13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 641

AN EXPERIMETAL STUDY ON THE FLUID PROPERTIES OF LIQUEFIED SAND DURING ITS FLOW

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SUMMARY

The authors investigated into the fluid properties of liquefied sand by carrying out flow tests of model grounds with various thicknesses under 1g condition. Following results were obtained; 1) The liquefied sand has the characteristic of non-Newtonian flow (non-linear viscous flow, pseudo-plastic flow, or Bingham flow) in which the viscosity of liquefied sand decreases with an increase in shear strain rate. 2) The viscosity of the liquefied sand is strongly affected by the thickness of the model ground, and is in proportion with 1.27~1.63 power of the thickness.

These experimental results can provide basic information for the prediction of ground surface displacements caused by lateral flows of actual grounds, as well as for the evaluation of external force from flowing liquefied sand on foundations.

INTRODUCTION

There are various theories proposed on the mechanism of the liquefaction-induced large ground displacements. One theory suggests that the liquefaction causes a notable decrease in the stiffness of ground and that the displacement is induced by the weight of the ground itself[1]. Another theory is saying that large ground displacements are caused by a liquid behavior of liquefied sand[2]. And, the third theory suggests that the displacement is caused by a slide of the upper ground on water film that is formed at the boundary between liquefied layer and non-liquefied layer[3].

In the first theory, liquefied ground is treated as a solid mass, thus the decrease in ground stiffness due to liquefaction should be evaluated. However, it is generally considered difficult to measure accurately the decrease rate that could explain the ground displacements observed during the past earthquakes. Although the second theory requires the clarification of the fluid characteristics of liquefied soil. There are few past studies on this subject[4].

Since the 1983 Nihonkai Chubu Earthquake, the authors have been studying the cases of lateral flow of the ground during earthquakes and have been carrying out experiments on the external force from the flowing liquefied sand, using models of piles and grounds. Based on the results of these experiments, the authors supposed that the liquid behavior of liquefied soil is attributable to the large ground displacements of several meters[4,5,6].

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Experiments on the effect of external force acting on a pile model placed in flowing liquefied soil showed that in the state of liquefaction in which the excess pore-water pressure ratio of the model ground reached approximately 1.0, the fluid force related with the flow velocity of the ground were prominent. Moreover, during the 1995 Kobe Earthquake, lateral movement of the ground behind quay walls ranged over 300 to 400 meters away from the quay walls, indicating that the ground displacements were caused even at a distance of 30 to 40 times of the thickness of the liquefied layer. From these facts, it is considered that the phenomenon of the lateral movement of the ground can be better explained by treating the liquefied soil as fluid.

This study is aimed at clarifying the fluid properties of liquefied sand through flow tests of saturated model grounds under 1g conditions. It is expected to provide basic knowledge in order to develop a prediction method for ground displacements caused by the flow.

FLOW TEST OF LIQUEFIED MODEL GROUND

Test Procedure

Figure1 shows the soil box and the model ground that were used for the flow test. The model ground is 530cm long in the flow direction, 103cm wide, and its thickness is 84~151cm. The walls at the upstream and the downstream ends as well as the sidewalls of the soil box are rigid. The downstream end of the soil box is cut diagonally in order to reduce the volume of the model ground. The ground material is a mixture of silica sands #5 and #6 with a mean grain size of 0.4mm and a coefficient of uniformity of 2.5. The model ground was prepared by laying the soil box horizontally off the shaking table, placing water in it, and then dropping the sands in the water. As **Table1** shows, the relative density of the model ground is in between 33%~35%. After the completion of the model ground, the soil box was placed on a base that had 8.7% inclination, and the model ground surface had the same gradient as the base.

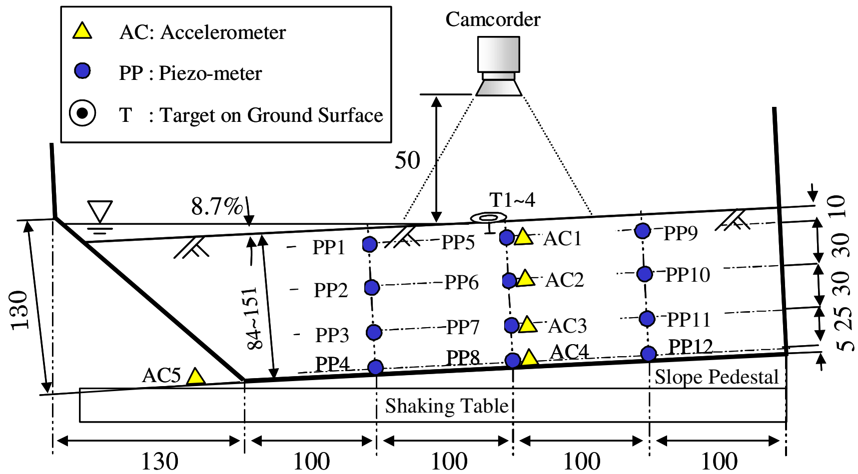
The ground water level had been set at the ground surface of the upstream end of the flow direction until right before the test. The level was lowered to the ground surface at the center of the flow direction before shaking started as shown in **Figure1 (a)**. Consequently, the sand layer of the upper half above the ground water level was saturated and was liquefied when the soil box was shaken.

The soil box was shaken horizontally, and perpendicularly to the flow direction, by a sine wave acceleration with amplitude of 600cm/s^2 and frequency of 6.0Hz. As the result, the model ground was liquefied, causing a flow toward the downstream direction. The model ground was shaken until the surface became almost flat.

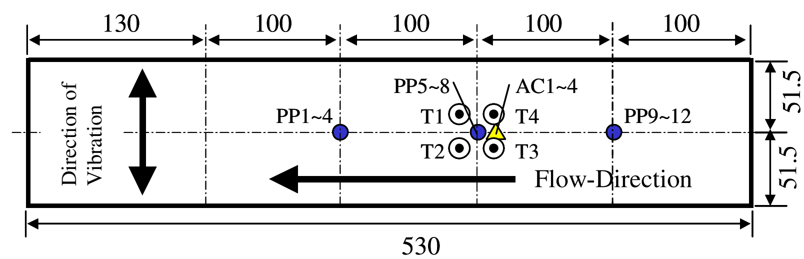
The displacements of circular targets, placed on the ground surface at the center of the soil box as shown in **Figure1 (c)** were recorded by a video camera. The time-history of the displacements was measured and the ground surface velocity was obtained by numerical differentiation. The targets are 3cm in diameter and are made of 1.0mm thick plastic boards that were fixed to the surface of the model ground by 15mm long pins.

Table1: Experimental Condition

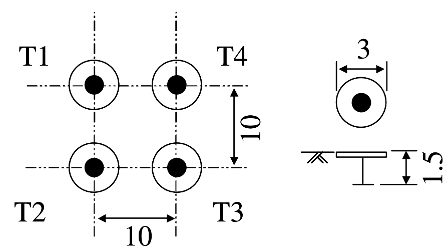
case	Thickness of Model Ground (cm)	Relative Density (%)	Ground Surface Gradient (%)	Input Acceleration (cm/s^2)	Frequency (Hz)
A1	84	34	8.7	597	6
A2	100	33	8.7	590	6
A3	119	33	8.7	679	6
A4	151	35	8.7	619	6



(a) Model Ground and Allocation of Sensor (Side View)



(b) Model Ground and Allocation of Sensor (Top View)



unit:cm

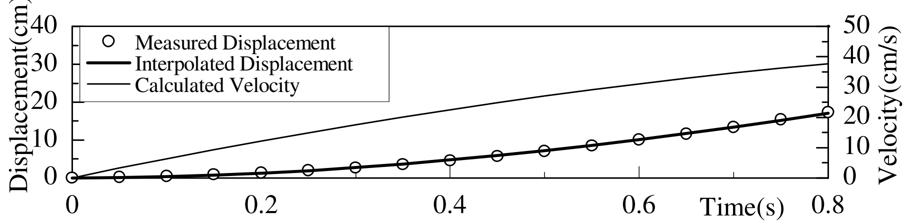
(c) Target on Ground Surface

Figure1: Model Ground for Experiment under 1G Condition

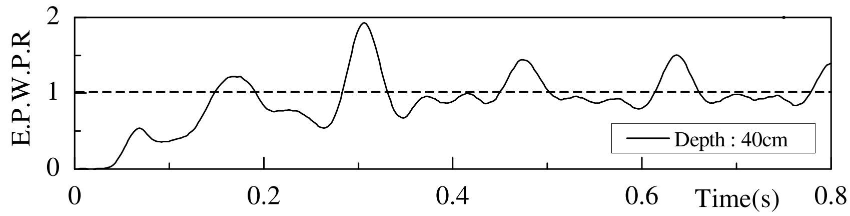
Estimation of the Coefficient of Viscosity

Figure2 shows the ground surface displacement and the ground surface velocity when the ground model is 100cm thick (**Table1**, case: A2), as well as the time-histories of the excess pore-water pressure ratios and the acceleration of the shaking table during the test. The ground surface velocity was obtained through an interpolation of the ground surface displacements by a cubic function and its differentiation.

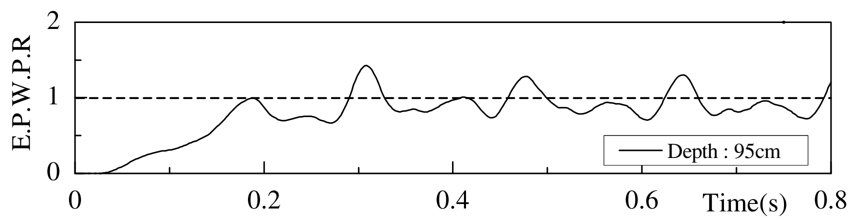
The excess pore-water pressure ratio record (PP6) at 60cm from the bottom of the soil box reached approximately 1.0 in 0.15 seconds after the shaking of the soil box started and the liquefaction was generated from the upper part of the model ground. After that, excess pore-water pressure ratio (PP12) at 5cm from the bottom of the soil box reached 1.0 in approximately 0.2 seconds as shown in **Figure2 (c)**. The ground surface displacement increased when the liquefaction had reached the bottom of the model ground. As demonstrated in **Figure2 (a)**, the displacements interpolated by cubic functions well coincide with the actual measurements.



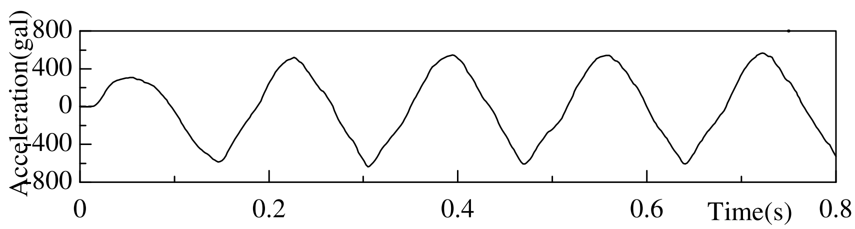
(a) Displacement and Velocity of Target (T1) on Ground Surface



(b) Excess Pore-Water Pressure Ratio (PP6)



(c) Excess Pore-Water Pressure Ratio (PP12)



(d) Base Accelerometer (AC5)

Figure2: An Example of Flow Test Result (case: A2, Thickness 100cm)

Figure3 represents the time-histories of the ground surface velocities in the tests in **Table1**. According to **Figure3**, even with different thickness (84cm-151cm), the time-histories of the ground surface velocities are mostly equal from the beginning of the flow up to around 0.5 seconds. This is because the initial tangent of velocity, i.e. the initial acceleration, expresses the component of gravitational acceleration in the flow direction caused by the ground surface inclination, which was constant in all tests. It is also due to a later-mentioned similitude law among the tests with different thicknesses, that both of the time and the velocity increase almost proportionally with the square root of the thickness of the model grounds. The coefficient of viscosity can be obtained by using the ground surface velocities shown in **Figure3** by the method described below.

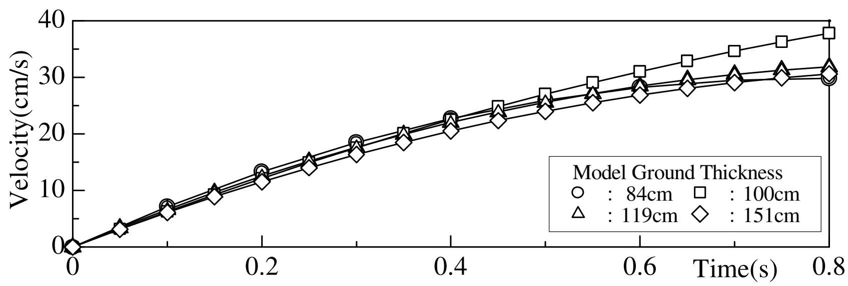


Figure3: Ground Surface Velocity

The ground flow at the center in the flow direction of the model ground is considered as a one-dimensional viscous flow as shown in **Figure4**. However, the ends of the soil box are fixed, and this boundary condition may influence the flow at the center of the soil box. The influence of the fixed boundary will be taken into the consideration by a correction mentioned later.

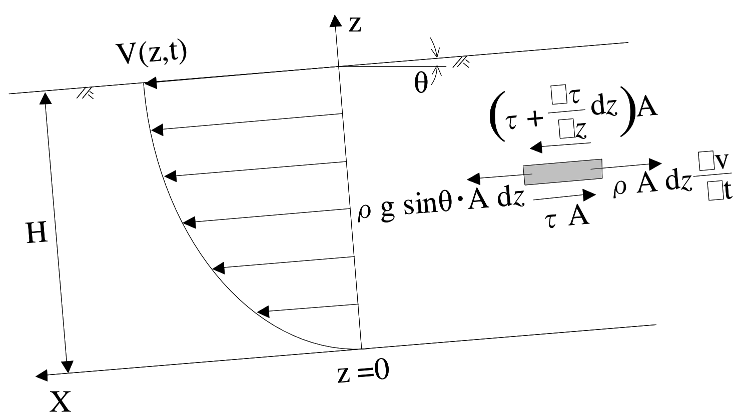


Figure4: One Dimensional Flow of Viscous Fluid

The equilibrium of a segment at depth z as shown in **Figure4** can be expressed by Equation (1).

$$\rho \frac{\partial V}{\partial t} - \frac{\partial \tau}{\partial z} = \rho\, g \sin\theta \qquad (1)$$

Where V and τ are the flow velocity and the shear stress in depth z, respectively. ρ, θ and g denote the density of the liquefied soil, the ground surface gradient, and the gravitational acceleration respectively. t is the time. The relationship between the shear stress τ and the shear strain rate $\dot{\gamma}$ is written by Equation (2).

$$\tau = \mu \dot{\gamma} \qquad (2)$$

Where μ is the coefficient of viscosity, which may be considered as an index of the resistivity of the liquefied sand against the flow.

Equation (2) can be written as follows by using the ground velocity *V (z, t)*:

$$\tau = \mu \frac{\partial V}{\partial z} \tag{3}$$

Then, by substituting Equation (3) into Equation (1), Equation (4) is obtained for the basic equation for one-dimensional viscous flow.

$$\rho \frac{\partial V}{\partial t} - \mu \frac{\partial^2 V}{\partial z^2} = \rho\, g \sin\theta \tag{4}$$

The flow velocity *V (z, t)* can be written as follows:

$$V(z,t) = \sum_{i=1,3,\cdots}^{\infty} q_i(t) \cdot \sin\frac{i\pi}{2H} z \tag{5}$$

And by giving the initial conditions of $t=0$, $V(z, 0)=0$, the time-history $V(t)$ of the ground surface velocity can be obtained as follow:

$$V(t) = \sum_{i=1,3,\cdots}^{\infty} 16 \frac{H^2}{(i\pi)^3} \cdot \frac{\rho g \sin\theta}{\mu} \left[1 - \exp\left\{ -\left(\frac{i\pi}{2H} \right)^2 \frac{\mu}{\rho} t \right\} \right] \sin\frac{i\pi}{2} \tag{6}$$

Equation (6) is a solution of one-dimensional viscous flow based on an assumption that the coefficient of viscosity of the liquefied sand is constant, without any effect from the shear strain rate, i.e., as linear viscous flow. The coefficient of viscosity can be estimated by using Equation (6) and the experimental values of the ground surface velocities shown in **Figure3**.

By substituting the measured value V_j of the ground surface velocity at time t_j into Equation (6), coefficients of viscosity μ_j that satisfy Equation (6) can be estimated. **Figure5** shows the coefficients of viscosity under a relationship with the shear strain rate. Here, the shear strain rate $\dot{\gamma}_j$ at time t_j is the average shear strain velocity of the model ground ; and is expressed as follows by using the thickness of the model ground H.

$$\dot{\gamma}_j = \frac{V_j}{H} \tag{7}$$

According to **Figure5**, the coefficient of viscosity decreases as the shear strain rate increases. From this, it may be concluded that liquefied sand has non-linear characteristic in which the coefficient of viscosity depends on the shear strain rate. The flow in which the coefficient of viscosity decreases as the shear strain rate increases is generally known as pseudo-plastic flow, or Bingham flow. The relationship between the shear stress and the shear strain rate of these flows is expressed in **Figure6**. Furthermore, larger the model ground thickness, the greater the coefficient of viscosity became, showing that the resistivity of the liquefied sand against the flow increases with the thickness of the model ground.

Equation (6) is based on the assumption that the coefficient of viscosity is constant independently with the shear strain rate, i.e. as linear viscous flow. Therefore, the coefficient of viscosity obtained using Equation (6) expresses the secant coefficient of viscosity in the relationship between the shear stress and the shear strain rate shown in **Figure6**.

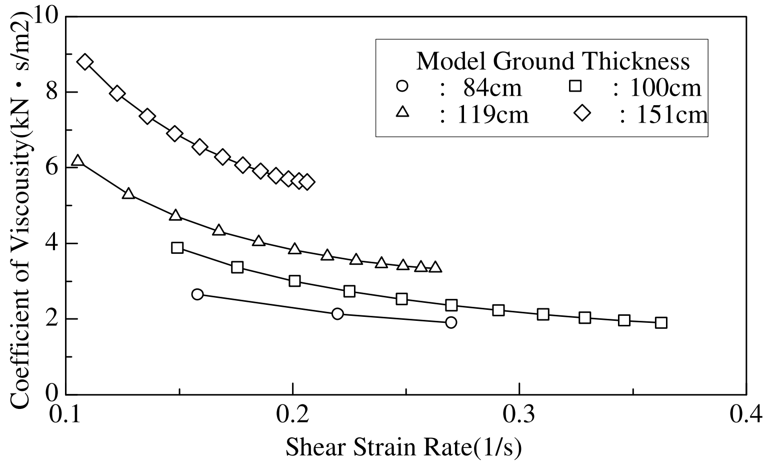


Figure5: Viscosity of Liquefied Sand

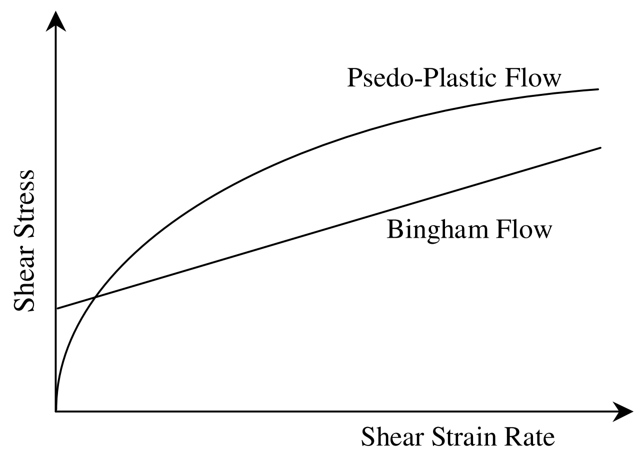


Figure6: Characteristics of Liquefied Sand as Non-Viscous Fluid

Examination of the Effects of Boundary Condition of Model Ground

Equation (6), which was used to estimate the coefficient of viscosity of the liquefied sand, express an one-dimensional flow solution under an assumption that the flow velocity changes only in the depth direction and assuming that boundary of the flow direction is infinite. However, as shown in **Figure1 (a)** the walls for the upper and the lower ends in the flow direction are rigid and the ground surface velocity at the center of the soil box is likely to have decreased by the effect of these boundary conditions. Therefore, the velocities were corrected by taking into the consideration the effect of these boundary conditions.

Figure7 shows the flow model that was used to analyze the flow of the model ground as two-dimensional viscous flow. The time-history of the ground surface velocity at the center of the soil box was obtained using PHOENICS, a computer soft ware for flow analysis. **Figure8** is the comparison between the result of two-dimensional analysis when the thickness of layer is 84cm and the result of one-dimensional analysis by Equation (6). The coefficient of viscosity used in both of one-and two-dimensional analyses is the mean coefficient of viscosity obtained as shown in **Figure5**. **Figure8** shows that the flow velocity obtained by two-dimensional analysis is lower than that by one-dimensional analysis. This is because the length of the flow direction is limited and the ground surface gradient decreased during the flow in the test.

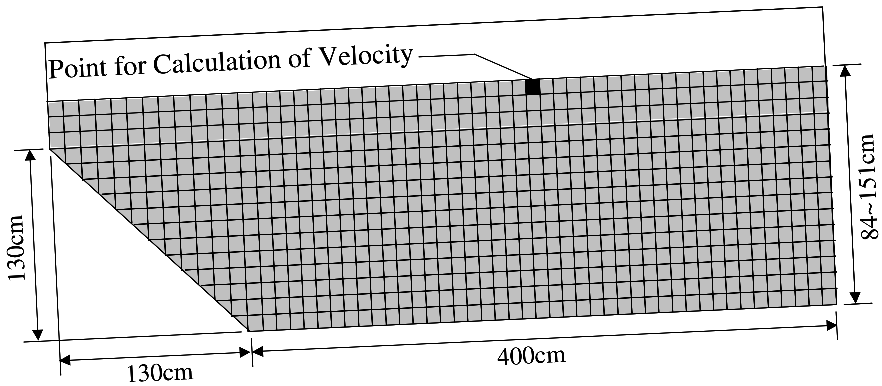


Figure7: Two-Dimensional Model for Flow Analysis

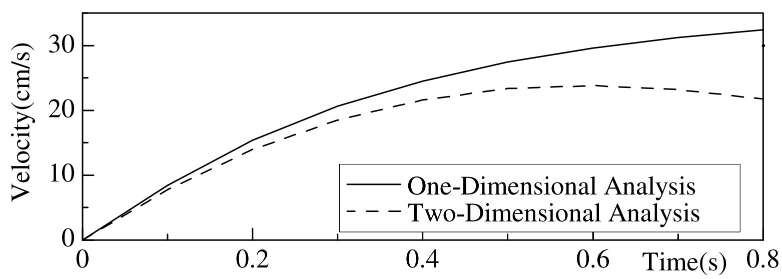


Figure8: Ground Surface Velocities of One-Dimensional and Two-Dimensional Analysies (case : A1,Thickness 84cm)

Figure9 shows the ratios of the velocities obtained by one-dimensional flow analysis to those by two-dimensional analysis in four test cases. Using these ratios, ground surface velocities shown in **Figure3** were corrected. The coefficients of viscosity were re-estimated using Equation (6) from the corrected ground surface velocities. As shown in **Figure10,** the coefficients of viscosity estimated from the corrected velocities also show non-linearity that they decrease as the shear strain rate increases. **Figure10** also shows that larger the model ground thickness, the greater the coefficient of viscosity.

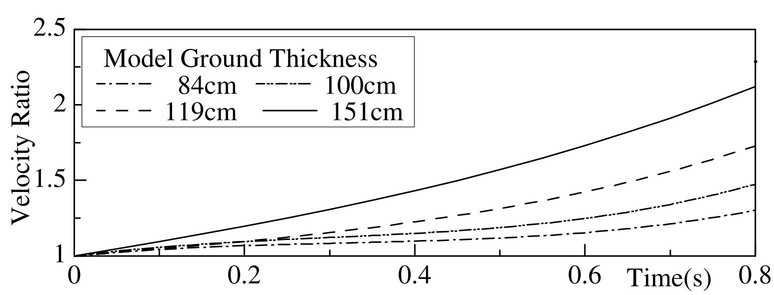


Figure9: The Ratios of Velocities by One-Dimensional Analysis to Those by Two-Dimensional Analysis

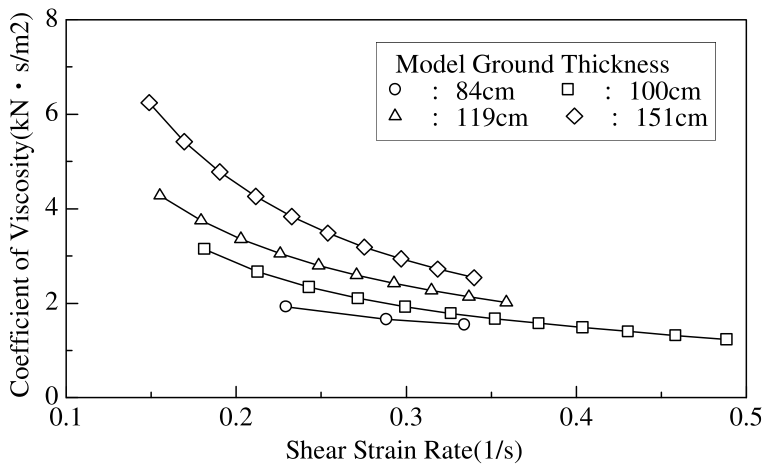


Figure10: Viscosity from Corrected Velocity

The flow tests were conducted by using a soil box with a horizontal width of 103cm as shown in **Figure1 (b)** , and it could be guessed that the existence of the side walls affected the flow.

Figure11 shows the distribution of the ground surface displacements during the test using model ground with a thickness of 119cm (Test: A3, **Table1**). According to **Figure11**, although the ground surface displacements decrease near the sidewalls, they are mostly uniform in the middle of the model ground, away from the sidewalls. The ground surface displacement used for estimating the coefficient of viscosity was measured at the center of the soil box; and therefore the effect of the sidewall boundaries is considered small.

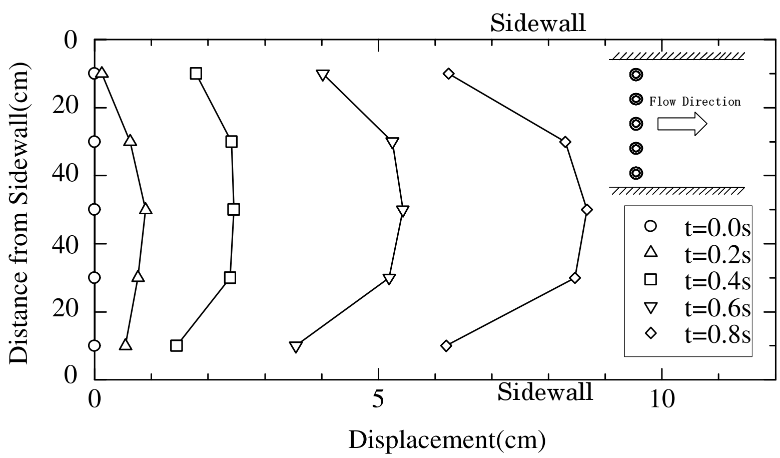


Figure11: Sidewall Effect for Viscous Flow

Effects of the Thickness of the Model Ground on the Coefficient of Viscosity

As mentioned previously, the coefficient of viscosity of liquefied sand shows the characteristics of Bingham flow or the pseudo-plastic flow that decreases as the shear strain velocity increases. Here, the non-dimensional shear strain rate $\bar{\dot{\gamma}}$ is defined and the coefficients of viscosity at same non-dimensional shear strain rate in the tests with different thicknesses are compared.

$$\bar{\dot{\gamma}} = \dot{\gamma}\sqrt{H/g} \tag{8}$$

The mean shear strain rate $\dot{\gamma}$ is obtained by dividing the ground surface velocity V by the thickness H. Therefore, non-dimensional shear strain rate becomes $\bar{\gamma} = V/\sqrt{gH}$

Figure12 shows the relationship between the coefficient of viscosity and the thickness of the model ground when the non-dimensional shear strain rate is 0.07~0.13. This figure also shows that the coefficient of viscosity of the liquefied sand increases as the thickness increases. The gradients of the lines in the figure, both axes of which are logarithmic are ranging between 1.27~1.63. Therefore the coefficient of viscosity of the liquefied sand increases in proportion with the 1.27~1.63 power of the thickness of the model ground.

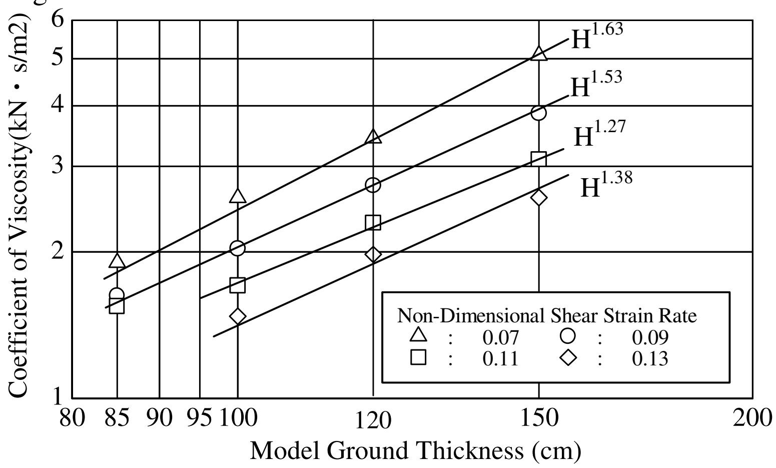


Figure12: Thickness Effect on Viscosity

CONSIDERATION ON THE SIMILITUDE LAW OF THE FLOW

The Relationship between Non-Dimensional Coefficient of Viscosity and Non-Dimensional Shear Strain Velocity Rate

The coefficient of viscosity can be expressed non-dimensionally by Equation (9)

$$\bar{\mu} = \mu / \rho\sqrt{g}H^{3/2} \qquad (9)$$

Figure13 shows the relationship between the non-dimensional coefficients of viscosity and non-dimensional shear strain rates. The results of four tests with different thicknesses are almost on the same line, demonstrating that even in tests of the model ground with different thicknesses, non-dimensional coefficients of viscosity are nearly equal when non-dimensional shear strain rates are equal.

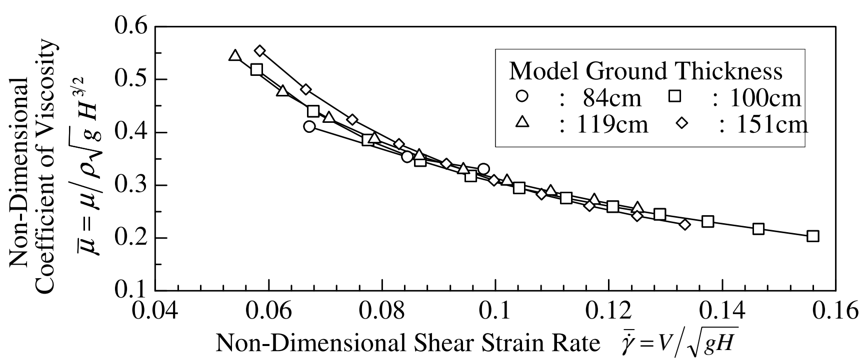


Figure13: Relationship between Non-Dimensional Coefficient of Viscosity and Non-Dimensional Shear Strain Rate

Simulate Law of the Flow Tests

As shown in **Figure13** non-dimensional coefficients of viscosity have mostly equal relationship with non-dimensional shear strain rate even in the tests with different thicknesses of model grounds. Based on this fact, the flow characteristics of the liquefied sand and the similitude among tests using the model ground of varying thickness are examined.

Equation (6) can be rewritten as

$$\frac{V}{\sqrt{gH}}=\sum_{i=1,3,\cdots}^{\infty}16\frac{\theta}{(i\pi)^3}\cdot\frac{\rho\sqrt{g}H^{3/2}}{\mu}\left[1-\exp\left\{-\left(\frac{i\pi}{2H}\right)^2\frac{\mu}{\rho}t\right\}\right]\sin\frac{i\pi}{2} \tag{10}$$

When the ground surface velocity at time t_j during the test using thickness H_1 is V_{1j}, and the coefficient of viscosity identified at that time is μ_{1j}, Equation (11) can be obtained.

$$\frac{V_{1j}}{\sqrt{gH_1}}=\sum_{i=1,3,\cdots}^{\infty}16\frac{\theta}{(i\pi)^3}\cdot\frac{\rho\sqrt{g}H_1^{3/2}}{\mu_{1j}}\left[1-\exp\left\{-\left(\frac{i\pi}{2H_1}\right)^2\frac{\mu_{1j}}{\rho}t_j\right\}\right]\sin\frac{i\pi}{2} \tag{11}$$

Similarly, in the test with thickness H_2, the ground surface velocity V_{2k} at time t_k and the estimated coefficient of viscosity μ_{2k}, at that time satisfy the following equation :

$$\frac{V_{2k}}{\sqrt{gH_2}}=\sum_{i=1,3,\cdots}^{\infty}16\frac{\theta}{(i\pi)^3}\cdot\frac{\rho\sqrt{g}H_2^{3/2}}{\mu_{2k}}\left[1-\exp\left\{-\left(\frac{i\pi}{2H_2}\right)^2\frac{\mu_{2k}}{\rho}t_k\right\}\right]\sin\frac{i\pi}{2} \tag{12}$$

The result in **Figure13** shows that if,

$$\frac{V_{1j}}{\sqrt{gH_1}}=\frac{V_{2k}}{\sqrt{gH_2}} \tag{13}$$

the non-dimensional coefficients of viscosity in two tests with different thicknesses are mostly equal, namely

$$\frac{\rho\sqrt{g}H_1^{3/2}}{\mu_{1j}}\doteqdot\frac{\rho\sqrt{g}H_2^{3/2}}{\mu_{2k}} \tag{14}$$

Therefore, the exponents of Equations (11) and (12) must be equal as follows;

$$\frac{\mu_{1j}}{H_1^2\rho}t_j\doteqdot\frac{\mu_{2k}}{H_2^2\rho}t_k \tag{15}$$

Figure14 shows the relationship between non-dimensional shear strain rate and the value $\mu t/H^2\rho$ of the exponential function in Equation (10). Although, when the non-dimensional shear strain rate under 0.08 the values were somewhat scattering, in the area above 0.08, those were mostly equal independently with the thickness of the model ground.

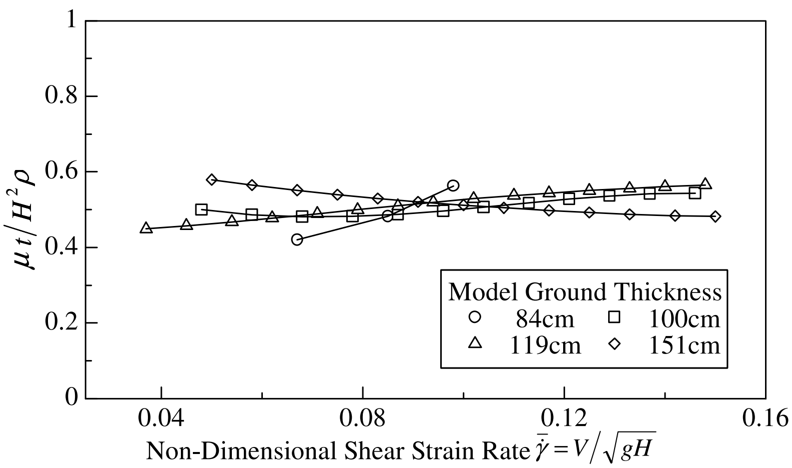


Figure14: Relation between Non-Dimensional Shear Strain Rate and $\mu t/H^2 \rho$ in Equation (10)

When substituting Equations (14) into Equation (15), the following relationship can be obtained.

$$\frac{t_{1j}}{t_{2k}} \fallingdotseq \sqrt{\frac{H_2}{H_1}} \tag{16}$$

And from Equations (13) and (14), the following relationship can be obtained;

$$\frac{V_{2k}}{V_{1j}} \fallingdotseq \sqrt{\frac{H_2}{H_1}} \tag{17}$$

$$\frac{\mu_{1j}}{\mu_{2k}} \fallingdotseq \left(\frac{H_1}{H_2}\right)^{3/2} \tag{18}$$

Equation (16), (17) and (18) show that even in two tests using model grounds with different thicknesses, at times t_{1f} and t_{2k} that satisfy Equation (16), the velocity is proportional to the square root of the thickness, and the coefficient of viscosity is proportional to 1.5 power of the thickness.

Figure3 showed that the time-histories of the ground surface velocities of the four tests with different thicknesses are nearly equal. This is considered due to the fact that both of the velocity and the time are proportional to the square root of the thickness of the model ground.

CONCLUSION

Following results on the fluid properties of liquefied sand during its flow were obtained from the flow tests of model grounds:

(i) The liquefied sand shows the characteristics of non-linear viscous flow , namely psedo-plastic flow on Bingham flow , in which the coefficient of viscosity decreases as the shear strain rate increases.

(ii) The coefficient of viscosity of liquefied sand increases proportionally to 1.27~1.68 power of its thickness of model ground. Namely the resistivity of liquefied sand against the flow increases with its thickness.

(iii) The flow tests of model grounds show that non-dimensional coefficients of viscosity are mostly equal even in the tests with different thicknesses when the non-dimensional shear strain rate are equal. This suggests that a similitude law among flow tests with different thicknesses of the model grounds, i.e. the time and the velocity are almost proportional to the square root of the thickness of the model ground and that the coefficients of viscosity are proportional to almost 1.5 power of the thickness.

One of the authors had proposed the following Equation (19) based on the cases studies on liquefaction-induced ground surface displacements.

$$D_s = 0.75\sqrt{H}\sqrt[3]{\theta} \tag{19}$$

In the above equation, D_s, H, and θ are the ground surface displacement (m), the total thickness (m) of the liquefied layer, and the ground surface gradient (%) respectively. According to this equation, the ground surface displacement is nearly proportional to the square root of the total thickness of the liquefied layer. However, if the coefficient of viscosity is assumed constant regardless of the thickness of the liquefied soil the ground surface flow velocity V are proportional to the square of the thickness of the liquefied soil H according to Equation (6). This clearly disagrees with the result of the case studies shown in Equation (19). Nevertheless, as shown by the model tests of this study, if the coefficient of viscosity of the liquefied sand is supposed to increase proportionally to 1.5 power of the thickness of the liquefied soil, the result of the case studies shown in Equation (19) can be considered consistent with the results of the flow tests on liquefied sand.

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