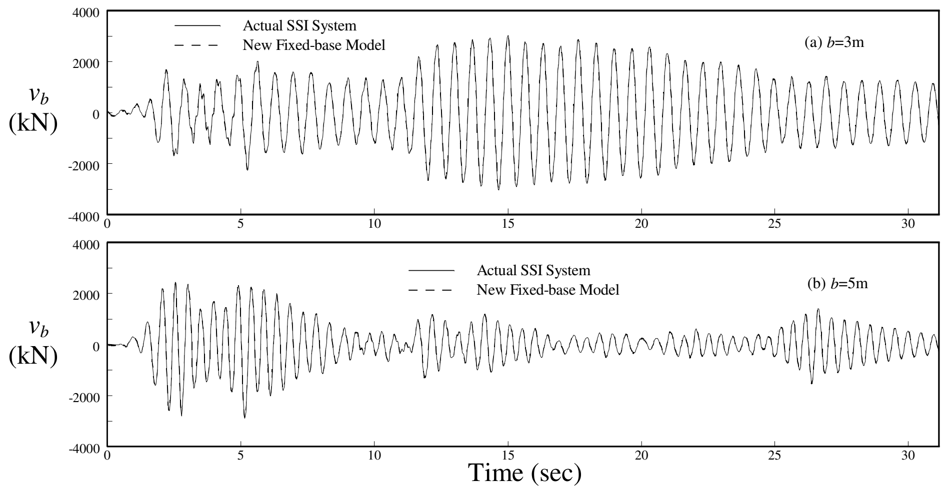

Figure 3. Comparison of base shear to El Centro earthquake

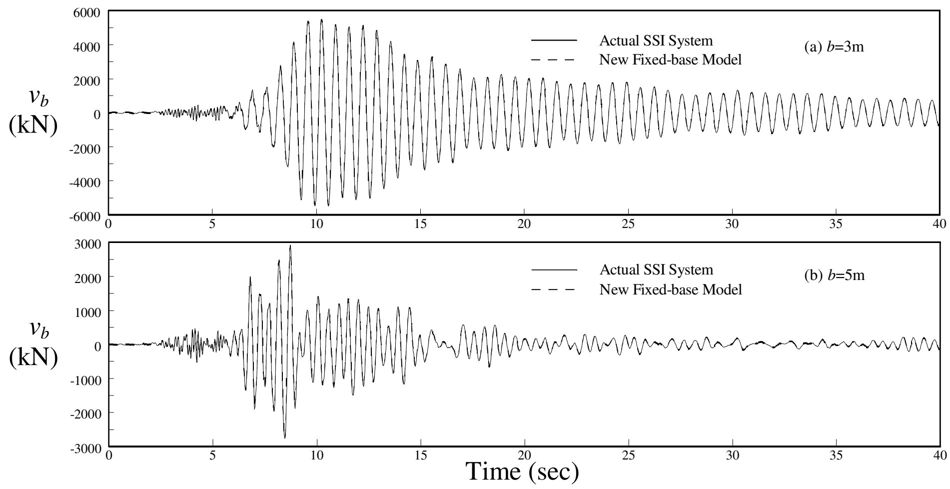

Figure 4. Comparison of base shear to Loma Prieta earthquake

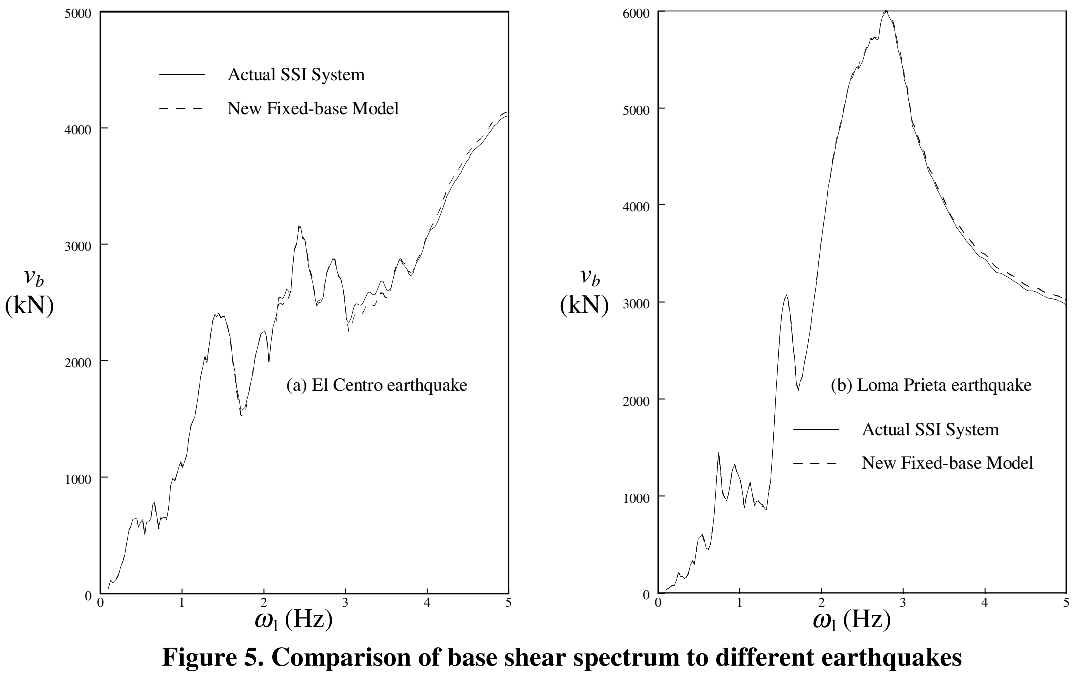

Figure 5. Comparison of base shear spectrum to different earthquakes