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DYNAMIC CHARACTERISTICS OF PILE FOUNDATION SYSTEMCONSIDERING NON-LINEARITY OF SURROUNDING SOIL MEDIUM

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SUMMARY

Soils may be regarded as being elastic only when the strain caused to them are as small as 10^{-5}. When subjected to strong earthquake motions, the soils around the foundation piles are believed to have entered a non-linear range. Because of these facts, it is considered important that the non-linearity of soils should be taken into consideration if the vibration characteristics of pile foundations are to be studied intelligently.

In this paper, the vibration characteristics of the piles are considered by applying the three-dimensional thin layer element method to a pile-boundary soil zone-outer soil system model and further by evaluating and analyzing the non-linearity of the soils within the boundary area around the piles by the use of the equivalent linearization method. Since the external force is dynamic and the soil strains vary on a minute-to-minute basis, the rigidity reduction ratio of the soils, etc. is evaluated by the equivalent linearization method that deals with them on an average basis. For determination of the relationships between the strains, shear rigidity and damping constants of soils, Ohsaki-Hara Model3), which was proposed by modifying Ramberg-Osgood Model by reflecting a large number of soil investigation results obtained in Japan on it was used.

INTRODUCTION

The rigidity and the hysteretic damping of soils vary as the levels of strains caused to them. Soils may be expected to remain within the elastic limit only when the strains caused to them are as small as 10^{-5}. Under strong earthquake motions, soils around the piles are presumed to be strained as to enter a non-linear region. Hence, it is highly important that the non-linearity of soils be taken into consideration if the vibration characteristics of pile foundations are to be studied intelligently.

A number of models are available which can be used to show the stress-strain relationship in soils. Those proposed by Ramberg-Osgood (R-O Model) and by Hardin-Drnevich (H-D Model) are well known. Ohsaki and Hara improved the R-O Model once and again by introducing into it the soil survey results achieved in Japan and finally developed and proposed what is now known as Ohsaki-Hara Model3). For analyses of the soil-pile foundation interactions, such techniques as the three-dimensional wave motion theory, the finite element method and the boundary element method can be used. M.H. El Naggar and M.Novak1) studied the responsecharacteristics of piles by considering the non-linearity of soils around foundation piles on the basis of the three-dimensional wave motion theory. In the analyses discussed in this paper, the authors studied the pile-boundary soil zone-outer soil ground system model by the use of the three-dimensional thin layer element method which combined with the finite element method for the vertical direction and the three-dimensional wave propagation theory for the horizontal direction. The non-linearity of soils was studied by the authors making use of the equivalent linearization method in which the convergence was completed on the basis of the shear rigidity and the damping constant of the soils as established corresponding to the strains of soils in the boundary area

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around the piles. In connection with the external force, the vibration characteristics of pile foundation were studied for the case wherein the pile head are subjected to the lateral excitation.

ANALYTICAL METHOD

Figure 1 shows the model used for the analyses. The model comprises the boundary area (Zone I) in a range of $R_0 < r < R_I$ within a radius R_I and the external soils (Zone II) lying outside the said Zone I thus forming a pile-boundary soil zone-outer soil ground system model. For the purpose of these analyses, the piles and the soils were assumed as cohesive elastic bodies and they were treated three dimensionally by the use of a cylindrical coordinate diameter system which enabled the thin layer element method to be used. In other words, the pile-boundary soil zone-outer soil ground system was divided into a number (L) of horizontal elements to enable the finite element method and the wave propagation theory to be used for the analyses in the vertical direction and the horizontal direction respectively, while the soils in the boundary soil zone (Zone I) was assumed to behave in a non-linear mode, the piles and the outer soil ground (Zone II) were assumed to behave in a linear mode. The horizontal excitation force (P) acting on the pile head position was taken as an external force. Further, the following assumptions were made in carrying out the analyses herein discussed.

(1) The soils around the piles maintain a circular shape even when they are being vibrated.

(2) The piles are vertical and their deformation is in accordance with Timoshenko's beam theory.

(3) Each thin layer element within the boundary soil zone (Zone I) and the outer soil ground is assumed to consist of three-dimensional visco-elastic medium which is isotropic and homogeneous.
(4) It is assumed that displacement varies in a linear mode along the depth of each thin layer element.

(5) It is assumed that each pile and Zone I are always in close contact and so are Zone I and Zone II and that all displacements and stresses in them are continuous.

Figure 2 shows in the form of a flow chart the analysis techniques used. The non-linearity of the soils is assessed by establishing the shear rigidity and the damping constant for the boundary soil zone (Zone I) in correspondence with the strains caused to the boundary area. The external force (lateral excitation applied to the pile head) is dynamic and the strains vary on a minute-to-minute basis; however, for the purpose of these analyses, the rigidity reduction ratio, etc. are dealt with on an average basis and the non-linearity of soils is taken into consideration by means of iterative computation in the process of equivalent linearization method. The condition of the convergence in the solutions of the equivalent linearization method is taken as 2%.

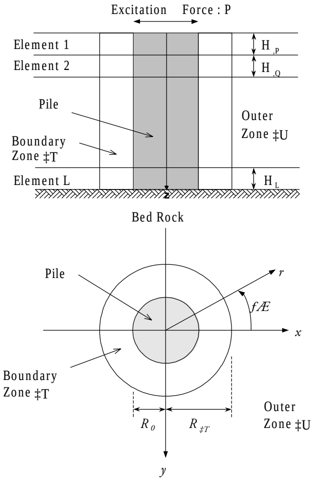


Figure1: Model used for analysis

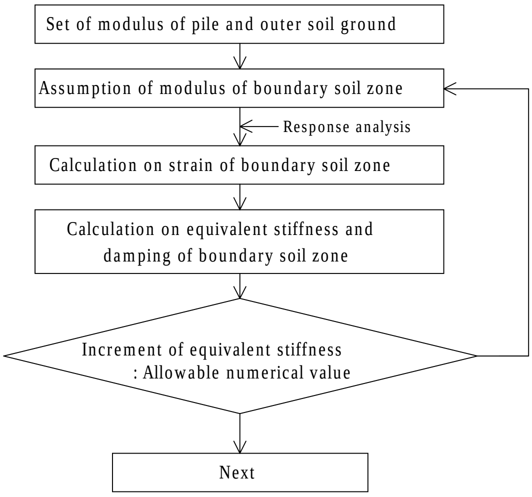


Figure 2: Flowchart of analysis method

EQUATIONS OF MOTION OF PILE-BOUNDARY SOIL ZONE-OUTER SOIL GROUND SYSTEM

For an element of the soil ground divided into a number (L) of layer elements, equations of motion of three-dimensional visco-elastic medium as expressed by a cylindrical coordinate system (r, θ, z) may be written as follows:

$$\begin{aligned}
&(\lambda^* + 2\mu^*)\frac{\partial \Theta}{\partial r} - \frac{\mu^*}{r}\frac{\partial (2\tilde{\omega}_z)}{\partial \theta} + \mu^* \frac{\partial (2\tilde{\omega}_\theta)}{\partial z} = \rho \frac{\partial^2 u_r}{\partial t^2} \\
&(\lambda^* + 2\mu^*)\frac{1}{r}\frac{\partial \Theta}{\partial \theta} - \mu^* \frac{\partial (2\tilde{\omega}_r)}{\partial z} + \mu^* \frac{\partial (2\tilde{\omega}_z)}{\partial r} = \rho \frac{\partial^2 u_\theta}{\partial t^2} \\
&(\lambda^* + 2\mu^*)\frac{\partial \Theta}{\partial z} - \frac{\mu^*}{r}\frac{\partial}{\partial r}(r \cdot 2\tilde{\omega}_z) + \frac{\mu^*}{r}\frac{\partial (2\tilde{\omega}_r)}{\partial \theta} = \rho \frac{\partial^2 u_z}{\partial t^2}
\end{aligned} \tag{1}$$

where, $\lambda^* = \lambda(1 + 2i\xi) \bullet \mu^* = \mu(1 + 2i\xi)$,

$$\begin{aligned}
&\Theta = \frac{1}{r}\frac{\partial}{\partial r}(r u_r) + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z}, \qquad 2\tilde{\omega}_r = \frac{1}{r}\frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{\partial z} \\
&2\tilde{\omega}_\theta = \frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}, \qquad 2\tilde{\omega}_z = \frac{1}{r}\frac{\partial}{\partial r}(r u_\theta) - \frac{1}{r}\frac{\partial u_r}{\partial \theta}
\end{aligned} \tag{2}$$

and λ, μ : Lamé’s constants, ξ : hysteretic damping constant, ρ : soil density, t : time

i : imaginary unit, u_r, u_θ, u_z : displacements in the directions of r , θ and z ,respectively,

As a next step, an equation expressing the pile motions under lateral and rotational excitations will be derived. If Timoshenko’s beam theory (a one-dimensional bending-shear beam theory) is applied to an element of a pile divided into a number (L) of elements, then, equations of motion of a pile may be written as follows:

lateral excitation: $$\frac{\partial}{\partial z}\left\{K_T G A \left(\frac{\partial\, {}_P u_H}{\partial z} + \frac{{}_P u_{RV}}{R_0}\right)\right\} = \rho_P A \frac{\partial^2\, {}_P u_H}{\partial t^2} \tag{3}$$

rotational excitation: $$\frac{\partial}{\partial z}\left(\frac{EI}{R_0}\frac{\partial\, {}_P u_{RV}}{\partial z}\right) - K_T G A \left(\frac{\partial\, {}_P u_H}{\partial z} + \frac{{}_P u_{RV}}{R_0}\right) = \frac{\rho_P I}{R_0}\frac{\partial^2\, {}_P u_{RV}}{\partial t^2} \tag{4}$$

where, ${}_P u_H$: horizontal displacement of an element, ${}_P u_{RV}$: rotational displacement of an element

E : Young’s modulus of piles, G : rigidity modulus of piles

A : cross sectional area of piles, I : secondary moment of inertia of piles

K_T : shape factor of the cross-section of a pile (0.85 if a pile has a circular cross section)

If an equation of motions for a pile-soil system is derived on the basis of the assumptions used in the analyses, such an equation may be written as follows:

$$([F]+[P]\sim\omega^2[M])\{V\}=\{Q\} \tag{5}$$

where, [F] : stiffness matrix of piles {V} : displacement vector of piles

[P] : impedance matrix of soils around piles {Q} : external force vector

[M] : mass matrix of piles ω : circular excitation frequency

INTRODUCTION OF SOIL NON-LINEARITY

In introducing non-linearity of soils, the relationships between shear strain (ƒÁ), shear rigidity (ƒÊ) and damping constant (ƒÌ) of soils are established by the use of the ƒÁ-ƒÊ • ƒÌ curve used in Ohsaki-Hara’s model which is based on the R-O model. Ohsaki-Hara’s model has a large number of soil survey results obtained in Japan reflected on it. The skelton curve, the equivalent shear rigidity and the equivalent damping constant are as expressed in equations (6), (7) and (8), respectivel

$$\text{ƒÁ} = \frac{S_u}{\text{ƒÊ}_0} S\{1+\text{ƒ¿}|S|^{\text{ƒÀ}}\} \qquad (6)$$

$$\frac{\text{ƒÊ}}{\text{ƒÊ}_0} = \frac{1}{1+\text{ƒ¿}|S|^{\text{ƒÀ}}} \qquad (7)$$

$$\text{ƒÌ} = \frac{2}{\text{ƒÎ}} \frac{\text{ƒÀ}}{\text{ƒÀ}+2}\left(1-\frac{\text{ƒÊ}}{\text{ƒÊ}_0}\right) \qquad (8)$$

where, S : standardized shear stress (= $\text{ƒÑ}/S_u$)

S_u : shear strength

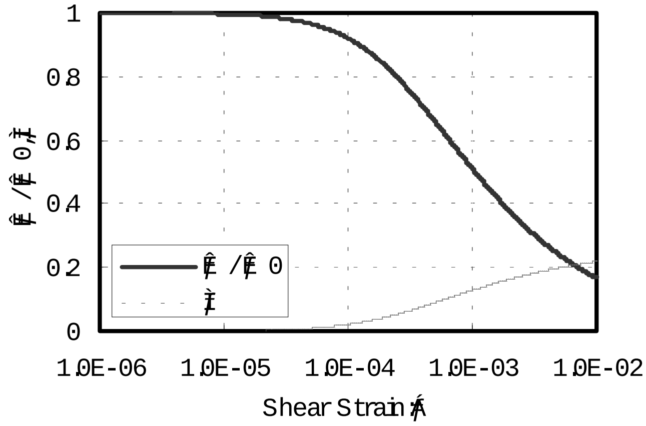


Figure 3: Strain-dependent soil properties for clays

ƒ¿ and ƒÀ are the values determined by a statistical technique on the basis of the soil survey results obtained in Japan. ƒ¿ and ƒÀ are often taken as 5.0 and 1.4, respectively for sand and as 10.0 and 1.6, respectively for clay. Those values are believed to coincide well with the dynamic test results. Figure 3 shows the ƒÁ-ƒÊ • ƒÌ curves for clays.

NUMERICAL ANALYSIS

A pile-soil system model simulating the ground consisting of a 30m thick alluvium clay surface stratum with in-situ concrete piles having a diameter (2 R_0) of 2m and a length of 30m provided in the stratum, underlain by a 30m thick lower stratum, was considered. Further, it was assumed that the said piles were subjected to the lateral excitation force caused by harmonic excitation as applied to the pile head. The Young’s modulus, the rigidity modulus and the density of the piles were taken as 20.6 kN/mm², 8.8 kN/mm² and 2400. kg/m³ , respectively. It was further assumed that each pile was directly underlain by a soil column whose properties are identical with those of the outer soil ground. Table 1 indicates the soil properties varying with depth. The S-wave velocities (Vs) of the lower stratum (pile bearing stratum) was taken as 400 m/s, whereas the two Vs =100m/s and 200 m/s were taken as parameters for the surface stratum. A total of 30 stratum elements, 20 for the surface stratum and 10 for the lower stratum, were considered. The ground damping constants were taken as 2%. Additionally the pile-soil boundary zone (Zone I: $R_0 < r < R_I$, where $R_I =$

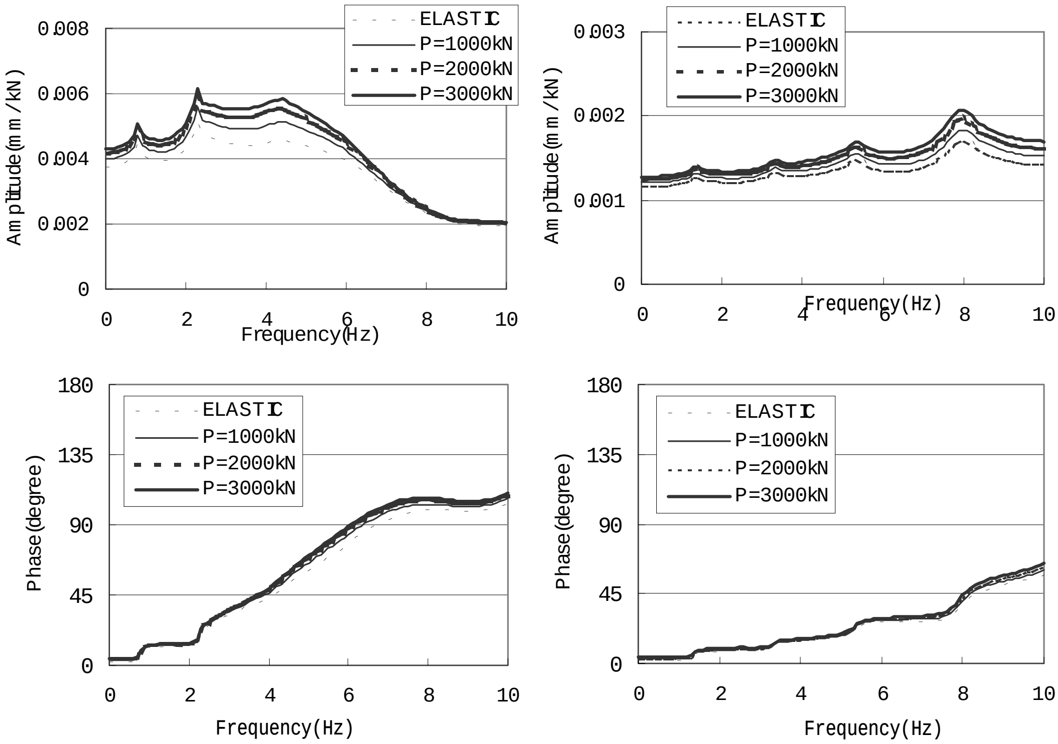
Table1: Soil properties varying with depth

Layer Number	Depth (m)	Medium	S-wave Velocities Vs (m/s)	P-wave Velocities Vp (m/s)	Densities of soils ƒ Ï kg/m³)
1 2 E E E E 20	0 1.5 3.0 E E 28.5 30.0 31.5 33.0 36.0 E E E E E 48.0 51.0 55.5 60.0	Pile	100. or 200.	1500.	1700.
21 22 23 24 E E E 27 28 29 30		Soil Column	400.	1800.	2000.

Table2: Predominant frequencies of the soils

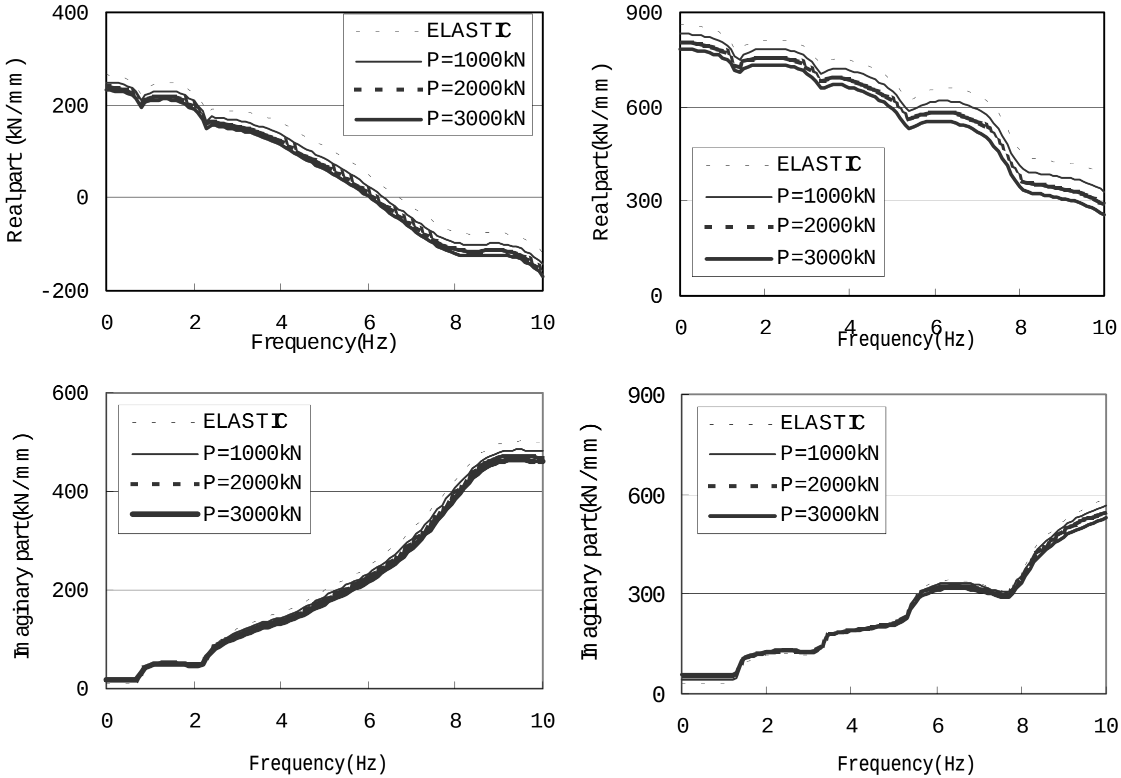
Mode	1st	2nd	3rd	4th
Vs=100m/s	0.90Hz	2.47Hz	3.73Hz	4.44Hz
Vs=200m/s	1.49Hz	3.69Hz	5.53Hz	8.30Hz

1.5 m was assumed) was considered with hysteretic damping due to the non-linearity of soils. As for the lateral excitation force acting on the pile head, three exciting forces, i.e., P = 1,000 KN, P = 2,000 KN and P = 3,000 KN, were taken as parameters. Further, in order to consider the effects of the non-linearity of soils, comparative analyses were made for a case where the soils around the piles (Zone I) were assumed to be linear.



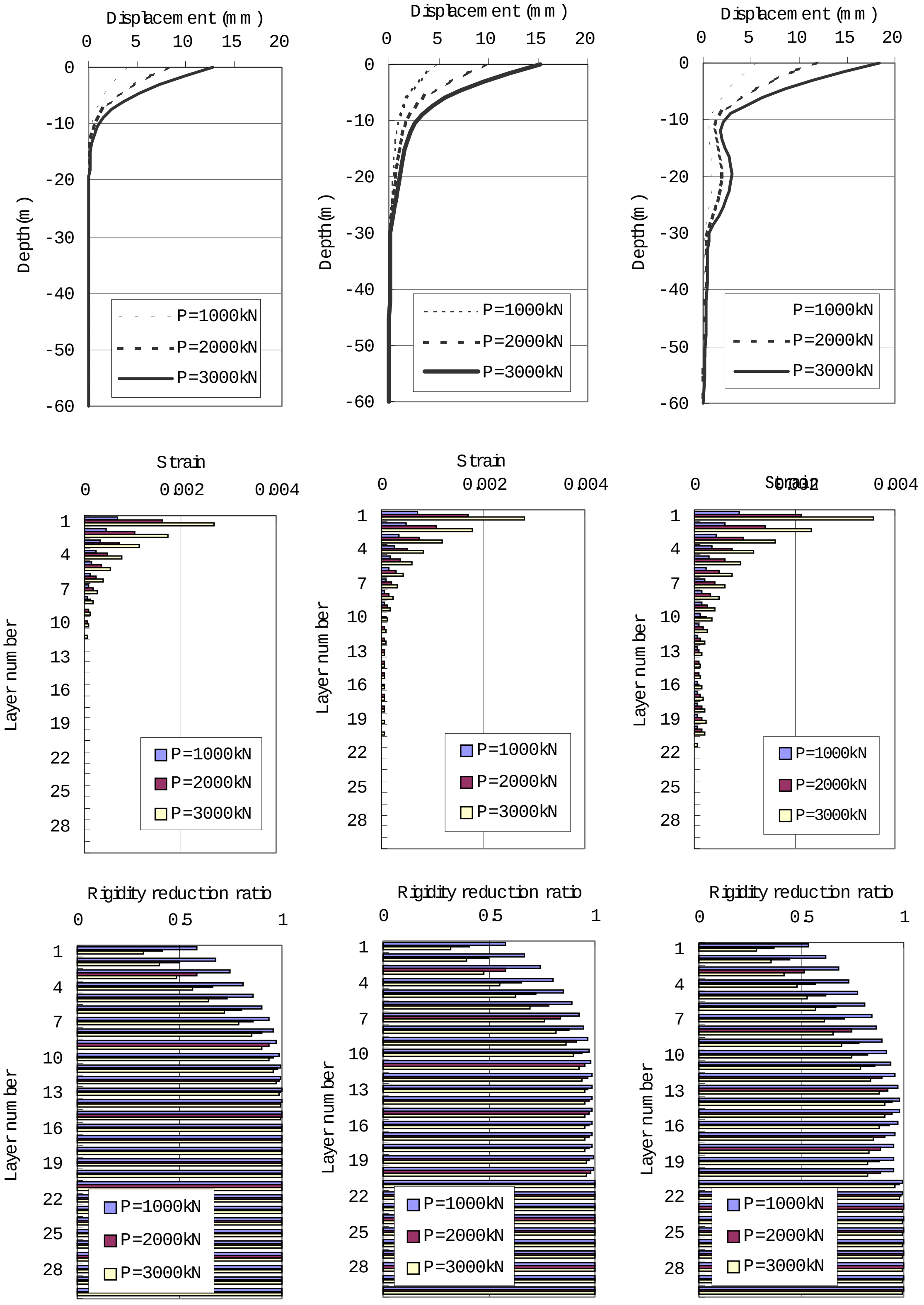
(a)Vs=100m/s

Figure 4: Displacement response characteristics at the pile head of soil-pile system



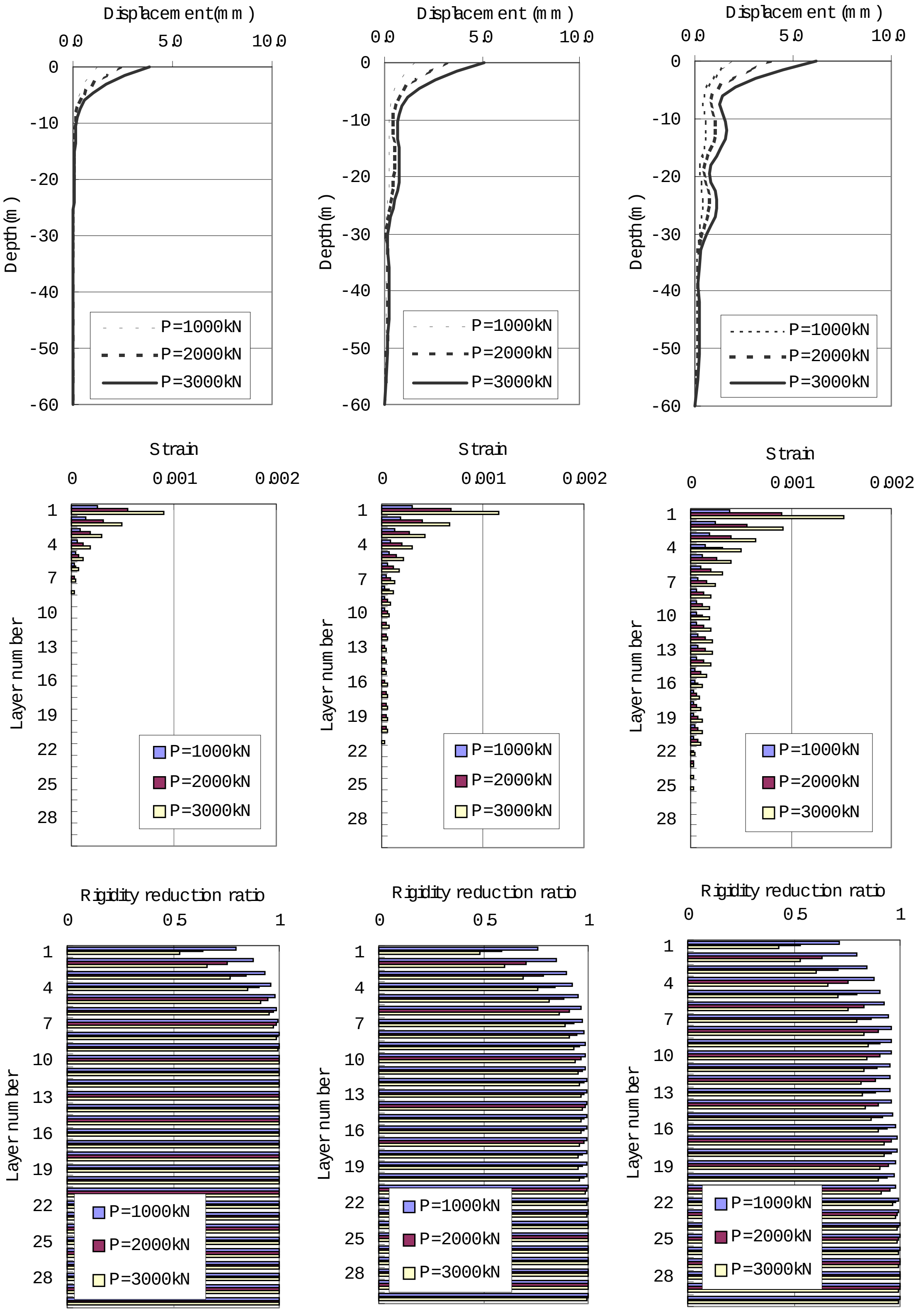
(a)Vs=100m/s

Figure 5: Impedance response characteristics at the pile head of soil-pile system



(a) f=0Hz (static)　　(b) f=0.8Hz　　(c) f=2.3Hz

Figure 6: Pile displacement, strain, and rigidity reduction ratio distributions (Vs=100m/s)



(a) f=0Hz(static)   (b) f=5.3Hz   (c) f=8.0Hz

Figure 7: Pile displacement, strain, and rigidity reduction ratio distributions (Vs=200m/s)

RESULTS OF NUMERICAL ANALYSIS AND CONSIDERATION

Table 2 indicates the predominant frequencies of soils for cases wherein the S-wave velocities (Vs) for the surface stratum are 100 m/s and 200 m/s. Figure 4 shows the displacement response characteristics of the pile head for the cases the Vs values of the surface stratum are 100 m/s and 200 m/s while Figure 5 shows the impedance response characteristics of the pile head. For convenience of comparison, the numerical results obtained for the case where the soils in the boundary soil zone around the piles (Zone I) were linear are also shown in Figure 5. The observation indicated that the lower was the S-wave velocity of the surface stratum and the larger was the external force, the greater the displacements tended to become. It was also observed that compared with the case where the soils in the boundary area around the piles are regarded as being linear, the non-linearity of soils tends to cause greater quantitative influence as the S-wave velocity of the surface stratum becomes lower the external force becomes greater. Further, it should be noted that there is a frequency range in which the displacements increase by 30-40%. However, neither the displacement response characteristics nor the impedance response characteristics were qualitatively affected by the difference in the level of the external force.

Figure 6 shows the pile displacement, the strain and the rigidity reduction ratio distributions in the soils around the piles when the surface stratum is subjected to excitations of 0Hz(static), 0.8 Hz and 2.3 Hz for Vs = 100m/s. The displacements and the strains as well as the rigidity reduction ratio increased as the measuring points approached the ground surface and as the applied external force increased. A certain influence of the secondary degree mode was observed in the displacement and the rigidity reduction ratio distributions at a state of 2.3 Hz.

Figure 7 shows the pile displacement, the strain and the rigidity reduction ratio distributions in the soils around the piles when the surface stratum was under the conditions of 0Hz(static), 5.3Hz and 8.0Hz for Vs = 200 m/s. The displacements, the strains as well as rigidity reduction ratio increased as the measuring points approached the ground surface and as the applied external force was increased. At a condition of 8.0Hz, the effect of the higher degree modes was apparently observed in the displacement and the rigidity reduction ratio distributions.

CONCLUSION

This paper describes the case wherein the vibration characteristics of the pile foundation subjected to lateral excitation force as applied to the pile head were investigated with consideration given to the non-linearity of soils in the boundary area around the piles by the use of the thin layer approach and the equivalent linearization method. The following are the findings obtained within the scope of parameters dealt with in this report.

(1)Quantitatively, the pile displacement increases as the S-wave velocity in the surface stratum decreases and also as the applied external force increases. Qualitatively, however, the displacement response characteristics of piles are not significantly affected by the non-linearity of soils.
(2)When the external forces P are in a range of 2,000~3,000kN which corresponds to them as caused by strong earthquake motions, a certain frequency range is seen wherein the pile displacements amplitude increases by 30~40% over the corresponding displacements seen when the soils are assumed to be linear, and this finding certainly requires reasonable engineering attention.
(3)Both the pile displacement and the rigidity reduction ratio of piles increase as the point of survey approaches the ground surface. This tendency is seen more apparently when the S-wave velocity is lower and the applied external force is greater.

(4)The displacement distribution pattern in direction of the depth of piles is rather insignificantly affected by the non-linearity of soils.

(5)There is a certain vibration frequency range wherein the displacement of piles at their intermediate portions increases due apparently to the effects of higher degree modes.The effects of higher degree modes on the pile-soil system are more remarkably seen as the applied external force increase.

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