STRENGTH REDUCTION FACTORS FOR MULTI-DEGREE-OF-FREEDOM SYSTEMS

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SUMMARY

Strength reduction factors of multi-degree-of-freedom (MDOF) structures are investigated. The study is based on the results of strength reduction factors computed for eight steel moment resisting-frame buildings undergoing different levels of inelastic deformation when subjected to 28 earthquake ground motions. The ground motions used in this study were recorded on different soil conditions corresponding to firm sites (site classes A, B, C and D according to the 1997 NEHRP Provisions). Four main variables are considered: a) the fundamental period of vibration of the building; b) the level of inelastic deformation in the building; c) number of stories; and d) soil conditions. Special emphasis is given to the ratio of the strength reduction factor that results in an adequate control of the maximum interstory displacement ductility ratio in a MDOF structure to the strength reduction factor corresponding to the same ground motion and same level of inelastic deformation in an equivalent single-degree-of-freedom (SDOF) system having a period of vibration equal to the fundamental period of the MDOF structure. Results indicate that strength reduction factors of MDOF structures are typically smaller than those corresponding to SDOF systems. Thus, the lateral strength of MDOF structures needs to be higher than the one obtained from SDOF systems. The ratio of the MDOF to SDOF strength reduction factor is primarily affected by the number of stories and the level of inelastic deformation. Finally, for the site conditions considered in this study, the soil type has only a small effect on MDOF reduction factors.

INTRODUCTION

Seismic codes allow structures to behave inelastically during severe earthquake ground motions. Reductions in design forces produced by nonlinear behaviour are typically accounted for in force-based earthquake resistant design through the use of strength reduction factors. The strength reduction factor due to nonlinear behaviour, R_μ, is the ratio of the lateral yielding strength required to maintain the structure elastic to the lateral yielding strength required to limit the displacement ductility demand μ less or equal to a maximum tolerable displacement ductility ratio μ_i. For design purposes, R_μ, corresponds to the maximum reduction in strength that can be used in order to limit the displacement ductility demand to the predetermined maximum tolerable ductility μ_i in a structure that will have a lateral strength equal to the design strength.

Several studies have been conducted to understand the behaviour of strength reduction factors in SDOF systems, but only a few have studied strength reduction factors in MDOF systems or the required modification to the results available from SDOF to be applicable to MDOF structures. A comprehensive review of previous studies on strength reduction factors for SDOF system has been presented by Miranda and Bertero [6].

The relationship between MDOF and SDOF system response was first studied by Veletsos and Vann [11]. The objective of their study was to identify the parameters which have a dominant influence on the response of MDOF elastoplastic systems and to estimate the significant features of the response of such systems in terms of the known response of similarly excited linear systems. The systems considered were cantilever structures of the

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shear-beam type with "n" equal masses connected by weightless springs in series from one to five degrees-of-freedom. This study concluded that the yield resistance required to limit the absolute maximum story deformation of an inelastic MDOF system to a prescribed ductility ratio could be estimated to a useful degree of approximation from the results on a linear analysis. For systems having two and three degrees of freedom, the relationship between the required yield deformation and the absolute maximum deformation of the associated linear system could be considered the same as that for a SDOF systems with the same frequency. Finally the study concluded that for systems having more than three-degrees of freedom the proposed design rules for SDOF systems were not sufficiently accurate and could lead to unconservative estimates of the required lateral yield resistance, and that errors tended to increase as the number of degrees of freedom increased.

Nassar and Krawinkler [7] studied three types of simplified MDOF models to estimate the modifications required to the inelastic strength demands obtained from bilinear SDOF systems in order to limit the story ductility demand in the first story of the MDOF systems to a prescribed value. The three types of MDOF models were "regular" two-dimensional frames with widely spaced elastic modal periods. The first type of model consisted of a "beam-hinge" model in which plastic hinges could be formed in beams only. The second model referred to as "column hinge" consisted of a model in which plastic hinges could be formed in columns only, and the third model, referred to as "weak story" model, allowed plastic hinges to form in columns of the first story only. The fundamental period of each model was given by $T = 0.02\ h_n^{3/4}$ where h_n is the total height of the building in feet. The base shear capacity of the MDOF systems was tuned to the inelastic strength demand of the corresponding SDOF system. The ductility demands in the MDOF models were compared to those of the SDOF systems when subjected to the same ground motions as those used for the SDOF systems. The study concluded that MDOF story ductility demands differ significantly from those of the corresponding SDOF systems. It was found that the required strength for specified target ductility ratios depend strongly on the type of failure mechanism. Furthermore, they observed that the deviation of MDOF story ductility demands from the SDOF target ductility ratios increased with period and target ductility ratio.

Hummar and Rahgozar [5] studied a ten-story shear frame with a uniform distribution of mass and interstory height. It was concluded that for high ductility values the displacement ductility demand in many stories of the MDOF system could be substantially higher that in the associated SDOF system and that in most of the systems the critical story was the lowest, although the uppermost stories could also exhibit larger ductility demands due to the participation of higher modes.

Chopra [1] studied the ratio of base shear seismic demands on a single bay 20-story building and its corresponding SDOF, both with natural period of 3.0 s. The base shear yield strength of the MDOF system was determined in conjunction with heightwise distribution forces according to the 1994 UBC code [10] to ensure that the first-story ductility demand did not exceed the allowable ductility. The study concluded that ratio between base yielding shear of the MDOF to that of the SDOF system, varied between 1 and 4, depending significantly on the allowable ductility and on the fundamental vibration period.

More recently, Seneviratna and Krawinkler [9] quantified seismic demands on MDOF systems and provided some modifications to SDOF results necessary to estimate the seismic demands for MDOF systems from elastic and inelastic spectra. They studied two types of lateral load resisting systems; moment-resisting frames (MRFs) and isolated structural walls. The types of MRFs considered and the expression used to obtain the fundamental period for each system were the same previously used by Nassar and Krawinkler [7]. This study concluded that, with exception of very short period structures, the maximum story ductility demand for MDOF frame models was higher than the target ductility ratio of the first mode SDOF system and that this amplification increases with increasing periods, illustrating the importance of higher mode effects.

The objective of this paper is to present the results of an ongoing study on strength reduction factors of MDOF systems. The structural models used in this study reproduce more realistically existing moment-resisting frame buildings than those used in previous investigations. The aim of the study is to provide modification factors on lateral strength demands derived from SDOF systems in order to estimate lateral strengths required in MDOF structures to adequately control maximum story displacement ductility demands when subjected to earthquake ground motions. The study attempts to also investigate the dispersion on the relationship between the base shear demands on MDOF structures to that of SDOF structures.

DESCRIPTION OF STRUCTURAL MODELS

The MDOF structures considered in this study were eight steel moment-resisting frame (SMRF) buildings four, eight, twelve and sixteen stories high. The eight buildings have the same plan, which consists of three bays on each direction as shown in Figure 1. The buildings were assumed to have a uniform mass distribution over their height and a non-uniform lateral stiffness distribution. Steel members in the buildings were designed using the lateral load distribution specified in the 1994 UBC [10] with member stiffnesses tuned to obtain fundamental periods of vibration for each structure representative of those obtained from earthquake records of instrumented existing SMRFs buildings. Additionally, with the exception of beam-to-column connections in the top floor, the steel sections of structural members was selected such that the sum of plastic section modulus, *Z*, of the columns framing into each beam-column joint was higher that the sum of plastic section modulus of the beams framing into the same joint.

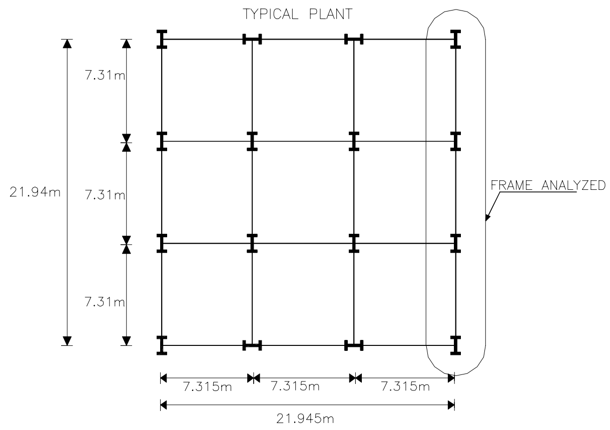

Figure 1. Plan of the multi-story buildings analyzed in this study

In order to study the influence of the period of vibration and the number of stories, for each building height two sets of target fundamental periods were considered: (a) flexible SMRFs buildings with fundamental periods given by $T = 1.78 \times 0.0853\, h_n^{3/4}$ where h_n is the total height of the building in meters (or $T = 1.78 \times 0.035\, h_n^{3/4}$ when h_n is given in feet); (b) rigid SMRFs buildings with fundamental periods given by $T = 1.07 \times 0.0853\, h_n^{3/4}$ (or $T = 1.07 \times 0.035\ h_n^{3/4}$ when h_n is given in feet). These fundamental periods were selected to approximately provide upper and lower bounds of those recently obtained from earthquake records of instrumented existing SMRFs in California [4]. It is important to notice that even the rigid MRFs considered herein have fundamental periods longer than those obtained using the expression recommended by the UBC [10] for SMRF buildings which overestimates the lateral-stiffness in order not to underestimate design forces from design spectra whose ordinates decrease with increasing periods.

The resulting flexible MRFs buildings barely satisfy the maximum interstory drift limitations of the 1994 UBC when subjected to a lateral load corresponding to zone 4 of that code. On the other hand, the rigid SMRFs buildings, which have more than two and a half times the lateral stiffness of the flexible SMRFs buildings, barely satisfy the maximum interstory drift limitations of the Mexico City building code [2] when applying lateral loads corresponding to the soft soil zone of Mexico City. In Figure 2 the elevations for each of the eight buildings are shown; the member sections used for columns and beams on each building are also shown in this figure. It is important to notice that the MDOF models considered in the study done by Nassar and Krawinkler [5] are significantly stiffer than those considered in this study (3.5 more times stiffer and 9.3 times stiffer than the rigid and flexible MRFs buildings considered in this investigation, respectively).

The first (fundamental), second and third mode periods of vibration for each system and the first mode effective modal mass M^* normalised by the total mass of the system M_T are given in Table 1. It can be observed that normalised first mode modal masses decrease with increasing number of stories and are slightly larger those of flexible SMRF buildings compared to those of the rigid SMRF buildings. Also in Table 1 is the lateral yield strength, V_{by}, normalised by the total weight, W, of the building and the yield roof displacement Δ_y normalised

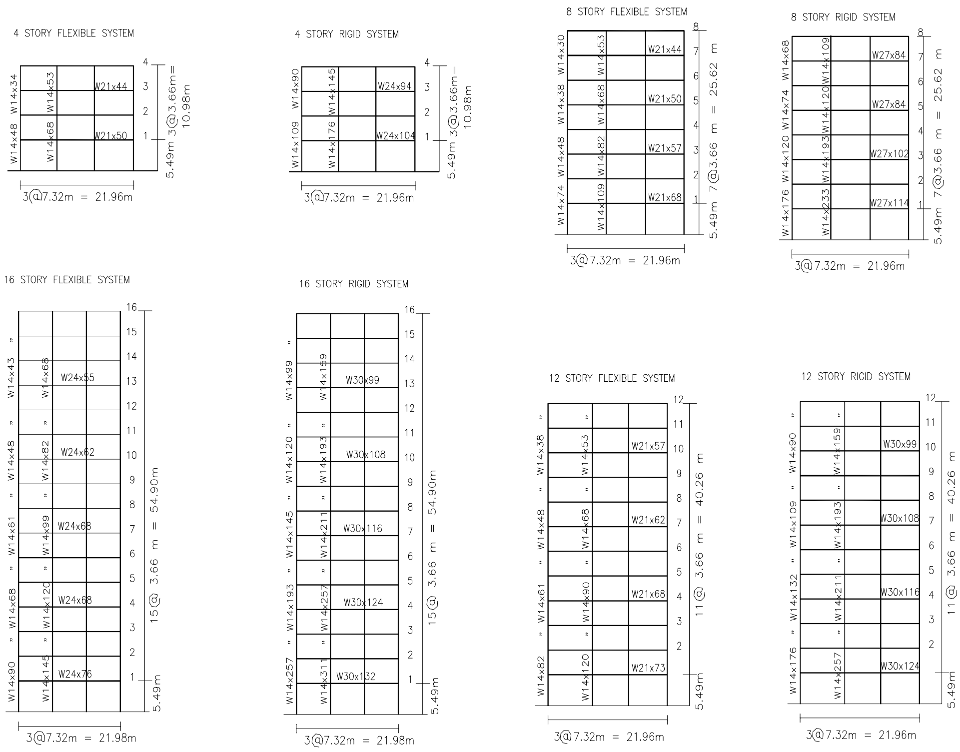

Figure 2. Elevations of the 4, 8, 12 and 16 story flexible and rigid buildings showing steel members.

by the total height of the building, h_n, ($\gamma_{y\ \mathrm{roof}} = \Delta_y/h_n$). The lateral strength V_{by} and the yield displacement Δ_y correspond to those of an equivalent elastoplastic behaviour obtained from the global strength-displacement relationship of the building (i.e. base shear vs. roof displacement) calculated with a pushover analysis. Note that because the moment of inertia, I, and the plastic section modulus are highly correlated for W shapes, rigid SMRFs buildings are not only significantly stiffer than flexible MRFs buildings but also significantly stronger.

For each building an equivalent SDOF was defined. The properties of these equivalent SDOF systems were set such that the weight of the SDOF system was the same as the total weight of the multistory building; similarly, the period of vibration and damping ratio of the SDOF systems were the same as the fundamental mode properties of the multistory building.

GROUND MOTIONS USED

Each structural model was subjected to a set of twenty-eight ground motion records listed in Table 2. The ground motions were recorded on various soil conditions corresponding to rock and firm sites with average shear wave velocities higher than 180 m/s, which correspond to site classes A, B, C and D according to the 1997 NEHRP Provisions [3].

Table 1. Dynamic characteristics of multi-degree-of-freedom systems studied

System	T_1 (s)	T_2 (s)	T_3 (s)	M_1^* / M_T	V_{by} / W	$\gamma_{y\ \mathrm{roof}}$ (%)
4 Story Flexible	1.23	0.39	0.20	0.98	0.32	0.91
4 Story Rigid	0.71	0.22	0.12	0.97	0.81	0.79
8 Story Flexible	1.92	0.68	0.39	0.95	0.22	0.99
8 Story Rigid	1.18	0.42	0.24	0.92	0.48	0.87
12 Story Flexible	2.61	0.91	0.53	0.89	0.15	0.89
12 Story Rigid	1.53	0.53	0.31	0.88	0.35	0.77
16 Story Flexible	3.09	1.00	0.55	0.87	0.15	1.00
16 Story Rigid	1.87	0.61	0.34	0.86	0.30	0.87

Table 2. List of ground motions used in this study

Earthquake	Station Name Location	Epicentral Distance km	Magnitude M_s	Components and Maximum Accelerations				Site Class
Whittier	Los Angeles, Griffith Park	22.3	6.1	0	-133.8	360	-121.4	A,B
Loma Prieta	Gilroy 1,Gavillan Coll.	10.9	7.1	90	433.6	360	426.6	A,B
Loma Prieta	San Francisco, Cliff House	57.4	7.1	0	-73.1	90	-105.7	A,B
Northridge	Los Angeles, Griffith Park	24.5	6.8	360	162.9	270	282.1	A,B
San Fernando	Glemdale,633 E Broadway	18.0	6.5	110	265.7	200	-209.1	C
Whittier	Garvey Reservoir A. Building	11.3	6.1	60	-367.1	330	-468.2	C
Loma Prieta	Corralitos, Eureka Canyon R.	2.2	7.1	90	469.4	360	617.7	C
Loma Prieta	Saratoga, Aloha Ave.	12.4	7.1	90	316.2	0	494.5	C
Northridge	Castaic Old Ridge Route	38.6	7.5	360	504.2	90	557.1	C
Kern County	Los Angeles, Hollywood PE	107.0	7.7	90	41.2	180	-58.1	D
San Fernando	Los Angeles, Hollywood Bldg	23.0	6.5	90	-207.0	180	167.3	D
Whittier	Vernon, Cmd Terminal	11.1	6.1	7	-267.3	277	-239.9	D
Loma Prieta	Gilroy 2, Hwy 101 Bolsa	12.6	7.1	90	316.3	0	394.2	D
Northridge	Los Angeles, Hollywood Bldg	22.5	6.8	360	381.4	90	227.0	D

RESULTS

Previous studies [1, 5, 7, 9, 11] have shown that the maximum interstory displacement ductility demand in MDOF structures may exceed significantly the target (or allowable) story ductility ratio if they are designed with a strength equal to the lateral strength required in a SDOF to control the maximum displacement ductility demand to the same allowable value. Thus, the R_μ factors computed from SDOF systems need to be modified for the design of MDOF structures in order to control the maximum story displacement ductility demand.

In a SDOF system the lateral strength required to avoid displacement ductility demands larger than the maximum allowable ductility ratio μ_i can be estimated from

$$V_{SDOF}(\mu = \mu_i) = \frac{V_{SDOF}(\mu = 1)}{R_\mu} \tag{1}$$

where $V_{SDOF}(\mu = 1)$ is the lateral strength required to maintain the SDOF system elastic and R_μ is the strength reduction factor derived from SDOF systems. $V_{SDOF}(\mu = 1)$ can be directly obtain from the design linear elastic response spectrum and R_μ whose variation with period, ductility ratio and soil conditions is now relatively well established can be taken from any of the various proposed equations described in Ref. 6.

For multi-storey buildings the lateral strength required to avoid story displacement ductility demands larger than the maximum allowable ductility ratio μ_i can be estimated from

$$V_{MDOF}(\mu = \mu_i) = \frac{V_{SDOF}(\mu = 1)}{R_\mu \; R_M} \tag{2}$$

where R_M reduction factor that takes into account the difference in lateral strength demands in MDOF structures to SDOF structures and is given by

$$R_M = \frac{V_{SDOF}(\mu = \mu_i)}{V_{MDOF}(\mu = \mu_i)} \tag{3}$$

Thus, R_M represents a modification factor to the strength reduction factor of SDOF systems so it can be applied to MDOF structures. The subscript M stands for the modification to take into account **M**DOF effects.

In this study for each ground motion and each building the modification factor R_M was calculated for target ductility ratios equal to 1.5, 2, 3, 4 and 5 as follows:

1. $V_{MDOF}(\mu = \mu_i)$ was computed by scaling the intensity of the ground motion until the maximum story displacement ductility ratio in the MDOF structure was, within a 1% tolerance error, equal to the target ductility. The scaling factor was obtained using an iterative procedure using Drain 2DX [8].
2. $V_{SDOF}(\mu = \mu_i)$ was computed by iteration on the lateral strength of the SDOF system when subjected to the same ground motion and same scale factor of step one until the displacement ductility ratio in the MDOF structure was, within a 1% tolerance error, equal to the target ductility.
3. The modification factor R_M was calculated using equation 3.

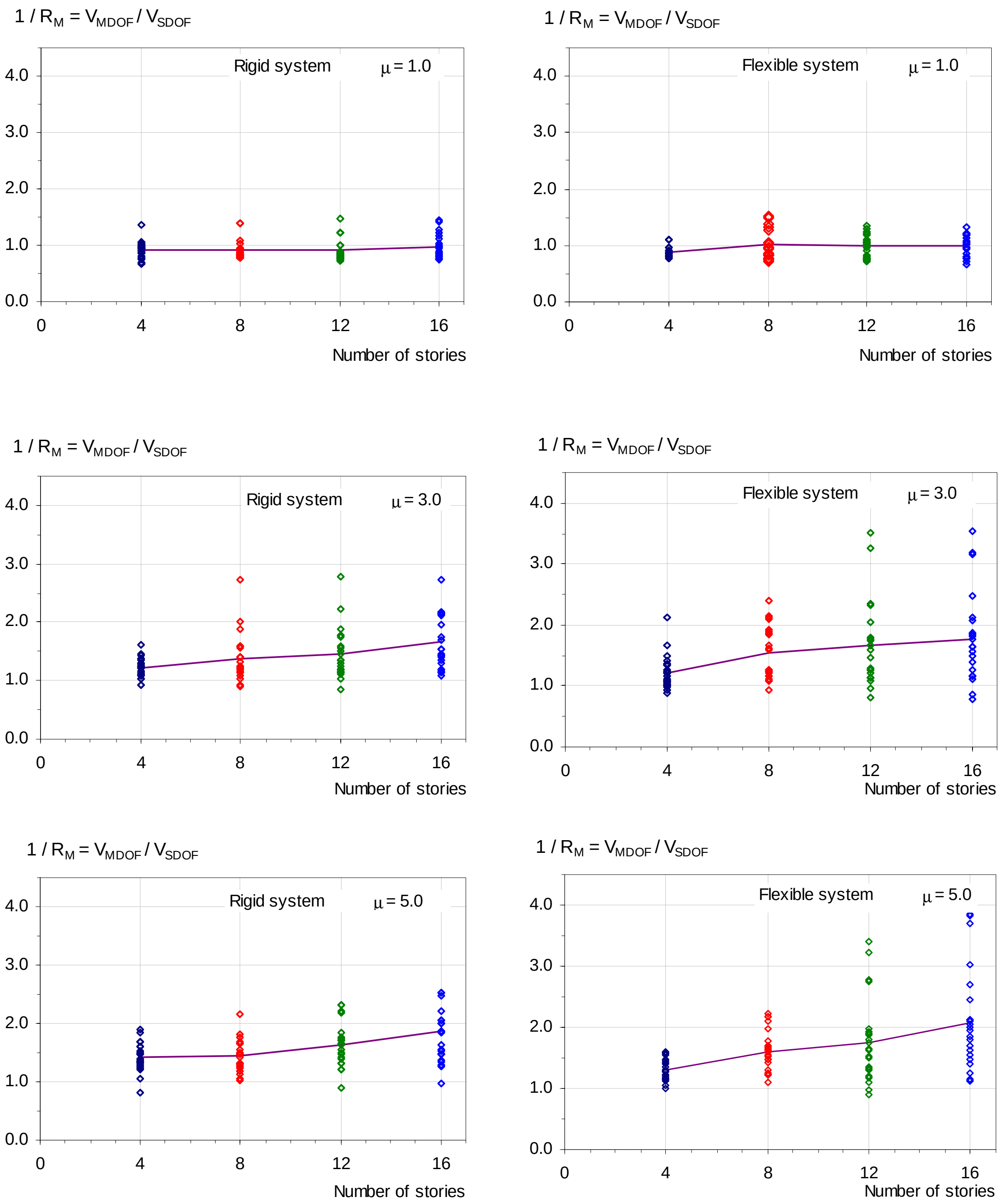

Figure 3. Ratios of lateral strength demands on MDOF to those on SDOF systems required to control maximum ductility demands for rigid and flexible buildings as a function of the number of stories.

Figure 3 shows the results for ductility ratios of one, three and five for the flexible and rigid SMRF buildings as a function of the number of stories. The inverse of the modification factor RM shown in this figure represents an amplification factor by which the lateral strength of the SDOF needs to be multiplied to control the maximum story displacement ductility in the MDOF structure to the target ductility ratio. It can be seen that for a ductility ratio of one there is a relatively small scatter on the results and mean values (shown in the continuous line) are approximately equal to one regardless of the number of stories. This means that for μ =1 the lateral strength of the MDOF is on average approximately equal to that of the SDOF system. However, as the level of inelastic deformation increases three main observations can be made: (1) mean modification factors increase with the number of stories; (2) for the same number of stories mean modification factors of flexible SMRF buildings are slightly larger than those of the rigid structures; and (3) dispersion increases with increasing number of stories and increasing ductility ratio. For the buildings and ground motions analysed in this study mean amplification values can be as high as 2.1 (for the 16-story flexible SMRF building), however values as large as 4.0 were computed for certain ground motions.

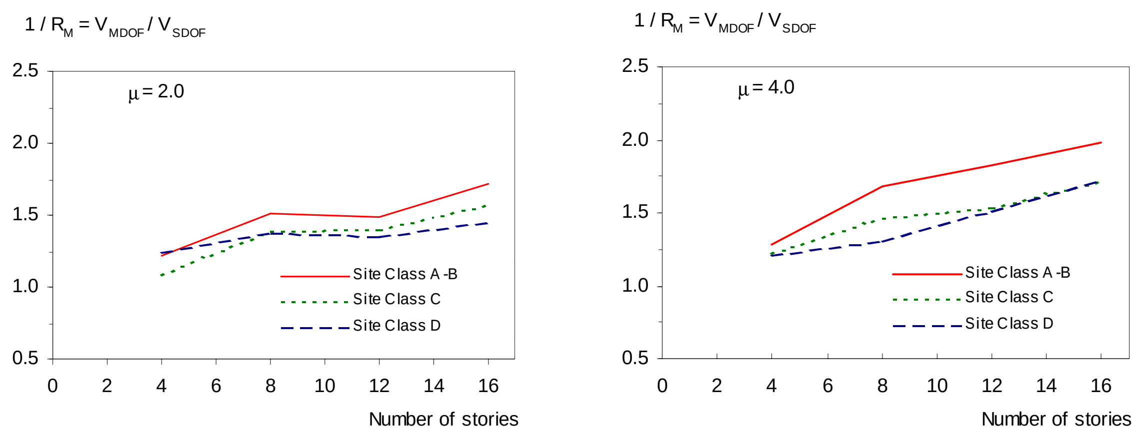

Figure 4. Influence of site conditions on ratios of lateral strength demands on MDOF to those on SDOF systems required to control maximum ductility demands.

The influence of site conditions is shown in figure 4, where mean amplification factors are shown as a function of the number of stories and the site conditions. It can be seen that the amplification factors for site classes A and B (rock) are slightly larger than those for site classes C and D (stiff soil). However, one must take into account that mean values shown in this figure correspond only to 8 records for sites classes A and B, and only 10 records for site classes C and D (each).

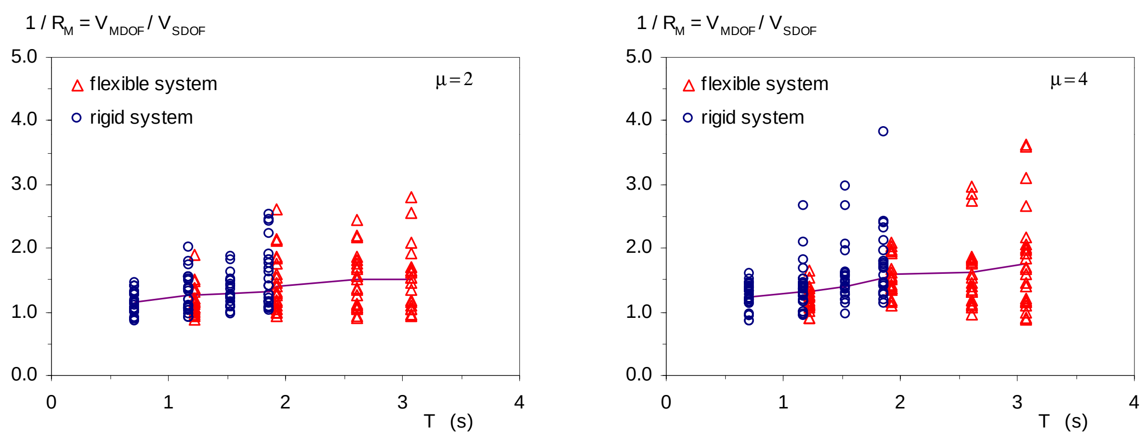

Figure 5. Influence of the fundamental period of vibration on the ratio of lateral strength demands on MDOF to those on SDOF systems required to control maximum ductility demands.

The influence of the fundamental period of vibration is shown in figure 5. Note that the fundamental period of the 8-story rigid building is practically the same as that of the 4-story flexible building. Similarly, the fundamental period of the 16-story rigid building is practically the same as that of the 8-story flexible building. It can be seen that for low levels of ductility (μ =2) the modification factors are, in general, similar for building with approximately the same period of vibration even though the number of stories is different. However, for high levels of ductility (μ =4) there are important differences in the modification factors for buildings with different number of stories despite their fundamental periods being practically the same, suggesting that ratios of MDOF to SDOF lateral strength demands are primarily affected by the number of stories and level of ductility and to smaller extent (and probably only indirectly) by the fundamental period of vibration.

CONCLUSIONS

The purpose of this study was to gain insight on modification factors that permit the estimation of lateral strength demands in MDOF required to control maximum story displacement ductility demands by using well established strength reduction factors of SDOF systems and elastic strength demands of SDOF systems. The following conclusions can be drawn from the results of this study:

(a) Lateral strengths required to control maximum story ductilities in multi-storey buildings are typically larger than those of SDOF systems having periods of vibration equal to the fundamental period of the MDOF structures.

(b) Mean ratios of MDOF to SDOF lateral strength demands increase with the number of stories. For the same number of stories mean ratios of flexible SMRF buildings are slightly larger than those of the rigid structures.

(c) A considerable dispersion of MDOF to SDOF lateral strength demands was computed. In general, dispersion increases with increasing number of stories and increasing displacement ductility ratio.

(d) From a limited number of results it appears that ratios of MDOF to SDOF lateral strength demands are more affected by the number of stories than by the fundamental period of vibration.

(e) For the site classes considered in this study, soil conditions at the site have only a small effect on mean ratios of MDOF to SDOF lateral strength demands.

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