TEST AND ANALYSIS OF RC CURVE BEAMS UNDER REVERSED CYCLIC IN-PLANE LOADS

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SUMMARY

Fifteen full scale RC curve beams are tested under two concentrated in-plane loads with one or multiple reversed cycles. The cross-section of curve beam is 25cm ×40cm with 110cm interior radius and 150cm exterior radius. f_c' changes from 22.4 to 34.4 Mpa; f_y of rebar is from 384.7 to 533.5 Mpa; f_y of stirrup and hanger is from 397.8 to 508.4 Mpa. ρ and ρ' vary from $14/f_y$ to ρ_b. All the specimens fail in fleuure or shear. After test, the flexural cracking load, flexural ultimate load, shear cracking load, shear ultimate load and effective cross-sectional area are calculated. In order to simplify the design process, equivalent rectangular stress block for ultimate bending capacity is also introduced in this paper. The analytical load-defelection curve under monotonic load is predicted and compared to the experimental result with reasonable accuracy.

INTRODUCTION

Arch roofs, arch bridges, tunnels and sewage ducts are very popular curve structures. However, most of the RC design codes do not specify clearly the design equations for curve structures. Curve beams can not be designed using equations for straight beams. Because the tangential strain across the curve beam cross-section is hypoblic instead of straight distribution when subjected to bending moment[1]. The purpose of this paper is to investgiate the flexural capacity, shear capacity and effective stiffness of RC curve beams under in-plane static loads. Finally, the equivalent stress block of RC curve beams at ultimate flexural state is also proposed.

EXPERIMENTAL TEST

Taable 1 is the parameters of the specimens. The cross-section of all specimens is 25cm ×40cm with interior radius 110cm and exterior radius 150cm. ρ and ρ' are from $14/f_y$ to ρ_b.for straight beam. Two concentrated in-plane loads are applied at one-third points under one or multuple reversed cycles. Fig.1 shows the experimental set up where two rollers are inserted at both supports. In general, member ductility would be larger than 2.5, if $\rho \le 0.5\rho_b$ and $\rho' > 0.5\rho$ for flexural curve beams as shown in Fig.2. Fig.3 and Fig.4 are the crack patterns at final states for flexural failure and shear failure.

ANALYTICAL STRENGTH OF CURVE BEAMS

Consider a segment of curve beam subjected to bending moment as shown in Fig.5. The radii of curvature of neutral axis befor and after bending are r_{na} and r_{na}'. Let r be the radius of curvature of any tangential fiber before bending. The tangential elongation of that fiber is $(r_{na}-r)(d\phi - d\phi')$. So the tangential compression strain of this fiber becomes :

$$\varepsilon = (r_{na}-r)(d\phi - d\phi')/(rd\phi)$$

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or

$$\varepsilon = \frac{(r_{na} - r)}{r}\left(1 - \frac{r_{na}}{r'_{na}}\right) \tag{1}$$

where r_{na} and r'_{na} are unknowns. However, for a prescribe strain ε_p at a given fiber r_p, Eq.(1) may be expressed as function of that prescribed condition :

$$\varepsilon = \frac{(r_{na} - r)}{r}\left(\frac{\varepsilon_p r_p}{r_p - r_{na}}\right) \tag{2}$$

For instance if $r_p = r_i$, $\varepsilon_p = +0.623\sqrt{f'_c}/E_c$, Eq.(2) gives the hypoblic strain distribution at the cross-section where positive cracking moment occurs. If $r_p = r_o$, $\varepsilon_p = 0.003 + 0.02b/z + 0.2\rho_s$, Eq.(2) gives the strain distribution at the cross-section where positive ultimate moment occurs. By try and error process, the location of neutral axis, r_{na}, may be found for equilibrium. Using Eq.(2), the tangential strain and stress of concrete and rebars at a cross-sectioncan can be calculated for any prescribed condition. Comparing the analytical solutions with experimental results, the error for the load of cracking moment is–20% to +18%; the error for the load of ultimate moment is –14% to +17%.

The ultimate shear capacity of curve beam, V_u, is similar to the ultimate shear capacity of straight beam. In other words, V_u is contributed by concrete V_c, stirrup V_s, dowel force of tensile rebars V_d, and component of tensile force of rebars. V_c is proposed by this paper as :

For axial compression

$$V_c = 0.175\left(1 + \frac{N}{14A_g}\right)\sqrt{f'_c}bd \tag{3}$$

For axial tension

$$V_c = 0.175\left(1 + \frac{N}{3.3A_g}\right)\sqrt{f'_c}bd \tag{4}$$

V_s is contributed by stirrups located within 45°diagonal shear cracks. Since all the stirrups are arranged in radial direction, they are not parallel with each other. So the gross angle of beam, α, to be cut by 45°diagonal crack, may be calculated from geometry relationship[2] :

$$\alpha = \sin^{-1}\left(\frac{R}{r_i}\cos\vartheta\right) - \sin^{-1}\left(\frac{R}{r_o}\cos\vartheta\right) \tag{5}$$

where ϑ is the angle between diagonal crack and centroid axis. If θ_s is angle of stirrups, A_v is cross-sectional area of stirrups, f_y is yield stress of stirrup, we get

$$V_s = \frac{\alpha}{\theta_s}A_v f_y \tag{6}$$

Dowel force of tensile rebars is proposed by statistic :

$$V_d = 0.03 \cdot \sqrt[3]{A_s f_y} \tag{7}$$

Since rebars are not perpendicular to the critical section of diagonal shear cracks, the component of tensile force, $T\alpha/2$, would have positive contribution if the inner rebars are tension. $T\alpha/2$ becomes negative if the outer rebars are tension. Consequently, ultimate shear capacity of RC curve beam is :

$$V_u = V_c + \frac{\alpha}{\theta_s}A_v f_y + V_d \pm \frac{1}{2}T\alpha \tag{8}$$

Comparing analytical results with test results, the error for loads of diagonal shear crack is from –16% to +16% and the error for loads of ultimate shear capacity is from –9% to +7%.

EFFECTIVE CROSS-SECTIONAL AREA OF CURVE BEAMS

For a curve beam with depth to radius of centroid axis larger than 0.3, it is considered as thick arch. Strain energy of bending, shear and axial force should be taken into account for calculation of deflection by Castigliano Theorem. That is :

$$\Delta = \frac{\partial U}{\partial P} = \int_0^S \left\{ \frac{Mm}{A\,\mathrm{Re}\,E} + \frac{1.2Vv}{AG} + \frac{Nn}{AE} - \frac{Mn+nM}{ARE} \right\} dS \tag{9}$$

In which $m = \partial M/\partial P$, $v = \partial V/\partial P$, $n = \partial N/\partial P$, A is effective cross-sectional area of curve beam, E is modulus of elasticity as code value, R is radius of centroid axis, e is eccentricity between centroid axis and neutral axis. Before flexural cracking, A is taken as A_g, gross cross-sectional area of curve beam. After flexural carcking, A is taken as A_{eff} :

$$A_{eff} = \left\{ \left(\frac{M_{cr}}{M_{\max}} \right)^3 + \left[1 - \left(\frac{M_{cr}}{M_{\max}} \right)^3 \right] \frac{I_{cr}}{I_g} \right\} A_g \tag{10}$$

In Eq.(10), the reduction ratio of (A_{eff}/A_g) is equal to the flexural rigidity reduction ratio of (I_{eff}/I_g) in ACI code. Eq.(10) valids until shear force at any section reaches V_y or reaches ultimate state of that section. V_y is the shear when softening of stiffness begins. V_y is proposed as :

$$V_y = V_c + 0.5V_s \pm 0.5T\alpha \tag{11}$$

Between V_y and ultinate load, the tangential A_{eff} is equal to $1/3$ of the secant A_{eff} at V_y point calculated by Eq.(10). After ultimate point, the tangential A_{eff} is $-1/3$ of the secant A_{eff} at V_y point for shear falure and $-1/10$ of the secant A_{eff} at V_y point for flexural falure. Comparison of analytical and experimental results for vertical deflection at arch crown and for horizontal deflection at roller support are shown in Fig.6. Chen[2] and Lee[3] had more details about the comparison of $P-\Delta$ curves.

EQUIVALENT RECTANGULAR STRESS BLOCK FOR ULTIMATE BENDING MOMENT OF CURVE BEAMS

In order to simplify the calculation of the ultimate flexural capacity of RC curve beam, it would be nice to find out the magnitude and location of the concrete compression resultant under ultimate bending moment. Because the stress strain relationship of compressive concrete is nonlinear at ultimate state. The strain distribution in the cross-section of curve beam is nonlinear too. So the integrate of compression stress over the cross-section is rather complicate. This paper tries to propose the simplified contour maps for the magnitude and location of the concrete compression resultant C_c.

Consider a curve beam with compression stress distributed in kd depth as shown in Fig.7(a). Fig.7(b) is the equivalent stress block with equivalent stress $\alpha \cdot f_c'$ and equivalent depth $\beta_1 kd$. α and β_1 may be calculated theoretically by :

$$\alpha\beta_1 = k_1 k_3 = \frac{C_c}{f_c' kd \cdot b} = \frac{1}{f_c' kd} \cdot \int_0^{kd} \sigma_{\theta,c}\, dy \tag{12}$$

$$\frac{\beta_1}{2} = k_2 = 1 - \frac{1}{kd} \cdot \frac{M_c}{C_c} = 1 - \frac{1}{kd} \cdot \frac{\int_0^{kd} \sigma_{\theta,c}(y) \cdot y\, dy}{\int_0^{kd} \sigma_{\theta,c}(y)\, dy} \tag{13}$$

For easy integrate of Eq.(12) and Eq.(13), let the stress-strain curve of compression concrete be the well-known Modified Hognestad's Equation [4], which is consisted of two parts :
1. Parabolic ascending part, $0 \le \varepsilon \le \varepsilon_0$:

$$\sigma_c(\varepsilon) = f_c'' \cdot \left[2 \cdot \left(\frac{\varepsilon}{\varepsilon_0} \right) - \left(\frac{\varepsilon}{\varepsilon_0} \right)^2 \right] \tag{14}$$

2. Linear descending part, $\varepsilon_0 < \varepsilon \le \varepsilon_u$

$$\sigma_c(\varepsilon) = f_c'' \cdot \left[1 - \gamma \cdot \left(\frac{\varepsilon - \varepsilon_0}{\varepsilon_u - \varepsilon_0} \right) \right] \tag{15}$$

in which, $\varepsilon_0 = 1.8 f_c''/E_c$ is the strain at maximum stress, $E_c = 4700\sqrt{f_c''}MPa$ is the code specified slope in ascending part, the ultimate strain $\varepsilon_u = 0.0038$, the decreasing factor of stress from ε_0 to ε_u in desending part $\gamma = 0.15$. Breen[5] and Attard[6] suggested that the maximum compression stress in flexure may be taken as the cylinder strength, that is $f_c'' = f_c'$, which gives a reasonably good agreement between the theoretical and experimental values.

If a curve beam is under positive bending moment, refer to y_1 coordinate as shown in Fig.5, the tangential flexural strain at ultimate state may be calaulated by :

$$\varepsilon_{\theta,p} = \varepsilon_{cu} \cdot \frac{r_o}{kd} \cdot \frac{y_1}{r_{na} + y_1} \tag{16}$$

in which $\varepsilon_{cu} = 0.003$ is taken as that given in code. Introducing Eq.(15) and Eq.(16) into Eq.(12) and Eq.(13) for a curve beam with positive moment, we get :

$$(\alpha\beta_1)_p = (k_1 k_3)_p = \left(\frac{\lambda_p - \omega}{\omega} \right) \cdot \xi_p \tag{17}$$

$$\left(\frac{\beta_1}{2} \right)_p = (k_2)_p = 1 - \left(\frac{\lambda_p - \omega}{\omega} \right) \cdot \frac{\zeta_p}{\xi_p} \tag{18}$$

where

$$\lambda_p = \omega \cdot \frac{r_o}{kd} \tag{19}$$

$$\omega = \frac{\varepsilon_{cu}}{\varepsilon_0} \tag{20}$$

$$z = \frac{\gamma}{\varepsilon_u - \varepsilon_0} \tag{21}$$

$$\xi_p = 1 - 2\lambda_p + \frac{\omega}{\lambda_p - \omega} + \lambda_p z\varepsilon_0 \left(\frac{1-\omega}{\lambda_p - \omega} \right) + 2\lambda_p (\lambda_p - 1) \cdot \ln\left(\frac{\lambda_p}{\lambda_p - 1} \right) + \lambda_p z\varepsilon_0 \cdot \ln\left(\frac{\lambda_p - 1}{\lambda_p - \omega} \right) \tag{22}$$

$$\zeta_p = -\frac{1}{2} + 3\lambda_p + \frac{1}{2}\left(\frac{\omega}{\lambda_p - \omega} \right)^2 \left[1 - z\varepsilon_0 (\lambda_p - 1) \right] + \frac{z\varepsilon_0}{\lambda_p - 1}\left(\frac{1}{2} - \lambda_p \right) + \lambda_p z\varepsilon_0 \left(\frac{\omega}{\lambda_p - \omega} \right) + \lambda_p (3\lambda_p - 2) \cdot \ln\left(\frac{\lambda_p}{\lambda_p - 1} \right) - \lambda_p z\varepsilon_0 \cdot \ln\left(\frac{\lambda_p - 1}{\lambda_p - \omega} \right) \tag{23}$$

Referring to y_2 coordinate in Fig.5, if a curve beam is subjected to negative bending moment, the tangential flexural strain may be calculated by another equation :

$$\varepsilon_{\theta,n} = \varepsilon_{cu} \cdot \frac{r_i}{kd} \cdot \frac{y_2}{r_{na} - y_2} \tag{24}$$

Similarily, Eq.(12) and Eq.(13) for a curve beam with negative bending moment can be written as :

$$(\alpha\beta_1)_n = (k_1 k_3)_n = \left(\frac{\lambda_n + \omega}{\omega}\right) \cdot \xi_n \tag{25}$$

$$\left(\frac{\beta_1}{2}\right)_n = (k_2)_n = 1 - \left(\frac{\lambda_n + \omega}{\omega}\right) \cdot \frac{\zeta_n}{\xi_n} \tag{26}$$

where

$$\lambda_n = \omega \cdot \frac{r_i}{kd} \tag{27}$$

$$\xi_n = -1 - 2\lambda_n + \frac{\omega}{\lambda_n + \omega} - \lambda_n z\varepsilon_0 \left(\frac{1-\omega}{\lambda_n + \omega}\right) - 2\lambda_n(\lambda_n + 1) \cdot \ln\left(\frac{\lambda_n}{\lambda_n + 1}\right) + \lambda_n z\varepsilon_0 \cdot \ln\left(\frac{\lambda_n + 1}{\lambda_n + \omega}\right) \tag{28}$$

$$\zeta_n = -\frac{1}{2} - 3\lambda_n + \frac{1}{2}\left(\frac{\omega}{\lambda_n + \omega}\right)^2 [1 + z\varepsilon_0(\lambda_n + 1)] - \frac{z\varepsilon_0}{\lambda_n + 1}\left(\frac{1}{2} + \lambda_n\right) + \lambda_n z\varepsilon_0 \left(\frac{\omega}{\lambda_n + \omega}\right) - \lambda_n(3\lambda_n + 2) \cdot \ln\left(\frac{\lambda_n}{\lambda_n + 1}\right) + \lambda_n z\varepsilon_0 \cdot \ln\left(\frac{\lambda_n + 1}{\lambda_n + \omega}\right) \tag{29}$$

Fig.8 shows the contour map of $\beta_1/2$ for positive bending. Fig.9 shows the contour map of $\alpha\beta_1$ for positive bending. Fig.10 and Fig.11 are the countour maps for negative $\beta_1/2$ and $\alpha\beta_1$ correspondingly. The resultant of equivalent stress block of curve beams may be find out easily from these countour maps.

CONCLUSIONS

1.Flexural cracking moment and ultimate moment may be calculated by force equilibrium process, if the hypoblic strain distribution is considered for the cross-section of curve beam.
2.Cracking shear strength may be calculated by Eq.(3) or Eq.(4) which are similar to ACI code equaction for straight beams. Ultimate shear capacity may be calculated by Eq.(8).
3.For thick arch, the effective cross-sectional area calculated by Eq.(10) would give the reasonable prediction of $P - \Delta$ curve under monotonic loading.
4.α and $\beta_1/2$ calculated by Eq.(17), Eq.(18), and Eq.(25), Eq.(26) provide a good prediction for rectangular stress block for curve beam.
5.At ultimate flexural state, the resultant of equivalent concrete compression stress block may be find out easily from the contour maps shown in Fig.8 to Fig.11.

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Table 1 : Specimen Parameters

Specimen No.	Tangential Reinforcement		Stirrup	f_c' (MPa)	Ultimate Load (KN)	Failure Mode
	Exterior	Interior				
CB31	2-D13	3-D13	D10@6°	24.3	+225.0	Flexure
CB32	2-D13	3-D19	D13@6°	18.6	+490.5	Flexure
CB33	2-D13	3-D25	D13@6°	24.1	+630.2	Flexure
CB37	3-D13	3-D25	D13@6°	25.3	+420.0	Shear
CB38	3-D19	3-D25	D13@6°	25.2	+750.3	Flexure
CB41	3-D13	2-D13	D13@6°	18.6	+158.0	Flexure
CB42	3-D25	2-D13	D13@6°	20.6	+180.0	Flexure
CB51	3-D25	3-D25	D10@4°	22.2	+727.2	Shear
CB52	3-D19	3-D19	D10@6°	31.2	+467.5	Flexure
CB53	3-D19	3-D19	D10@$4^\circ,8^\circ$	32.5	+473.3	Shear
CB54	3-D25	3-D25	D10@4°	22.2	-886.0	Shear
CB56	3-D19	3-D19	D10@$4^\circ,8^\circ$	32.5	-609.9	Shear
CB60	3-D19	3-D19	D10@$3^\circ,6^\circ$	24.1	+380.2	Shear
CB62	3-D19	3-D19	D10@6°	34.4	+398.8	Flexure
CB65	3-D25	3-D25	D10@6°	24.1	+525.0	Shear

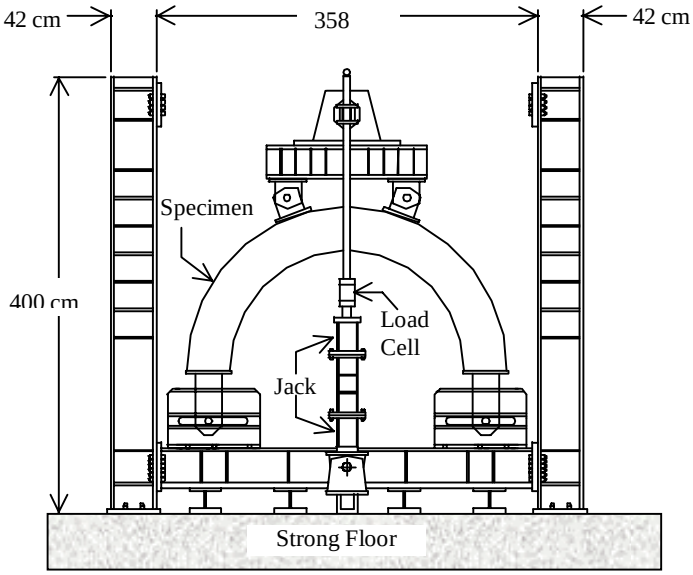

Fig.1 Experimental Set Up

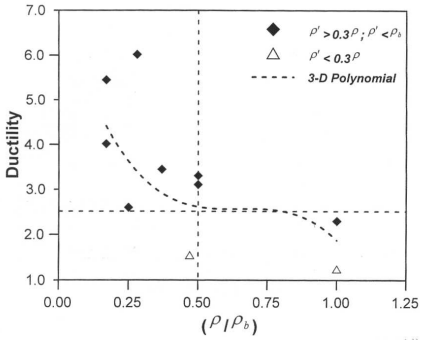

Fig.2 Change of Curve Beam Ductility

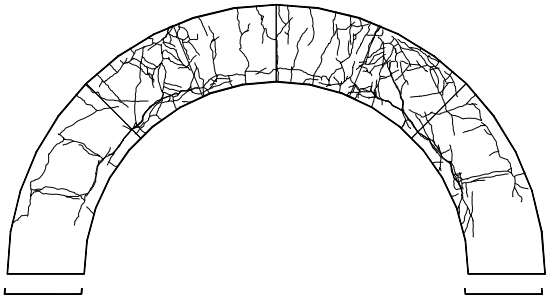

Fig.3 Final Cracks for Flexural Failure

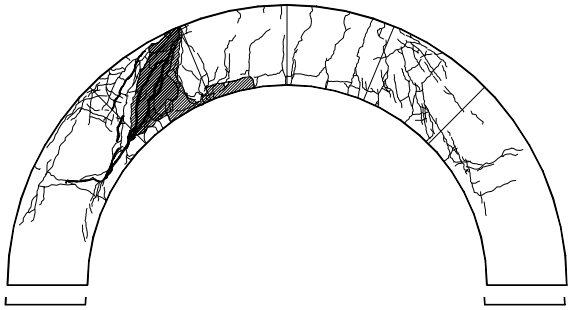

Fig.4 Final Cracks for Shear Failure

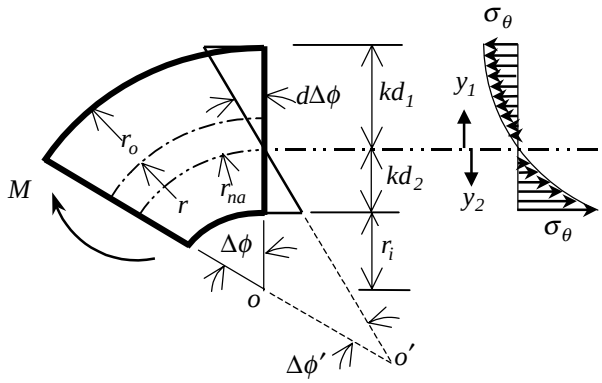

Fig.5 Tangential Stress of Curve Beam

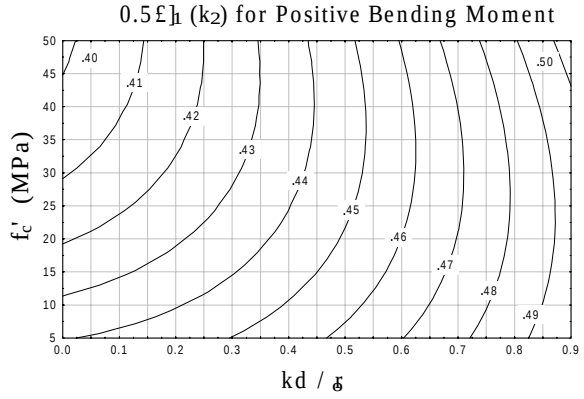

Fig.6 Comparison of $P-\Delta$ Curves for CB65

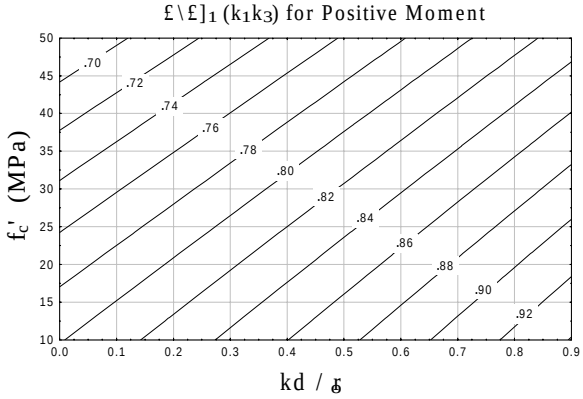

Fig.7 Equivalent Stress Block of Curve Beam

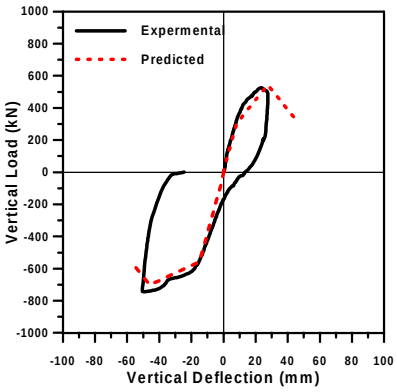

Fig.8 Countour of $0.5\beta_1$ Under Positive Bending Moment

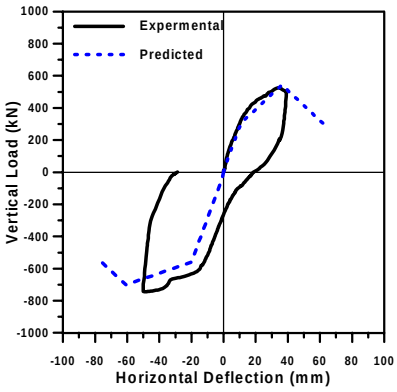

Fig.9 Countour of $\alpha\beta_1$ Under Positive Bending Moment

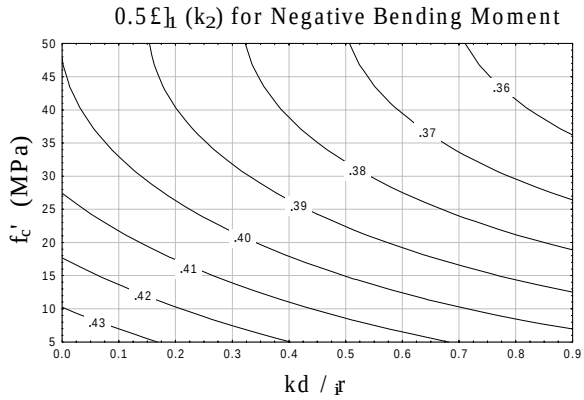

Fig.10 Countour of $0.5\beta_1$ Under Negative Bending Moment

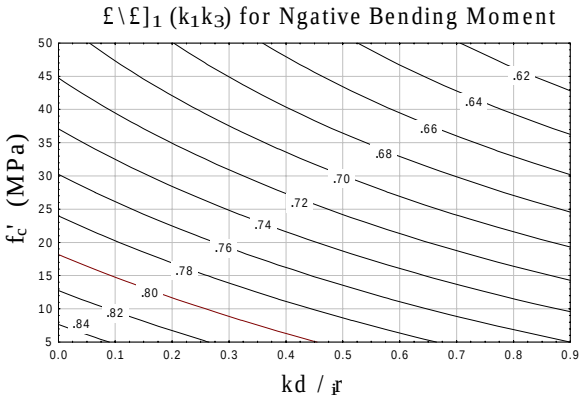

Fig.11 Countour of $\alpha\beta_1$ Under Negative Bending Moment