ANALYSIS OF FLUID-SATURATED POROUS MEDIA IN TWO DIMENSIONS UNDER EARTHQUAKE LOAD

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SUMMARY

The analysis of dynamic transient phenomena in fluid-saturated porous media is of great interest in geotechnical engineering and engineering seismology. In the present paper, the response problem of fluid-saturated porous media in two dimensions excited by earthquake load is described. The fluid saturated porous media is modeled as a two-phase system consisting of elastic solid and incompressible fluid phase. This approach uses the solid displacement, the fluid displacement and the pore pressure as the field variables. Based on the fluid dynamic and elastic solid equations under the quasi-microcosmic continuum condition, the application of finite element spatial discretization with Galerkin•s method yields a set of uncoupled linear matrix equations. Time integration of the resulting semi-discrete finite element equations is performed by using Wilson-• method. Then, the procedure is applied to a simple two-dimensional problem of non-homogeneous fluid saturated porous medium. Some conclusions are gained from the numerical results. 1) Inherent period is different between the two-phase media and the single- phase media; 2) The damping effect of fluid is significant; 3) The response of solid skeleton is more sensitive than that of pore pressure; 4) The effect of different porosity of the calculated example is significant. The larger the porosity, The smaller the response of the pore pressure and the fluid velocity relative to solid skeleton; 5) The effect of different permeability is smaller than that of the porosity.

INTRODUCTION

Scholars have very early developed the research of the porous liquid-solid two-phase medium behavior and properties. Biot has established the dynamic equation of the saturated porous medium in 1956, and has established on the foundation for the wave dynamic theory in the research heterogeneous porous medium. Owing to the mathematical difficulty that the couple feature of the solid liquid two-phase dynamic equation of saturated soil under the earthquake exciting and the non- homogeneous character of saturated soil, the solution can be gained under only a few special condition. Men Fulu (1981)[2] have given some solution with certain boundary condition. Since what the solid skeleton appears under the earthquake effect strongly nonlinear, and makes this problem become more complicated, many researchers have adopted the numerical method to attempt to solve it. Chiefly go on in accordance with homogeneous saturated soil of two dimensions and the non-homogeneous saturated soil of one dimension in existing research. The finite element method is among them applying more methods. Ghaboussi and Wilson (1972)[1] have firstly established variation formula and leads the finite element equation on the foundation of Biot's dynamic equation, and its variables is the solid and liquid displacement. Neglecting the coupling inertial item in Biot's equation, Zienkiewicz and Simon (1984) etc [7] [4] [5] has established several kinds of finite element equations of different form described by the different variables with Galerkin's method. Their calculating the analysis example is in one dimension. The Sandhu etc (1990)[3] have given out a finite element equation and numerical calculation method described by field variable of solid displacement, liquid relatively displacement of skeleton and porous pressure, but the dynamic analysis of the saturated porous medium is only gone on in one dimension. Yiagos and Prevost (1991)[6] have carried on the earthquake response analysis of long dam which is composed of the horizontal elastic-plastic two-phase soil layer with the finite element method in two dimensions.

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In the present paper, the saturated soil is modeled as a two-phase porous midium system consisting of an elastic skeleton and an incompressible fluid phase. This approach uses the solid displacement, the fluid displacement and the pore pressure as the field variables. based on the two-phase coupled differential equations, the application of Galerkin's principle and Wilson - • method yields a three-field finite element integration procedure. The procedure can not only calculate the response of solid displacement and fluid displacement but also give out the response of pore pressure. Then, the procedure is applied to a simple two-dimensional example of non-homogeneous fluid-saturated porous medium.

BASIC EQUATION AND IT DISCRETIZATION

2.1 Differential equation of saturated medium in two dimensions

If disregards gravity, the differential equation of saturated aperture medium is.
As for the solid phase

$$\begin{cases} \frac{\partial \sigma_{xx}}{\partial x}+\frac{\partial \tau_{xy}}{\partial y}-b\frac{\partial}{\partial t}(u_x-U_x)-\rho_1\frac{\partial^2}{\partial t^2}u_x=0 \\ \frac{\partial \tau_{xy}}{\partial y}+\frac{\partial \sigma_{yy}}{\partial y}-b\frac{\partial}{\partial t}(u_y-U_y)-\rho_1\frac{\partial^2}{\partial t^2}u_y=0 \end{cases} \tag{1}$$

As for the liquid phase

$$\begin{cases} \frac{\partial \sigma}{\partial x}+b\frac{\partial}{\partial t}(u_x-U_x)-\rho_2\frac{\partial^2}{\partial t^2}U_x=0 \\ \frac{\partial \tau}{\partial y}+b\frac{\partial}{\partial t}(u_y-U_y)-\rho_2\frac{\partial^2}{\partial t^2}U_y=0 \end{cases} \tag{2}$$

The continuity equation is

$$S\frac{\partial}{\partial t}(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y})+\frac{\partial}{\partial t}(\frac{\partial U_x}{\partial x}+\frac{\partial U_y}{\partial y})-\frac{1}{fE_w}\frac{\partial \sigma}{\partial t}=0 \tag{3}$$

When liquid is incompressible, the equation (3) becomes

$$S\frac{\partial}{\partial t}(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y})+\frac{\partial}{\partial t}(\frac{\partial U_x}{\partial x}+\frac{\partial U_y}{\partial y})=0 \tag{4}$$

Where, $\sigma_{xx},\sigma_{yy},\tau_{xy}$ is solid stress tensor, σ is average active water pressure, u_x,u_y is solid displacement tensor, U_x,U_y is liquid displacement tensor, $\rho_1=\gamma_1(1-f)$ is mass of bulk of solid, γ_1 is mass density of bulk of solid, f is the porosity, $\rho_2=\gamma_2 f$ is liquid mass of bulk solid medium, γ_2 is mass density of bulk of liquid, $S=\frac{1-f}{f}$ $b=\frac{f^2\rho_2 g}{k_\Phi}$ k_Φ is the coefficient of permeability.

When the liquid is incompressible, the constitutive equation is

$$\sigma_{xx}-S\sigma=\lambda e+2Ge_{xx}\ \ \sigma_{yy}-S\sigma=\lambda e+2Ge_{yy}\ \ \psi_{xy}=Ge_{xy} \tag{5}$$

Where

$$e_{xx}=\frac{\partial u_x}{\partial x},\ e_{yy}=\frac{\partial u_y}{\partial y},\ e_{xy}=(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}),\ e=e_{xx}+e_{yy} \tag{6}$$

Here, λ is Lame's constant, G is shear modulus.

2.2 The application of Galerkin's principle

Insert (5) and (6) into (1) it is gotten the solid dynamic equation expressed by displacement.

$$\begin{cases}(\lambda+G)\frac{\partial}{\partial x}(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y})+G\nabla^2 u_x-b\frac{\partial}{\partial t}(u_x-U_x)+\frac{\partial(S\sigma)}{\partial x}-\rho_1\frac{\partial^2}{\partial t^2}u_x=0\\(\lambda+G)\frac{\partial}{\partial y}(\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y})+G\nabla^2 u_y-b\frac{\partial}{\partial t}(u_y-U_y)+\frac{\partial(S\sigma)}{\partial y}-\rho_1\frac{\partial^2}{\partial t^2}u_y=0\end{cases} \quad (7)$$

Where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$

The boundary condition of displacement is $u_x=\overline{u}_x \bullet u_y=\overline{u}_y\ U_x=\overline{U}_x\ U_y=\overline{U}_y$ (8a)

The boundary condition of stress is

$$\sigma_{xx}n_x+\tau_{xy}n_y=\overline{X},\ \tau_{xy}n_x+\sigma_{yy}n_y=\overline{Y},\ \sigma n_x=p_x,\ \sigma n_y=p_y \quad (8b)$$

The Approximate solution is chose as

$$u_x^n=\sum_{i=1}^{n}a_iF_i(x,y,t)\bullet u_y^n=\sum_{i=1}^{n}b_iF_i(x,y,t)\bullet U_x^n=\sum_{i=1}^{n}a_i^{'}F_i(x,y,t)$$

$$U_y^n=\sum_{i=1}^{n}b_i^{'}F_i(x,y,t)\bullet\sigma^n=\sum_{i=1}^{n}c_iF_i(x,y,t) \quad (9)$$

Where a_i b_i $a_i^{'}$ $b_i^{'}$ c_i is uncertain function or the parameter, F_i is belong to a set of complete function and it is chosen must let u_x^n u_y^n U_x^n U_y^n satisfy boundary condition (8a). Insert them into (7), (2) and (4), it is gain:

$$\begin{aligned}&(\lambda+G)\frac{\partial}{\partial x}(\frac{\partial u_x^n}{\partial x}+\frac{\partial u_y^n}{\partial y})+G\nabla^2u_x^n-b\frac{\partial}{\partial t}(u_x^n-U_x^n)+S\frac{\partial\sigma^n}{\partial x}-\rho_1\frac{\partial^2}{\partial t^2}u_x^n\\&=R_1(x,y,t,a_i,b_i,a_i^{'},b_i^{'},c_i,(i=1,\cdots,n))\end{aligned} \quad (10a)$$

$$\begin{aligned}&(\lambda+G)\frac{\partial}{\partial y}(\frac{\partial u_x^n}{\partial x}+\frac{\partial u_y^n}{\partial y})+G\nabla^2u_y^n-b\frac{\partial}{\partial t}(u_y^n-U_y^n)+S\frac{\partial\sigma^n}{\partial y}-\rho_1\frac{\partial^2}{\partial t^2}u_y^n\\&=R_2(x,y,t,a_i,b_i,a_i^{'},b_i^{'},c_i,(i=1,\cdots,n))\end{aligned} \quad (10b)$$

$$\frac{\partial\sigma^n}{\partial x}+b\frac{\partial}{\partial t}(u_x^n-U_x^n)-\rho_2\frac{\partial^2}{\partial t^2}U_x^n=R_3(x,y,t,a_i,b_i,a_i^{'},b_i^{'},c_i,(i=1,\cdots,n)) \quad (11a)$$

$$\frac{\partial\sigma^n}{\partial y}+b\frac{\partial}{\partial t}(u_y^n-U_y^n)-\rho_2\frac{\partial^2}{\partial t^2}U_y^n=R_4(x,y,t,a_i,b_i,a_i^{'},b_i^{'},c_i,(i=1,\cdots,n)) \quad (11b)$$

$$S\frac{\partial}{\partial t}(\frac{\partial u_x^n}{\partial x}+\frac{\partial u_y^n}{\partial y})+\frac{\partial}{\partial t}(\frac{\partial U_x^n}{\partial x}+\frac{\partial U_y^n}{\partial y})=R_5(x,y,t,a_i,b_i,a_i^{'},b_i^{'},c_i,(i=1,\cdots,n)) \quad (12)$$

Where, $R_1, R_2, R_3, R_3, R_4, R_5$ is the error item, if $u_x^n, u_y^n, U_x^n, U_y^n, \sigma^n$ is truly solution, these R_i is consistently equal to zero in the close region $(D+\Gamma)\bullet D$ is the whole area, Γ is its boundary), Generally, R_i is not equal to zero.

In order to determinate $a_i \bullet b_i \bullet a_i^{'} \bullet b_i^{'} \bullet c_i$ •carries on the righting integration on (10) to (12) and goes on the integration of parts, then it gain 5n united equations of $a_i \bullet b_i \bullet a_i^{'} \bullet b_i^{'} \bullet c_i$.

$$\iint_D [\lambda \frac{\partial F_i}{\partial x}(\frac{\partial u_x^n}{\partial x}+\frac{\partial u_y^n}{\partial y})+2G\frac{\partial F_i}{\partial x}\frac{\partial u_x^n}{\partial x}+G\frac{\partial F_i}{\partial y}(\frac{\partial u_x^n}{\partial y}+\frac{\partial u_y^n}{\partial x}) + F_i b\frac{\partial}{\partial t}(u_x^n-U_x^n)+S\frac{\partial F_i}{\partial x}\sigma^n+F_i\rho_1\frac{\partial^2}{\partial t^2}u_x^n]dD=\int_\Gamma (X+SP_x)F_i d\Gamma \tag{13a}$$

$$\iint_D [\lambda \frac{\partial F_i}{\partial y}(\frac{\partial u_x^n}{\partial x}+\frac{\partial u_y^n}{\partial y})+2G\frac{\partial F_i}{\partial y}\frac{\partial u_x^n}{\partial y}+G\frac{\partial F_i}{\partial x}(\frac{\partial u_x^n}{\partial y}+\frac{\partial u_y^n}{\partial x}) + F_i b\frac{\partial}{\partial t}(u_y^n-U_y^n)+S\frac{\partial F_i}{\partial y}\sigma^n+F_i\rho_1\frac{\partial^2}{\partial t^2}u_y^n]dD=\int_\Gamma (Y+SP_y)F_i d\Gamma \tag{13b}$$

$$\iint_D [\frac{\partial F_i}{\partial x}\sigma^n - F_i b\frac{\partial}{\partial t}(u_x^n-U_x^n)+F_i\rho_2\frac{\partial^2}{\partial t^2}U_x^n]dD=\int_\Gamma F_i P_x d\Gamma \tag{14a}$$

$$\iint_D [\frac{\partial F_i}{\partial y}\sigma^n - F_i b\frac{\partial}{\partial t}(u_y^n-U_y^n)+F_i\rho_2\frac{\partial^2}{\partial t^2}U_y^n]dD=\int_\Gamma F_i P_y d\Gamma \tag{14b}$$

$$\iint_D [S\frac{\partial}{\partial t}(\frac{\partial u_x^n}{\partial x}+\frac{\partial u_y^n}{\partial y})+\frac{\partial}{\partial t}(\frac{\partial U_x^n}{\partial x}+\frac{\partial U_y^n}{\partial y})]F_i dD=0 \tag{15}$$

Here following boundary conditions is used

$$\lambda(\frac{\partial u_x^n}{\partial x}+\frac{\partial u_y^n}{\partial y})n_x+G(\frac{\partial u_x^n}{\partial x}n_x+\frac{\partial u_x^n}{\partial y}n_y)+G(\frac{\partial u_x^n}{\partial x}n_x+\frac{\partial u_y^n}{\partial x}n_y)=\sigma_{xx}n_x+\tau_{xy}n_y=\overline{X} \tag{16a}$$

$$\lambda(\frac{\partial u_x^n}{\partial x}+\frac{\partial u_y^n}{\partial y})n_y+G(\frac{\partial u_x^n}{\partial y}n_x+\frac{\partial u_y^n}{\partial y}n_y)+G(\frac{\partial u_y^n}{\partial x}n_x+\frac{\partial u^n}{\partial y}n_y)=\sigma_{xx}n_y+\tau_{xy}n_x=\overline{Y} \tag{16b}$$

$$\sigma n_x = P_x \quad \sigma n_y = P_y \tag{17}$$

Where; n_x n_y is respectively the direction cosine about X and Y axle of outer normal of boundary. Because F_i is a complete function set, when $n \to \infty$; the approximate solution will tend towards to accurate solution.

2.3 Finite element discretization of three field

Dividing the space into finite (for example M) sub-area or element D_m, the shape of these sub-areas is determined by a set of nodes location completely. In each element, the solid displacement field and liquid displacement field and the average pore pressure field is determined by the value of node

$$u_x^m(x,y,t) = \{\Phi^m\}^T \{u_x(t)\}, \quad u_y^m(x,y,t) = \{\Phi^m\}^T \{u_y(t)\}$$

$$U_x^m(x,y,t) = \{\Phi^m\}^T \{U_x(t)\}, \quad U_y^m(x,y,t) = \{\Phi^m\}^T \{U_y(t)\}$$

$$\sigma^m(x,y,t) = \{\Phi^m\}^T \{\sigma(t)\} \tag{18}$$

where, $u_x^m, u_y^m, U_x^m, U_x^m$ is respectively the solid and liquid displacement of element m which locates at point (x , y), σ^m is average water pore pressure, $\{\Phi^m\}$ is shape function set, $\{u_x(t)\}, \{u_y(t)\}, \{U_x(t)\}, \{U_y(t)\}, \{\sigma(t)\}$ is respectively the displacement vector of solid and liquid and the average pore pressure vector at the node.

Relation (6) of strain - stress can be written as

$$[e_{xx}, e_{yy}, e_{xy}]^T = [\frac{\partial u_x}{\partial x}, \frac{\partial u_y}{\partial y}, (\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x})]^T = L\{\Phi^m\}\{u(t)\} = B^m\{u(t)\} \tag{19}$$

Where, L is the differentiation operator matrix of plane problem, and B is train matrix.

$$L = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial y} \\ 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix}^T$$

$$B^m = L\{\Phi^m\}$$

The solid stress can be gotten by the equation (5)

$$[\sigma_{xx}^m, \sigma_{yy}^m, \sigma_{xy}^m]^T = D^m B^m\{u(t)\} - [S,S,0][\Phi^m]\{\sigma(t)\} = S^m\{u(t)\} - E^m\{\sigma(t)\} \qquad \bullet 20 \bullet$$

where; D^m is the elasticity matrix and S^m is called the stress matrix.

$$S^m = D^m B^m$$

For any node j of element m , takes the broad sense displacement form for

$$\{\Psi_j(t)\} = [u_x^j(t), u_y^j(t), U_x^j(t), U_y^j(t), \sigma^j(t)]^T \tag{21}$$

Put (18) into (13) , (14) and (15) and go on the integration of parts, it can gain the equation as follow

$$[M]\{\ddot{\Psi}(t)\} + [C]\{\dot{\Psi}(t)\} + [K]\{\Psi(t)\} = \{F(t)\} \tag{22}$$

Where, $[K]$ is stiffness matrix , $[C]$ is damping matrix , $[M]$ is mass matrix , $[F]$ is load matrix.

EXAMPLE OF NON_HOMOGENEOUS SATURATED SOIL IN TWO DIMENSIONS

According to the formula mentioned above, the computer program has been made with FORTRAN language. Choose the calculation model shown in Fig. 1, it is composed of 48 elements, 36 nodes. All parameters of soil are chosen as follows

Part of soil: Lame's constant $\lambda_I : 2.0 \times 10^8 N \cdot m^{-2}$, Shear modulus

$G_I : 0.8\times10^8 N\cdot m^{-2}$

Solid density $\rho_{sI} : 2.0\times10^3 g\cdot m^{-3}$ Liquid density $\rho_{fI} : 1.0\times10^3 g\cdot m^{-3}$

Part of soil: Lame's constant $\lambda_{\mathrm{II}} : 3.0\times10^8 N\cdot m^{-2}$ Shear modulus $G_{II} : 1.2\times10^8 N\cdot m^{-2}$

Solid density $\rho_{sII} : 2.5\times10^3 g\cdot m^{-3}$ Liquid density $\rho_{fII} : 1.0\times10^3 g\cdot m^{-3}$

The input wave is shown in Fig. 2. The period of input wave is chose to be 0.2,0.4 and 0.8 second. The dynamic response analysis is carried out respectively for saturated two-phase medium and single-phase medium. Part of calculated results are shown in Fig. 3 to Fig. 6.

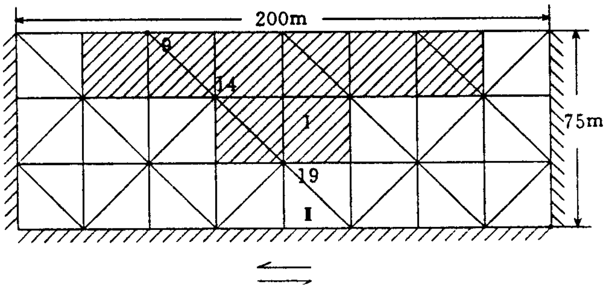

Fig. 1 Calculated model of non-homogeneous saturated soil

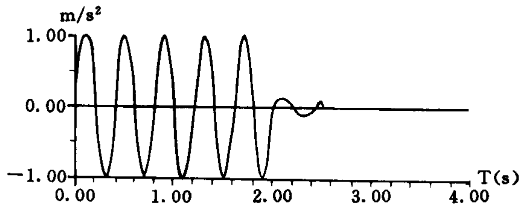

Fig. 2 Inputed load

By the figures it can see that.

1. The Fig.3 is the horizontal acceleration response curve of skeleton when porosity is $f_I = 0.2, f_{II} = 0.4$ and permeability is $K_I = 0.5\times10^{-3} m/s$ U $_x^m$ . .From the figure, it shows that:1). The result shows the basic features of forced vibration of model; 2). The reaction quickly attenuates while the input load become smaller; 3). The magnifying multiple of reaction is different according to the different period of input wave. 4). The difference of the response phase feature of saturated soil is quite significant when the input load's period T is 0.4s or 0.2s but 0.8s. By comparing, that difference of the single-phase soil is smaller.5) After unloading, the attenuation of saturated soil's response is slower than that of the single-phase soil. This indicates that the influence of the interaction damp of two-phase medium.
2. Fig. 4 is the response curve of fluid velocity related to skeleton and Fig.5 is the response curve of pore pressure (permeability $K_I = 1.0\times10^{-3} m/s$, $K_{II} = 0.5\times10^{-3} m/s$ and load's period T = 0.8s). As for the single-phase soil, the porosity enlarges, the response of relative velocity and pore pressure reduce, otherwise are enlarged. As for the saturated soil, the reaction has the difference along with the different porosity and the difference position. The variation character of node 14 and node 19 is different(show in Fig. 4)
3. Fig.6 is the response curve of pore pressure (porosity $f_I = 0.4, f_{II} = 0.2$ and load's period T = 0.8s). Being shown such as the figure, the influence of different permeability is not so clear to the reaction of pore pressure.

4.

CONCLUSIONS

1. Inherent period is different between the two-phase media and the single- phase media.
2. The damping effect of fluid is significant.
3. The response of solid skeleton is more sensitive than that of pore pressure.
4. The effect of different porosity of the calculated example is significant. The larger the porosity, The smaller the response of the pore pressure and the fluid velocity relative to solid skeleton.

The effect of different permeability is smaller than that of the porosity.

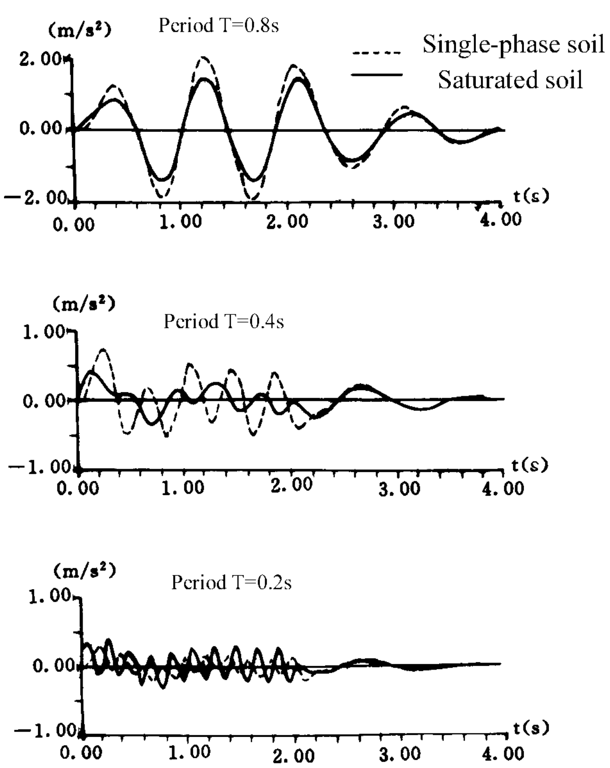

Fig.3 horizontal acceralation response curve of solid at node 19

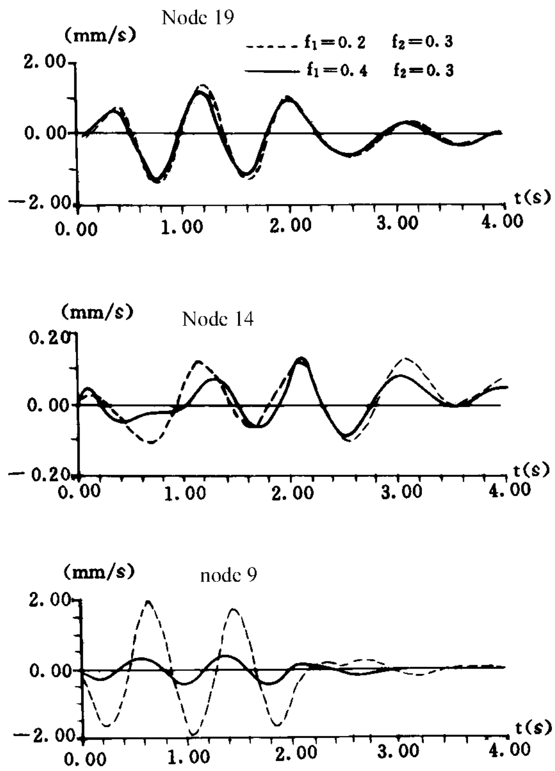

Fig.4 Relative velocity of liquid to solid (T=0.8s)

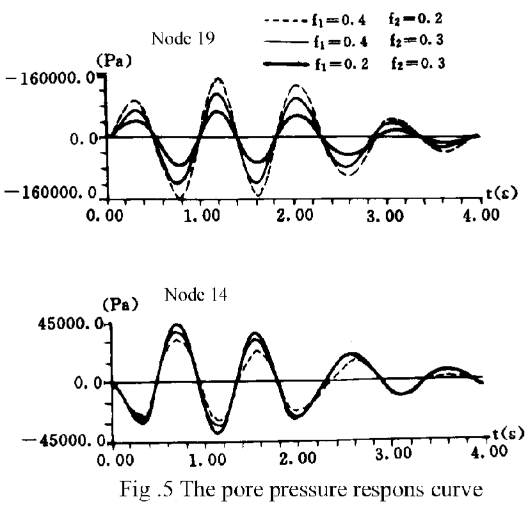

Fig .5 The pore pressure respons curve (T=0.8s)

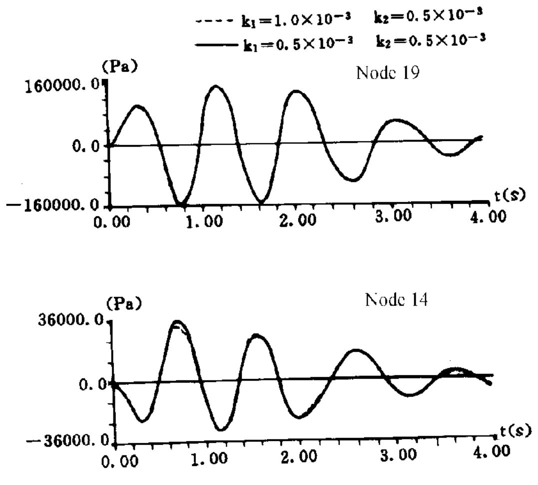

Fig.6 The pore pressure response curve (T=0.8s)

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