SEISMIC ANALYSIS AND DESIGN OF BUILDING STRUCTURES WITH SUPPLEMENTAL LEAD DAMPERS

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SUMMARY

The response behaviour of building structures incorporating a new type of damping device (lead shear damper developed by Penguin Engineering Ltd) was under investigation. For regular and symmetrical frame structures, a satisfactory distribution of these supplemental dampers in the stories has been determined. For such a distribution of the dampers, the structure with supplemental dampers will behave predominantly in its first mode. This leads to a simplified method using an equivalent SDOF system that is able to predict the response of the MDOF structure for preliminary design. Optimal damping levels due to supplemental dampers have been found. A displacement-based method to determine the strength levels of the dampers in the storeys suitable for preliminary design is outlined.

INTRODUCTION

A basic principle in structural design when seeking to minimise the effects of severe earthquake excitations is to allow the structure to absorb and dissipate energy through structural ductility. However, ductile structures may undergo very large inelastic deformation so that they may be severely damaged after strong earthquake excitations. Recently, more emphasis has been given to the development of cost-effective devices for dissipating seismically induced energy in the structure while keeping the structure's response as much as possible in the elastic range. These energy-dissipating devices provide large supplemental damping to the structure and significantly reduce the seismic demand of the structure.

The lead damper, Penguin Vibration Damper (PVD), developed by Penguin Engineering, is a compact damping device. The damping of this device is achieved through deformation of a lead core [Monti et al, 1996].

ANALYTICAL MODEL OF THE DAMPER AND THE STRUCTURE

All the results obtained through the testing programme of this device have shown it to behave as an almost perfectly plastic device [Monti et al, 1996]. A bi-linear model has been used to represent the force-deformation relationship of the dampers [Lin, 1999].

A 12-storey 3-bay reinforced concrete frame structure was used for this study. The supplemental dampers were connected to the structure by means of diagonal braces (Figure 1).

DISTRIBUTION OF THE YIELD STRENGTHS OF THE DAMPERS IN THE STRUCTURE

The purpose of the research outlined here was to find out a satisfactory distribution of the dampers rather than the optimal distribution. Two parameters were used to measure the structural demand and response. They are peak interstorey drift and peak base shear. A code compatible earthquake El Centro 1940 NZS4203 was adopted here for time history analyses [Lin, 1999].

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For a given common yield strength of the damper in the first storey of the structure, four types of distributions of the damper yield strengths in the structure were compared. The yield strength distributions of the dampers in the structure for these cases are chosen to be proportional to the storey shear due to four types of lateral load distribution (while the damper yield strengths at the 1st storey are the same for these cases). These four types of lateral load distribution are: only one lateral force acting on the top level (case-I), parabolic load distribution (case-II), inverted-triangular load distribution (case-III), uniform load distribution (case-IV). These are shown in Figure 2.

The comparison of these four cases is shown in Table 1 and the values in brackets show the differences of the parameters of these cases compared to case-III. The peak interstorey drift is a minimum for case-III and its peak base shear is also close to the minimum, hence case III is close to the optimal case. Similar results can also be found with other earthquake excitations [Lin, 1999; Lin et al, 1998a]. Thus a satisfactory distribution of the yield strengths of the dampers in the structure can be taken to be proportional to the storey shear force due to an inverted-triangular lateral load pattern. There is no need to have the same yield strength for all dampers in the structure. It has also been found that for this type of distribution of the strength levels of the dampers, all devices can reach their inelastic (yielding) stage simultaneously. This characteristic will lead to maximum energy dissipation and cause the structure to behave mainly in its first mode [Lin, 1999].

PUSHOVER ANALYSES OF THE STRUCTURE WITH DAMPERS

Pushover analyses have been performed and compared for the structure with and without dampers [Lin, 1999]. For the structure with dampers, two types of damper yield strength distribution were considered: case I (the same yield strength for all dampers) and case III (satisfactory distribution). For carrying out the simplified nonlinear static analysis and displacement-based design of the structure with dampers, the deflected shapes of the structure with dampers are necessary. These deflected shapes were obtained from pushover analyses. The normalised deflection shape vectors of the structure with the supplemental dampers (both case-I and III) and the original structure without dampers are shown in Figure 3. It can be seen that for the satisfactory distribution of damper yield strengths, the deflected shape of the structure with the dampers is very close to that of the original structure without dampers, hence the deflected shape can be taken as that of the original structure without dampers. This makes analysis and design much easier.

It can also be seen that for the distribution of the same yield strength for all the dampers in the structure, the difference in the deflected shape of the structure with dampers and the non-damped structure becomes much larger. This shows another advantage for the satisfactory distribution.

It has also been found that for a satisfactory distribution of dampers in the structure, the dynamic peak response of the MDOF structure with dampers can be predicted effectively by its equivalent SDOF system [Lin, 1999; Lin et al, 1998a]. The relationship between the characteristics of a MDOF structure and its equivalent SDOF system is shown in detail in the references [Fajfar et al, 1987; Lin, 1999 and Qi et al, 1991].

THE EQUIVALENT VISCOUS DAMPING RATIO AND THE EFFECTIVE PERIOD OF THE STRUCTURE WITH DAMPERS

Sdof System

The SDOF system includes both the original structural frame system and the supplemental damping system. The two systems act in parallel and can be described as a dual system (see Figure 4).

The relationship between energy dissipation per cycle E_d, equivalent viscous damping ξ and maximum elastic strain energy E_s has been proposed by Clough [1993] as:

$$\xi = \frac{E_d}{4\pi E_S}$$

The initial elastic stiffness of the original structure is K_s. The yielding force of the original structure is P_y. The yielding displacement of the original structure is Δ_{y0}. P is the elastic force in the original structure at a given response displacement Δ if the structure remains elastic. rK_s is the post yielding stiffness of the original structure. An elastic-perfectly-plastic hysteresis model is adopted to represent the behaviour of supplemental damping system. The initial elastic stiffness of the supplemental damping system is designated as S_RK_s, and the yielding force of the supplemental damping system is designated as $F_R P$. (Figure4). F_R and S_R are the force and stiffness factors respectively. The equivalent viscous damping ratio and the effective period of the dual system can be expressed as [Lin, 1999 and Lin et al, 1998a,b]:

$$\xi = \frac{E_d}{4\pi E_s} = \frac{2}{\pi} \frac{F_R\left(1 - \frac{F_R}{S_R}\right)}{F_R + \frac{1}{\mu} + r\left(1 - \frac{1}{\mu}\right)} \tag{1}$$

$$T_{eff} = 2\pi\sqrt{\frac{M}{K_{eff}}} = T_0\sqrt{\frac{\mu}{\mu F_R + 1 + r(\mu - 1)}} \tag{2}$$

The term $\left[1 - \frac{F_R}{S_R}\frac{1}{\mu}\right]$ in Equation (1) is very close to 1 for lead dampers.

The original viscous damping ξ_0 and the equivalent viscous damping due to the inelastic deformation of the structure $\Delta\xi_0$ can be estimated by [Shibata et al, 1975]:

$$\xi_s = \xi_0 + \Delta\xi_0 = 0.05 + \frac{1 - 1/\sqrt{\mu}}{5} \tag{3}$$

Mdof Structure

The modal strain energy method has been adopted to estimate the amount of equivalent structural damping provided by the supplemental dampers [Zhang et al, 1989]. The equivalent viscous damping can be estimated according to this formula (only the fundamental mode is of interest):

$$\xi_i = \frac{E_d{}^i}{4\pi E_s{}^i} \tag{4}$$

where E_d^i is the energy dissipated by the supplemental dampers per cycle for the *ith* vibration mode, E_s^i is the strain energy of the structure with the supplemental dampers for the *ith* vibration mode, ξ_i is the equivalent viscous damping ratio for the *ith* vibration mode.

The effective period (first mode) of the MDOF structure with the supplemental dampers at the target displacement x_t can be calculated from the Rayleigh method:

$$T_{eff} = 2\sqrt{\frac{\Sigma\left(W_i u_i{}^2\right)}{g\Sigma\left(F_i u_i\right)}} = 2\pi\sqrt{\frac{\sum_{i=1}^{N} m_i \phi_i{}^2}{\sum_{i=1}^{N} F_i \phi_i} x_t} \qquad \left(where\ u_i = \phi_i x_t\right) \tag{5}$$

For regular frame structure with a satisfactory distribution of dampers (the damper yield strengths in the structure are proportional to the shear forces due to the inverted-triangular lateral load pattern), the equivalent viscous damping ratio and the effective period of the structure with dampers can be expressed as [Lin, 1999]:

$$\xi = \frac{2}{\pi}\frac{F_R}{\left[F_R + \frac{1}{\mu} + r\left(1 - \frac{1}{\mu}\right)\right]} \tag{6}$$

$$T_{eff} = T_0 \sqrt{\frac{\mu}{\mu F_R + 1 + r(\mu - 1)}}$$
(7)

where F_R = the force factor for the MDOF structure = $F_{yd1} \cos\theta / F_0$
(8)

F_{yd1} = the damper yield strength at the 1st floor, and

F_0 = elastic base shear of the MDOF undamped structure if the structure remained elastic at the target displacement.

$$= \left(P \sum_{i=1}^{N} \psi_i \right) / \left(\sum_{i=1}^{N} \phi_i \psi_i \right)$$
(9)

F_R in Equations (6) and (7) for a MDOF structure is equivalent to F_R in Equations (1) and (2) for the SDOF system. The only difference between Equations (6) and (1) is the term $\left[1 - \frac{F_R}{S_R} \frac{1}{\mu}\right]$. This difference is very small for lead dampers, hence it can be concluded that the equivalent viscous damping ratio and the effective period of the structure with dampers can be easily calculated by its equivalent SDOF system through Equations (6) and (7) [Lin, 1999 and Lin et al, 1998b].

SIMPLIFIED STATIC METHOD OF ANALYSIS OF STRUCTURES WITH SUPPLEMENTAL DAMPERS

From above it can be seen that the peak dynamic response of the MDOF structure with the supplemental dampers can be predicted effectively by its equivalent SDOF system. And the equivalent viscous damping ratio and the effective period of the structure with dampers can be easily calculated by its equivalent SDOF system. Hence, SDOF system is a good tool for simplified analysis and design purposes. The SDOF method makes it possible to adopt spectral analysis for design.

Based on these results, a simplified static analytical method can be adopted to predict the dynamic response of the MDOF structure with the supplemental dampers.

Due to the fact that the equivalent viscous damping of supplemental dampers and the effective period of structures with supplemental dampers vary with respect to the displacement response of structures, some iterations might be needed to obtain the response of structures since the displacement response is not known prior to analysis. A pushover analysis is needed to obtain the base shear-roof displacement relationship during structural inelastic deformation.
The analysis procedure can also be obtained as follows:

Step 1. Conduct a pushover analysis of the undamped structure. The base shear and the roof displacement of the original structure at yield can be obtained. The ratio of the post-yield stiffness to the initial stiffness can be obtained as well.

Step 2. Make an initial estimate for the roof displacement (x_{t0}) of the structure with the supplemental dampers. The deflected shape $\{\phi\}$ can be obtained by the displacement profile corresponding to the estimated displacement from the result of step 1. The initial assumed target ductility μ is also obtained.

Step 3. The force factor F_R of the structure with the supplemental dampers can be calculated. From Equation (6) the equivalent viscous damping ξ_d due to the supplemental dampers can be calculated. The equivalent viscous damping ratio (ξ_0+$\Delta\xi_0$) due to inelastic deformation and original damping can also be estimated by Equation (3) for the guessed displacement. The total equivalent viscous damping ratio ξ_t (=ξ_d +ξ_0+$\Delta\xi_0$) is known.

Step 4. The effective period T_{eff} of the structure with the supplemental dampers can be obtained from Equation (7).

Step 5. The displacement for the equivalent SDOF system (or spectral displacement) x^* for the effective period T_{eff} and equivalent viscous damping ratio of ξ_t can be obtained directly from the displacement spectra.

Step 6. The target roof displacement x_{t1} can be obtained [Lin, 1999 and Qi et al, 1991] by:

$x_{t1} = \frac{L^*}{M^*} x^*$ where $M^* = \sum_{i=1}^{N} m_i \phi_i^2$, $L^* = \sum_{i=1}^{N} m_i \phi_i$, and x^* is the spectral displacement.

Let the new target roof displacement x_t be: $x_t = \frac{x_{t1} + x_{t0}}{2}$

Step 7. Compare x_t with x_{t0}. If they are close enough, they are the target roof displacement. Then go to step 8. If the difference is large, iteration is needed. We need to use a new estimated roof displacement ($x_{t0} = x_t$) and go back to step 3.

Step 8. For the target displacement obtained in step 7, the effective period of the structure T_{eff} and the total equivalent viscous damping ratio ξ_t can be obtained. The spectral acceleration value S_a can be obtained from the acceleration spectra for the T_{eff} and ξ_t. The peak base shear of the structure with the supplemental dampers (MDOF) can be calculated from the spectral acceleration of its equivalent SDOF system as follows:

$$Q = L^* \frac{\sum_{i=1}^{N} \psi_i}{\sum_{i=1}^{N} \phi_i \psi_i} S_a \left(T_{eff}, \xi_t \right)$$

where Q is the peak base shear, $S_a(T_{eff},\xi_t)$ is the spectral acceleration for the effective period T_{eff} and the equivalent viscous damping ratio ξ_t , while ψ is the normalised vector of the inverted-triangular lateral load pattern.

The final displacement shape of the structure at the target displacement can be obtained from the result of the pushover analysis of the original structure in step 1. Then the peak interstory drift index $IDI_{\max}$ can be calculated as follows:

$$IDI_{\max} = \left[\frac{\phi_i - \phi_{i-1}}{h} \right]_{\max} \Delta_f$$

The 12-storey model structure shown in Figure1 was used as the example. The initial period of the undamped structure is 1.99s. The damper yield strength at the 1st level is 211kN. The yield strengths of dampers in upper storeys are taken to be satisfactory distribution. $S_R = 10$, $\cos\theta = 0.91$. The simplified nonlinear static analysis of this model structure was carried out following the procedure mentioned above. Time history analysis of the same structure was also performed. The comparisons of the peak structural response of these two methods are shown in Table 2. The results of the simplified nonlinear static analysis are very close to those of the time-history analysis except for the peak base shears. This is because the base shear has a significant contribution from the higher modes while the simplified method is based on the first mode response. However, for the design of the structure with supplemental dampers the main concern is the displacement. For a preliminary design this method of analysis of the structure with the supplemental dampers should be sufficiently accurate.

THE OPTIMAL DAMPING LEVEL DUE TO THE SUPPLEMENTAL DAMPERS

From the equivalent SDOF system it can be seen that when the strength levels of the dampers increase, the effective period of the structure with the supplemental dampers will decrease and the equivalent viscous damping due to the dampers will increase. This will lead to a reduction of the spectral displacement. However, the influence of the strength levels of the dampers on the response spectral acceleration is not that straightforward. When the strength levels of the dampers increase, on the one hand, the equivalent viscous damping will tend to increase, this will lead to a lower value of the acceleration response; on the other hand, the effective period of the structure with the supplemental dampers will tend to reduce, this will result in a higher value of the acceleration response. The acceleration response of the structure reflects the response level of the base shear of the structure. There exists a certain level of damping to minimise the acceleration response. For different ductility the structure might experience during earthquake excitations, it has been show that this optimal damping ratio due to the supplemental dampers is 15%-17% [Lin, 1999 and Lin et al, 1998b].

PROCEDURE FOR THE DISPLACEMENT-BASED METHOD FOR CHOOSING PARAMETERS FOR THE SUPPLEMENTAL DAMPERS

It is known that of the two characteristic parameters of the supplemental dampers (the force factor F_R and the stiffness factor S_R), the force factor F_R dominates the dynamic behaviour of the structure with the supplemental dampers as long as the stiffness factor S_R exceeds some particular level. Hence in the displacement-based method we focus on the choice of the force factor F_R for the supplemental dampers. We already know that a satisfactory distribution of the dampers occurs when the distribution of the strength of the dampers in every storey along the height of the structure is proportional to the shear distribution developed due to an inverted triangular distribution of lateral load. It is assumed that the original structures are regular and symmetrical in mass and stiffness distribution. Hence the peak response of the MDOF structure with the supplemental dampers can be effectively predicted by its equivalent SDOF system. This gives good grounds for the displacement-based method [Priestley, 1995 and Qi et al, 1991] to be adopted in the choice of the force factor F_R of the supplemental dampers at the preliminary design stage.

The procedure for the displacement-based method can be established as follows [Lin, 1999 and Lin et al, 1998a]:

Step 1. Check the original structure to see whether the maximum interstorey drift meets the requirements of the design or not (time-history analysis or nonlinear static analysis can be adopted for this purpose). If yes, there is no need of any supplemental dampers. If not, go to step 2.

Step 2. The initial displacement shape ϕ_0 can be obtained from the nonlinear pushover analysis of the original structure (at the yielding displacement of the original structure). The yielding base shear and the yielding roof displacement of the original structure (converted to its equivalent SDOF system) can be obtained.

Step 3. Given the required maximum interstorey drift ratio, the first target displacement at roof level (Δ_f) of the structure with the supplemental dampers can be obtained (from the initial displacement shape). Compare this first target displacement with that of the initial displacement shape. If there is a big difference, some iteration is needed until the target displacement obtained from nonlinear pushover analysis also meets the maximum interstorey drift requirement. Then the target displacement and the constant displacement shape ϕ (at the target displacement) can be obtained. The target ductility μ of the original structure can also be calculated. The target spectral displacement (for the equivalent SDOF system) can be obtained. The elastic force of the original structure P at the target displacement (converted to its equivalent SDOF system) can be calculated.

Step 4. Choose the optimal damping ξ_d of 15-17% of critical due to supplemental dampers. Calculate the initial viscous damping and the effective damping of the structure due to inelastic deformation of the original structure ($\xi_0+\Delta\xi_0$) at the target ductility. The total equivalent viscous damping ratio

ξ_t ($=\xi_0+\Delta\xi_0+\xi_d$) can then be obtained.

Step 5. From the generated displacement spectra, knowing the equivalent viscous damping ξ_t and the target spectral displacement, the maximum effective period of the structure with supplemental dampers $T_{\max}$ can be obtained to meet the requirement of the maximum interstory drift ratio.

Step 6. For the given equivalent viscous damping ratio ξ_d (15-17%) due to the supplemental dampers and the stiffness factor S_R, the force factor F_R can be obtained from ξ—F_R relationship (Equation (6)).

Step 7. The effective period of the structure with supplemental dampers corresponding to the F_R factor from Step 6 and the target ductility μ can be calculated.

Step 8. Compare $T_{\max}$ and T_{eff} : if $T_{eff} \leq T_{\max}$, the assumed optimal equivalent viscous damping ratio ξ and the force factor obtained meet the design requirement. If $T_{eff} > T_{\max}$, the assumed optimal equivalent viscous damping ratio (thus the corresponding force factor F_R) is too small. Letting $T_{eff} = T_{\max}$, a modified force factor F_R' and the corresponding equivalent viscous damping ratio ξ_d' can be obtained.

Step 9. From the force factor F_R (or F_R') obtained in Step 8 and the elastic force P at the target displacement of the original structure obtained in Step 3, the yielding force of damping system for the equivalent SDOF system can be calculated as: $F_{yd} = F_R P$

Step 10. The strength level of the damper at the 1st storey can be obtained by the relationship between the base shear of the MDOF structure and its SDOF system. Through Equations (8) and (9), it can be obtained as:

$$F_{yd1} = \frac{F_{yd}^*}{\cos\theta} \frac{\sum \psi_i}{\sum \phi_i \psi_i} = \frac{F_R P}{\cos\theta} \frac{\sum \psi_i}{\sum \phi_i \psi_i}$$

The strength levels of the dampers at the upper storeys can be determined by the satisfactory distribution mentioned above.

An example for determining the parameter of the dampers in a structure following the above procedure is shown in Lin et al [1998a] and Lin [1999].

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Table 1. Comparison of the response for the structure with four different distributions of the yield strength of the dampers under El Centro 1940 NZS4203 compatible earthquake

	Peak interstorey drift (cm)		peak base shear (kN)	
Case-I	1.984	(22.39%)	1502.7	(7.98%)
Case-II	1.760	(8.58%)	1442.0	(3.62%)
Case-III	1.621	(0)	1391.6	(0)
Case-IV	1.842	(13.63%)	1307.2	(-6.07%)

Table 2. Comparison of the peak responses of the structure with the supplemental dampers for a simplified non-linear static analysis and time history analysis under the El Centro 1940 NZS4203 compatible earthquake

	Roof displacement (cm)	Base shear (kN)	Interstorey drift (cm)
Simplified static analysis	17.33	1074.54	2.09
Time history analysis	17.74	1262.7	2.087
Difference	2.37%	17.51%	0.14%

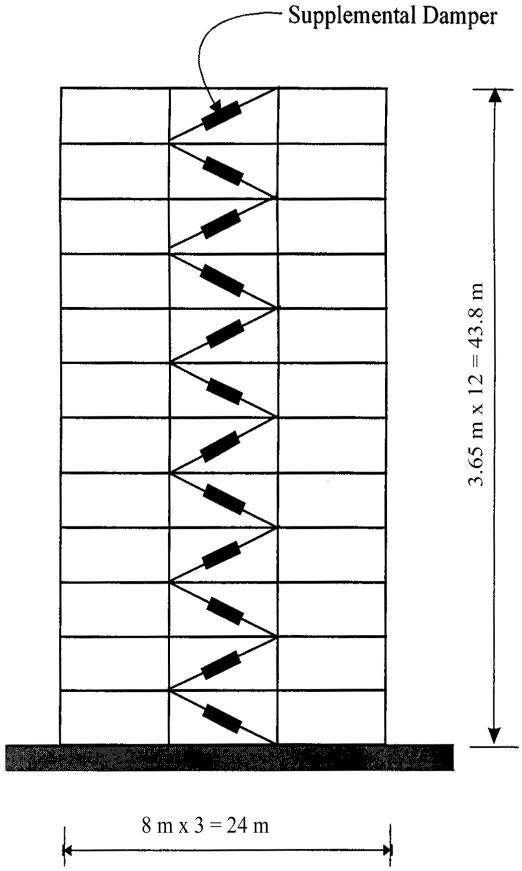

Figure 1 12-storey Frame Structure with Supplemental Dampers

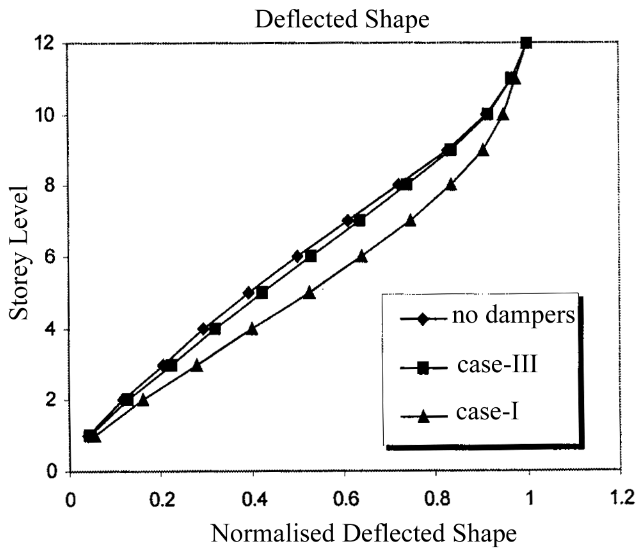

Figure 2 Four Types of Lateral Load Distribution

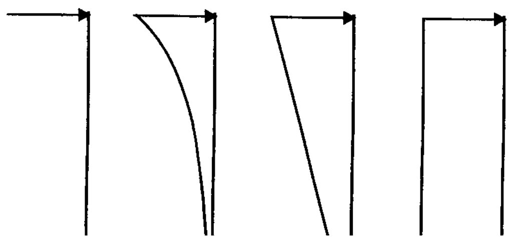

Figure 3 Comparison of the Deflected Shapes of the Structure with and without Dampers

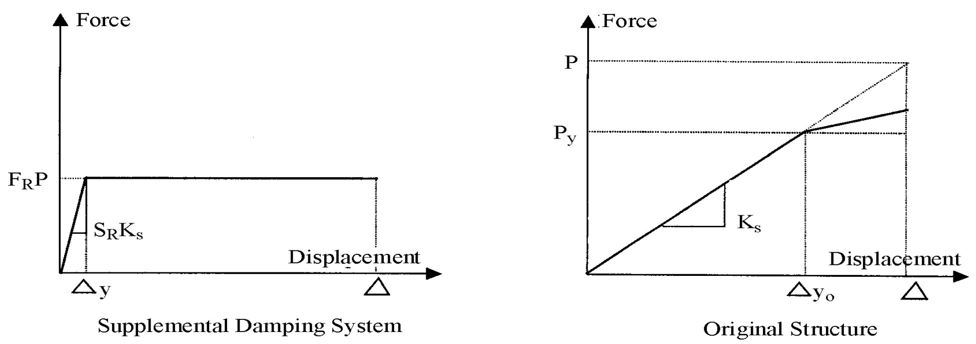

Figure 4 Force-Displacement Relationship for the Supplemental Damping System and the Undamped Original Structure