13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 569

PARALLEL COMPUTATION OF 3D WAVE PROPAGATION BY SPECTRAL STOCHASTIC FINITE ELEMENT METHOD

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SUMMARY

Strong ground motion plays an important role for efficient seismic design and prevention of earthquake disasters. It is impossible, however, to simulate or predict perfectly accurate earthquake ground motions because many influential factors have uncertainty. It is essential therefore to quantitatively estimate the effect of these factors on the ground motions and take them into consideration. Monte Carlo simulation is a powerful tool for consideration of uncertainty. However, computation of MCS is expensive in terms of computational resources and more efficient methods are also in demand.

We present application of Spectral Stochastic Finite Element Method (SSFEM) to the three dimensional wave propagation analysis in the uncertain field. SSFEM represents a spatial distribution of uncertain parameters by Karhunen-Loève (KL) expansion. Stochasticity of the solution is represented by Polynomial Chaos (PC) expansion. The system equation with stochasticity is then projected on Polynomial Chaos functionals to generate a set of equations. Since PC functionals consist an orthogonal basis of the probability space called homogeneous chaos, solution of the projected equations assures the best approximation of the system equation in terms of the error of the norm defined in that homogeneous chaos space.

Since equations projected to PC functionals are not independent of each other, computation of SSFEM involves treatment of huge matrices. We propose a scheme for parallel computation with MPI for an efficient computation of SSFEM. In this scheme, each of the projected equations is considered as a separate domain which has interaction with each other. We utilize a non-iterative time integration scheme for a nonlinear dynamic FEM analysis. This enables an efficient parallel computation of SSFEM by treating the interaction between domains of SSFEM as nonlinearity of the system.

The proposed scheme is applied to a three-dimensional wave propagation in the uncertain media. Efficiency and accuracy of the scheme are verified based on the computational results. It is shown that the presented scheme considerably reduces the computation time without losing accuracy. It indicates that SSFEM with MPI would be a powerful option for strong motion simulation with quantitative consideration of uncertainty.

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INTRODUCTION

For efficient and rational seismic design, it is important to estimate earthquake ground motions with sufficient accuracy. However uncertainty in the estimation of ground motion is inevitable since many influential factors have uncertainty and most of those factors are difficult to determine or investigate through the survey. Therefore consideration of the effect of such uncertainties is essential in seismic design and damage estimation.

In this paper, we consider uncertainty of material property of the ground. Ground property is not uniform and it is impossible to obtain the perfect information about the distribution of the ground property such as stiffness.

Wave propagation in the random media have been studied for decades and various results have been reported [1, 2, 3]. Most of them, however, are pursuing analytical solutions for rigorous or approximate expressions and they are not necessarily suitable for engineering problems where various practical conditions such as ground layer structures and boundary conditions have to be taken into consideration.

One of the most powerful tools for such problems is a Monte Carlo simulation, but it requires huge computational resources and more efficient methods are demanded. To compute such problems efficiently, we can use stochastic finite element methods (SFEM) by Yamazaki *et al.*[4] and Spanos & Ghanem [5]. We have proposed to use Spectral Stochastic Finite Element Method (SSFEM) which was originally proposed by Ghanem and Spanos [6] and SSFEM has been applied to two dimensional wave field [7]. In this paper, we apply SSFEM to three dimensional wave field.

This paper proposes a method for the parallel computation of SSFEM to the three dimensional analysis of wave propagation in the field with uncertainty. First part presents formulation of SSFEM applied to three dimensional wave field. Then the algorithm for parallel computation of dynamic SSFEM analysis is described. To illustrate the performance of the proposed formulation, numerical simulations are conducted on the computer with MPI. Compare of computational results will verify the efficiency of the proposed scheme.

SPECTRAL STOCHASTIC FINITE ELEMENT METHOD

This section describes the formulation of wave propagation analysis using Spectral Stochastic Finite Element Method (SSFEM). We introduce two schemes to represent stochastic processes that play important roles in the formulation of SSFEM; Karhunen-Loève expansion and polynomial chaos expansion. The formulation of wave propagation analysis by SSFEM is presented using these stochastic representation schemes. More information about utilization of these representations in the formulation of SSFEM are also found in the references [6, 8, 9, 10].

Karhunen-Loève Expansion

In SSFEM, uncertain parameters are regarded as stochastic processes whose spatial distributions are represented by Karhunen-Loève expansion. The Karhunen-Loève expansion is a representation of a stochastic process in terms of uncorrelated random variables. When applied to a stochastic process whose covariance function is known, Karhunen-Loève expansion can provide the optimal representation of the original process in the mean-square sense.

Let us consider the domain S and a stochastic process $G(x, \theta)$ defined in S where $x \in S$ denotes the spatial coordinate and θ denotes an event in the probability space. Assume covariance function of the value at

arbitrary points $x_1, x_2 \in S$ is given as $C(x_1, x_2)$. Then Karhunen-Loève expansion of a stochastic process $G(x, \theta)$ is given as

$$G(x,\theta) = \bar{G}(x) + \sum_{i=1}^{\infty} \xi_i(\theta)\sqrt{\lambda_i}\, f_i(x) \tag{1}$$

where $\bar{G}(x)$ is a mean value of $s(x, \theta)$ at x and $\xi_i(\theta)$'s are orthonormal random variables. Scalars λ_i's and functions $f_i(x)$'s are respectively given as eigenvalues and normalized eigenfunctions of the integral equation

$$\int_S C(x_1, x_2) f_i(x_2) dx_2 = \lambda_i f_i(x_1). \tag{2}$$

It is impossible to expand Eq.(1) to an infinite order in practice and summation of Eq.(1) is truncated at a finite order. The truncation order of the summation is referred to as KL order and denoted by N_{KL} in this paper. Truncated summation of Eq.(1) does not generally represent the original stochastic process perfectly. However, it is known that Karhunen-Loève expansion gives the optimal approximation of the original stochastic process in the sense of mean-square, when the summation is truncated at a given finite order. Accuracy of truncated Karhunen-Loève expansion is discussed by Ghanem [6, 11].

Polynomial Chaos Expansion

Since it is impossible to obtain the information of covariance function of the solution in advance, solution process can not be represented by Karhunen-Loève expansion. Instead, the solution process is represented by polynomial chaos expansion. Displacement vector $\boldsymbol{u}$ for example is given as

$$\boldsymbol{u} = \sum_{i=0}^{\infty} \boldsymbol{u}_i \Psi_i(\boldsymbol{\xi}) \tag{3}$$

where $\Psi_i(\boldsymbol{\xi})$ denotes i-th polynomial chaos and its argument $\boldsymbol{\xi}$ is a set of orthonormal Gaussian stochastic variables:

$$\boldsymbol{\xi} = (\xi_1, \xi_2, \dots,)^\top. \tag{4}$$

For the purpose of practical computation, we take an integer N_{PC} and the expansion is truncated after the N_{PC}-th term. The value of N_{PC} is disussed below.

Polynomial chaos $\Psi_n(\boldsymbol{\xi})$ are given as multivariate Hermite Polynomials which can be constructed as

$$\Psi_n(\boldsymbol{\xi}) = e^{\frac{1}{2}\boldsymbol{\xi}^\top \boldsymbol{\xi}} (-1)^m \frac{\partial^n}{\partial \xi_{i_1^n} \partial \xi_{i_2^n} \cdots} e^{-\frac{1}{2}\boldsymbol{\xi}^\top \boldsymbol{\xi}} \tag{5}$$

where a multiindex $(i_1^n, i_2^n, \dots,)$ denotes a set of all possible combination of non-negative integers whose summation is equal to a certain integer m where m is a non-negative integer taken as $0, 1, \dots$ Suppose we have two independent random variables, ξ_1 and ξ_2, then possible combination of (i_1^n, i_2^n) for $n = 0, 1, 2, 3, \dots,$ are

$$(i_1^n, i_2^n) = (0,0), (1,0), (0,1), (2,0), (1,1), (0,2), \cdots \tag{6}$$

and corresponding polynomial chaos are obtained as

$$\begin{aligned} &\Psi_0(\boldsymbol{\xi}) = 1, \quad \Psi_1(\boldsymbol{\xi}) = \xi_1, \quad \Psi_2(\boldsymbol{\xi}) = \xi_2, \\ &\Psi_3(\boldsymbol{\xi}) = \xi_1^2 - 1, \quad \Psi_4(\boldsymbol{\xi}) = \xi_1\xi_2, \quad \Psi_5(\boldsymbol{\xi}) = \xi_2^2 - 1, \dots \end{aligned} \tag{7}$$

They are orthogonal to each other with respect to the Gaussian measure.

The space spanned by $\{\Psi_n(\boldsymbol{\xi})\}$ for which $m = \sum_k i_k^n$ $(m \geq 0)$ is called an m-th order homogeneous chaos (HC). In this paper, the direct sum of the homogeneous chaos with the order less or equal to N_{HC} is referred to as a homogeneous chaos truncated at the N_{HC}-th order.

Polynomial chaos consist an orthogonal basis of the homogeneous chaos [12, 13]. When the HC is truncated at a finite order, the number of terms of polynomial chaos to consist the orthogonal basis of the HC is also finite. Let N_{PC} denote the total number of terms excluding the 0-th order term $\Psi_0(\boldsymbol{\xi})$. Then N_{PC} is uniquely determined by the truncation order of HC, N_{HC}, and the number of independent stochastic variables in consideration. Suppose we have N_{KL} stochastic variables, then N_{PC} is given as

$$N_{PC} = \sum_{\ell=1}^{N_{HC}} \frac{1}{\ell!} \frac{(N_{KL} + \ell - 1)!}{(N_{KL} - 1)!}. \tag{8}$$

Since N_{PC} is uniquely determined by N_{KL} and N_{HC}, we will specify the cases by KL order and HC order without explicitly mentioning to N_{PC}. For example, if we truncate Karhunen-Loève expansion at 2nd order and homogeneous chaos by 2nd order, we simply write KL=2 and HC=2, implicitly meaning that we have $N_{PC} = 5$.

A mean value, variance and probability density function (pdf) of the solution expressed in the from of PC expansion can be easily evaluated. A mean value is given as the 0-th term. For example, a mean value of displacement vector $\boldsymbol{u}(\theta)$ in Eq.(3) is given as

$$\langle \boldsymbol{u}(\theta) \rangle = \boldsymbol{u}_0. \tag{9}$$

We denote n-th component of $\boldsymbol{u}$ by u^n, and coefficient of i-th PC of $\boldsymbol{u}^i$ by u_i^n. Then the variance of n-th component of $\boldsymbol{u}$ is obtained as

$$\langle \{u^n - \langle u^n \rangle\}^2 \rangle = \sum_{i=1}^{N_{PC}} \langle \Psi_i(\boldsymbol{\xi}) \Psi_i(\boldsymbol{\xi}) \rangle (u_i^n)^2 \tag{10}$$

where the orthogonality of polynomial chaos, that is, $\langle \Psi_i(\boldsymbol{\xi}) \Psi_j(\boldsymbol{\xi}) \rangle = 0$ for $i \neq j$ is taken into consideration. Pdf can be simply evaluated in a Monte Carlo Simulation-like manner. Generate numerous sets of independent Gaussian variables ξ_i $(i = 1, 2, \dots)$ and estimate the value of Eq.(3) for each of them. Statistical distribution of those values represents pdf of Eq.(3).

Wave Propagation Analysis by SSFEM

Let us derive the formulation of SSFEM for the wave propagation problem in the uncertain media. We consider an FEM formulation of equation of motion as

$$Ma(t, \theta) + K(\theta)u(t, \theta) = p(t) \tag{11}$$

where M and K denote mass and stiffness matrices; $a(t, \theta), u(t, \theta)$ and $p(t)$ are acceleration vector, displacement vector and external force vector, respectively.

We assume that the shear stiffness has a Gaussian uncertainty. We expand the shear stiffness $G(x, \theta)$ by KL expansion as in Eq.(1). We truncate the expansion of Eq.(12) at a finite KL order N_{KL} and we obtain

$$G(x, \theta) = \bar{G}(x) + \sum_{i=1}^{N_{KL}} \xi_i(\theta)\, G_i(x) \tag{12}$$

where x denotes spatial coordinate; θ denotes an event in the probability space; $\bar{G}$ denotes the expected value of $G(x,\theta)$; $\xi_i(\theta)$ are orthogonal Gaussian random variables. $G_i(x)$ is given as

$$G_i(x) = \sqrt{\lambda_i}\, f_i(x) \tag{13}$$

where λ_i and $f_i(x)$ are an eigenvalue and an eigenfunction of the Eq.(2) with $C(x,y)$ representing the correlation function of $G(x,\theta)$.

Let $K_n(\theta)$ denote a stiffness matrix corresponding to the shear stiffness $G_n(x,\theta)$, then the stiffness matrix $K(\theta)$ can be expressed as

$$K(\theta) = K_0 + \sum_{n=1}^{N_{KL}} \xi_n(\theta) K_n. \tag{14}$$

Since it is impossible to obtain the correlation function of solution a priori, KL expansion is not applicable for the solution such as displacement, velocity and acceleration. They are expressed by PC expansion as in Eq.(3) where summation is truncated at N_{PC}.

Substitute in Eq.(11) the stiffness matrix and state variables which are expressed in the stochastic expansion representation, we obtain

$$M \sum_{i=0}^{N_{PC}} a_i(t)\Psi_i(\boldsymbol{\xi}) + \sum_{n=0}^{N_{KL}} \xi_n(\theta) K_n \sum_{i=0}^{N_{PC}} u_i(t)\Psi_i(\boldsymbol{\xi}) = p(t). \tag{15}$$

To approximate Eq.(15) in the space of homogeneous chaos truncated at a finite order, we project Eq.(15) on the polynomial chaos $\Psi_j(\boldsymbol{\xi})$. Since PC consist an orthogonal basis, this gives the best approximation of Eq.(15) in the homogeneous chaos under consideration. This process gives equations to be solved.

$$\sum_{i=0}^{N_{PC}} \left\langle \Psi_i(\boldsymbol{\xi})\Psi_j(\boldsymbol{\xi}) \right\rangle M a_i(t) + \sum_{n=0}^{N_{KL}} \sum_{i=0}^{N_{PC}} \left\langle \xi_n \Psi_i(\boldsymbol{\xi})\Psi_j(\boldsymbol{\xi}) \right\rangle K_n u_i(t) = \left\langle p(t)\Psi_j(\boldsymbol{\xi}) \right\rangle \tag{16}$$

It can be also shown that PC expansion coefficients of acceleration $a_j(t)$, velocity $v_j(t)$ and displacement $u_j(t)$ satisfy the relation of differentiation as

$$a_j(t) = \frac{\mathrm{d}}{\mathrm{d}t} v_j(t) = \frac{\mathrm{d}^2}{\mathrm{d}t^2} u_j(t). \tag{17}$$

Therefore Eq.(16) can be treated as an ordinary second order differential equation and it can be numerically solved by conventional time integration schemes.

PARALLEL COMPUTATION OF SSFEM

Suppose that our problem has N_{DOF} degree of freedom and that the number of terms of polynomial chaos to consist the basis of HC under consideration is N_{PC}. Then the matrices of SSFEM to represent Eq.(16) consist of $(N_{PC}+1)\times(N_{PC}+1)$ blocks where each block is of the size of the matrix of deterministic FEM analysis, $N_{DOF}\times N_{DOF}$. It can be found recognized that SSFEM requires manipulation of such huge matrices and it can make the computation expensive.

We propose an efficient algorithm for parallel computation of dynamic analysis of SSFEM. This is enabled by using a non-iterative time integration scheme for FEM and also exploiting the sparseness of the matrices of SSFEM.

Non-Iterative Time Integration Scheme [14]

In order to implement an efficient computation of dynamic analysis of SSFEM, we utilize a non-iterative time integration (NITI) scheme [14]. NITI scheme is implemented by combining implicit and explicit time integration schemes, taking advantage of benefits of both schemes. Computational stability of NITI scheme is comparable with that of implicit schemes under certain conditions, which allows us to take a large time step to reduce the total number of updating steps. NITI scheme does not require iterative process even when applied to nonlinear dynamic problems and computation load is decreased considerably.

In the following, we present formulation of NITI scheme applied to the dynamic analysis of a nonlinear system, using central difference method as an explicit scheme and Newmark β method as an implicit scheme. Let us consider an ordinary equation of motion as

$$M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = f(t,u) \tag{18}$$

where M, C and K denote mass, damping and stiffness matrices respectively; u is displacement; $f(t,u)$ is an external force and it is a function of time t and displacement u. Nonlinear problems that have displacement-dependent stiffness can be written in the same expression by shifting the effect of nonlinearity to the right-hand side of the equation. The equation is discretized assuming the time step as Δt and letting subscript k denote the value at the k-th time step, $t_k = k\Delta t$.

Let us illustrate the process by updating the time level from $t = t_n$ to $t = t_{n+1}$. We estimate the external force at time $t = t_{n+1}$ using the displacement u_n at the current time level $t = t_n$. Updating the equation using Newmark β method, we obtain the prediction of the displacement $\tilde{u}_{n+1}$, velocity $\tilde{\dot{u}}_{n+1}$ and acceleration $\tilde{\ddot{u}}_{n+1}$ as

$$\tilde{\ddot{u}}_{n+1} = -\ddot{u}_n - \frac{4}{\Delta t}\dot{u}_n + \frac{4}{\Delta t^2}(\tilde{u}_{n+1} - u_n) \tag{19}$$

$$\tilde{\dot{u}}_{n+1} = -\dot{u}_n + \frac{2}{\Delta t}(\tilde{u}_{n+1} - u_n) \tag{20}$$

$$\tilde{u}_{n+1} = \left(K + \frac{2}{\Delta t}C + \frac{4}{\Delta t^2}M\right)^{-1} \cdot \left\{ f(t_{n+1}, u_n) + M\left(\frac{4}{\Delta t^2}u_n + \frac{4}{\Delta t}\dot{u}_n + \ddot{u}_n\right) + C\left(\frac{2}{\Delta t}u_n + \dot{u}_n\right)\right\} \tag{21}$$

These prediction requires correction to compensate the difference of the external force $f(t_{n+1}, u_{n+1})$ and the assumed external force $f(t_{n+1}, u_n)$. Conventional implicit scheme requires iteration to estimate compensation terms. NITI scheme estimate the difference using the central difference method (CDM). Since CDM is an implicit scheme, compensation can be taken into consideration without iteration process. Difference of the external force Δf_u is simply given as

$$\Delta f_u = f(t_{n+1}, \tilde{u}_{n+1}) - f(t_{n+1}, u_n) \tag{22}$$

Let $\Delta\ddot{u}_{n+1}$, Δu_{n+1} and $\Delta\dot{u}_{n+1}$ denote the responses of the system due to the external force Δf_u, and they are given as

$$\Delta\ddot{u}_{n+1} = \left(M + \frac{\Delta t}{2}C\right)^{-1} \Delta f_u \tag{23}$$

$$\Delta\dot{u}_{n+1} = \left(M + \frac{\Delta t}{2}C\right)^{-1} \Delta f_u \frac{\Delta t}{2} \tag{24}$$

$$\Delta u_{n+1} = 0 \tag{25}$$

Summation of the prediction and the correction yields the response at time level $t = t_{n+1}$ as

$$\ddot{u}_{n+1} = \tilde{\ddot{u}}_{n+1} + \Delta\ddot{u}_{n+1} \tag{26}$$

$$\dot{u}_{n+1} = \tilde{\dot{u}}_{n+1} + \Delta\dot{u}_{n+1} \tag{27}$$

$$u_{n+1} = \tilde{u}_{n+1}. \tag{28}$$

Eq.(25) indicates that displacement does not change in the compensating process and consequently the external force $f(t_{n+1}, u_{n+1})$ does not change either. Therefore the relationship

$$f(t_{n+1}, \tilde{u}_{n+1}) = f(t_{n+1}, u_{n+1}) \tag{29}$$

holds and, requiring no further interaction, Eqs.(26)–(28) give the solution for the time level $t = t_{n+1}$.

Parallelization of Computation

Matrices used in SSFEM analysis are huge and sparce. They consist from blocks, each of which is of the size of matrices of ordinary FEM analysis. We write these blocks as

$$M^{ij} = \langle \Psi_i(\boldsymbol{\xi})\Psi_j(\boldsymbol{\xi})\rangle M \tag{30}$$

$$K^{ij} = \sum_{n=0}^{N_{KL}} \langle \xi_n \Psi_i(\boldsymbol{\xi})\Psi_j(\boldsymbol{\xi})\rangle K_n \tag{31}$$

Correspondingly, external force vector should be also divided into blocks as

$$p^j(t) = \langle p(t)\Psi_j(\boldsymbol{\xi})\rangle \tag{32}$$

Diagonal blocks of mass matrix are nonzero since $\langle \Psi_i^2 \rangle \neq 0$, while its non-diagonal blocks are zero matrices because $\langle \Psi_i \Psi_j \rangle = 0\,(i \neq j)$. Similarly, diagonal blocks of SSFEM stiffness matrices are non-zero matrices due to the terms with $n = 0$. We decompose the stiffness matrix into the matrix consisting of the diagonal blocks ($i = j$ in Eqs.(30) and (31)) and the matrix consisting of the non-diagonal blocks ($i \neq j$). The matrix containing diagonal blocks of the stiffness matrix K is denoted by K^{D} and the matrix with the rest blocks is by K^{ND}. By using these matrices, Eq.(16) can be written as

$$M\ddot{u} + K^{\mathrm{D}}u = f(t, u) - K^{\mathrm{ND}}u. \tag{33}$$

Exploiting the fact that both mass matrix M and stiffness matrix K^D are block diagonal, Eq.(33) can be separated into

$$M^{ii}\ddot{u}^i(t_{n+1}) + K^{ii}u^i(t_{n+1}) = p^i(t_{n+1}) - \sum_{j,\, j\neq i} K^{ij}u^j(t_{n+1}) \tag{34}$$

where i denotes the order of polynomial chaos to which the stochastic equation is projected and Eq.(34) represents the equation of motion projected to the i-th polynomial chaos. Although Eq.(34) gives an independent equation for each i, since non-diagonal block matrix K^{ND} is not empty indicating the existence of interactions, they are not independent of each other.

Now, for an efficient parallel computation, we can apply NITI scheme by regarding the second term of the right-hand side of Eq.(34) as an external force which is a function of displacement.

The application procedure is simple. First, we solve Eq.(34) assuming the right hand side takes the value at time $t = t_n$ to obatain

$$M^{ii}\ddot{u}^i(t_{n+1}) + K^{ii}u^i(t_{n+1}) = p^i(t_{n+1}) - \sum_{j,\, j\neq i} K^{ij}u^j(t_n). \tag{35}$$

This computation can be conducted separately on different CPUs in parallel. Matrix used in the computation on each CPU is of the size of deterministic FEM analysis. We obtain the tentative value of displacement $\tilde{u}_{n+1}$ for the time at $t = t_{n+1}$. We update the interaction using these displacement values to obtain the increment of the external force as

$$\Delta f_u^i = -\sum_{j,j\neq i} K^{ij}\{\tilde{u}^j(t_{n+1}) - u^j(t_n)\}. \tag{36}$$

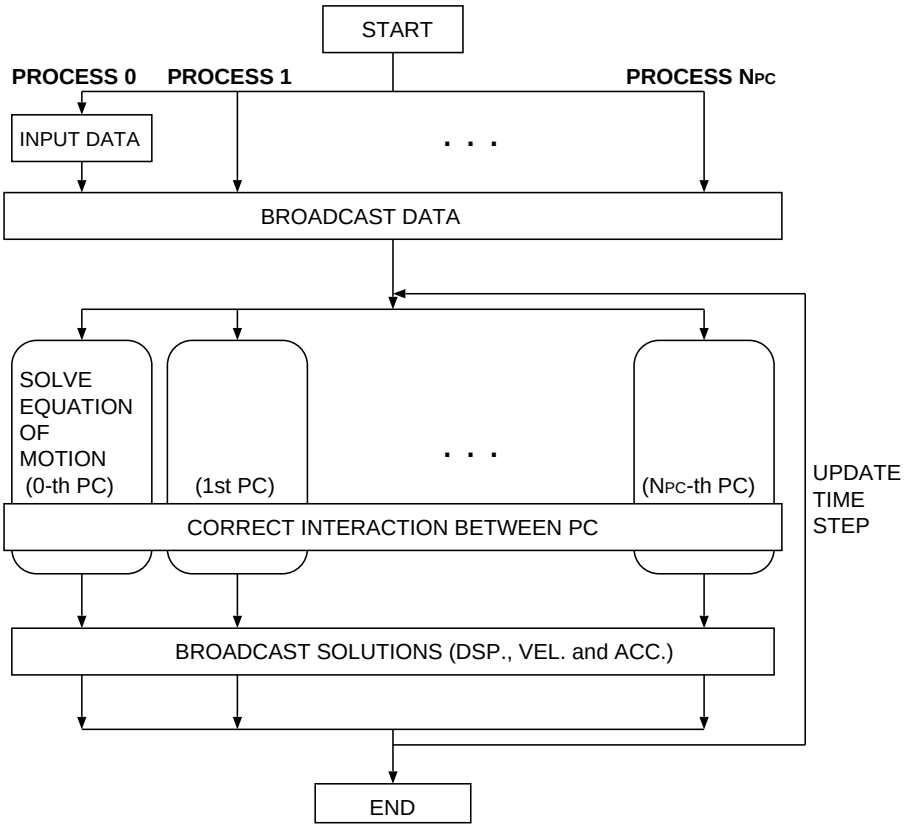


Figure 1: Flowchart of parallel computation of SSFEM.

Velocity and acceleration are respectively updated by Eqs.(23) and (24). Substituting them in Eqs.(26) – (28) gives the value at time level $t = t_{n+1}$. This computation can be also conducted on each CPU in parallel. The proposed scheme works efficiently especially because stiffness matrix K of SSFEM are sparse. The interaction terms are not dominant and consideration of them does not require huge amount of computation.

The total flow of the proposed parallel computation algorithm is summarized in **Figure 1**. The flow is assumed to have N_{PC} processes to run parallelly. The 0-th process obtain the input data and distribute the required information to all other processes. In every step of update, each process independently updates the state variables on each of the spaces projected to N_{PC} polynomial chaos, assuming the interaction of the previous time level. The interaction is then estimated based on the computed displacement and their values are cast to other processes. Based on the updated interaction, process on each processors parallelly compensates velocity and acceleration. Updating process is conducted on each CPU independently except communication of interaction forces. Communication has to be conducted only once for every time step and most of the computation process can be conducted highly independently.

NUMERICAL SIMULATIONS

The performance of the proposed scheme is studied by applying it to the computation of SSFEM analysis of the three dimensional wave propagation in the uncertain field.

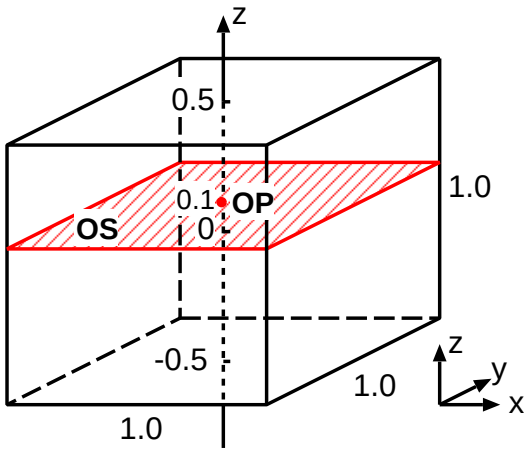


Figure 2: Problem Setup. Observation point OP $(x, y, z) = (0.0, 0.0, 0.1)$ and observation plane OS $z = 0.1$

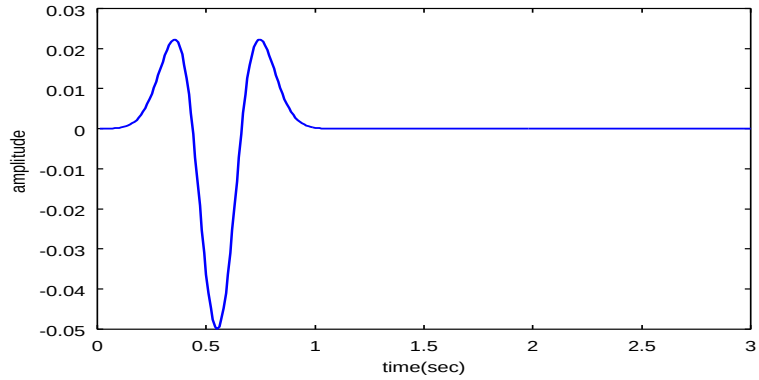


Figure 3: Time history of the input motion

Problem Setup

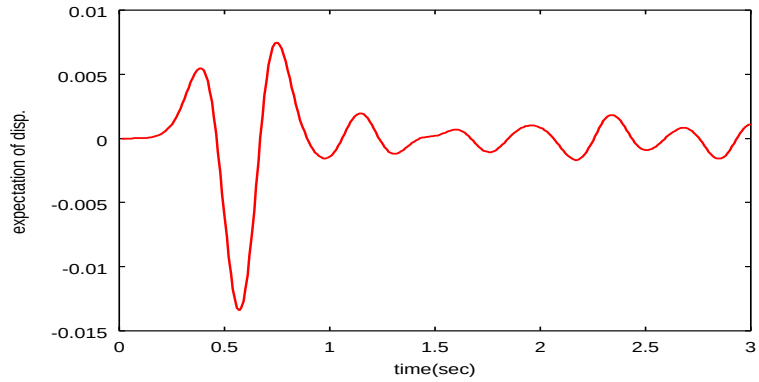
The domain of $[-0.5, 0.5]\times[-0.5, 0.5]\times[-0.5, 0.5]$ is considered and finite element model is generated by dividing the domain into $40\times40\times40 = 64,000$ elements and total degree of freedom is 207,763. **Figure 2** illustrates the model under consideration. We set an observation point OP as $(x, y, z) = (0.0, 0.0, 0.1)$ and an observation plane OS $z = 0.1$ in the model. Displacement at this point and on this plane will be discussed later.

All parameters except time are given as dimensionless. The unit weight is 1.0 and Poisson's ratio is 0.25. Shear stiffness G is assumed to have uncertainty with Gaussian distribution. Its expectation $\bar{G}$ is set as 0.1 which gives corresponding shear wave velocity as 0.1[/sec]. Correlation function of stiffness is assumed as

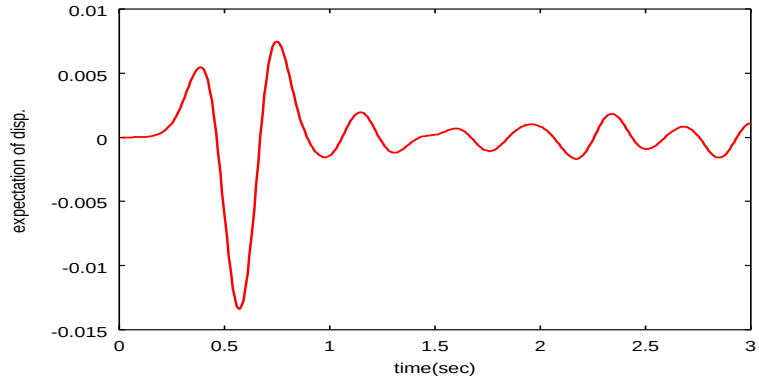
$$C(x_1, y_1, z_1, x_2, y_2, z_2) = \left(\gamma\bar{G}\right)^2 \exp\left\{-\frac{(|x_1 - x_2| + |y_1 - y_2| + |z_1 - z_2|)}{b}\right\} \tag{37}$$

where b denotes a correlation length and γ denotes a magnitude of randomness. They are assumed as $b = 1.0$ and $\gamma = 0.1$.

Ricker wavelet with a dominant frequency of 2 [Hz] is given as an explosive source at the center of the domain. The time history of input motion is plotted in **Figure 3**. Time step is set as $\Delta t = 0.01$ [sec] and analysis was conducted for 300 steps.



(a) single CPU.



(b) Parallel computation with 6 CPUs.

Figure 4: Time history of the expectation of z-component of response displacement at an observation point OP $(0, 0, 0.1)$

KL order and HC order are set as $N_{KL} = 2$ and $N_{HC} = 2$. It determines the highest order of PC as $N_{PC} = 5$ and the total number of PC to be considered is 6 and we utilize the same number (6) of CPUs in the parallel computation.

Computation Results

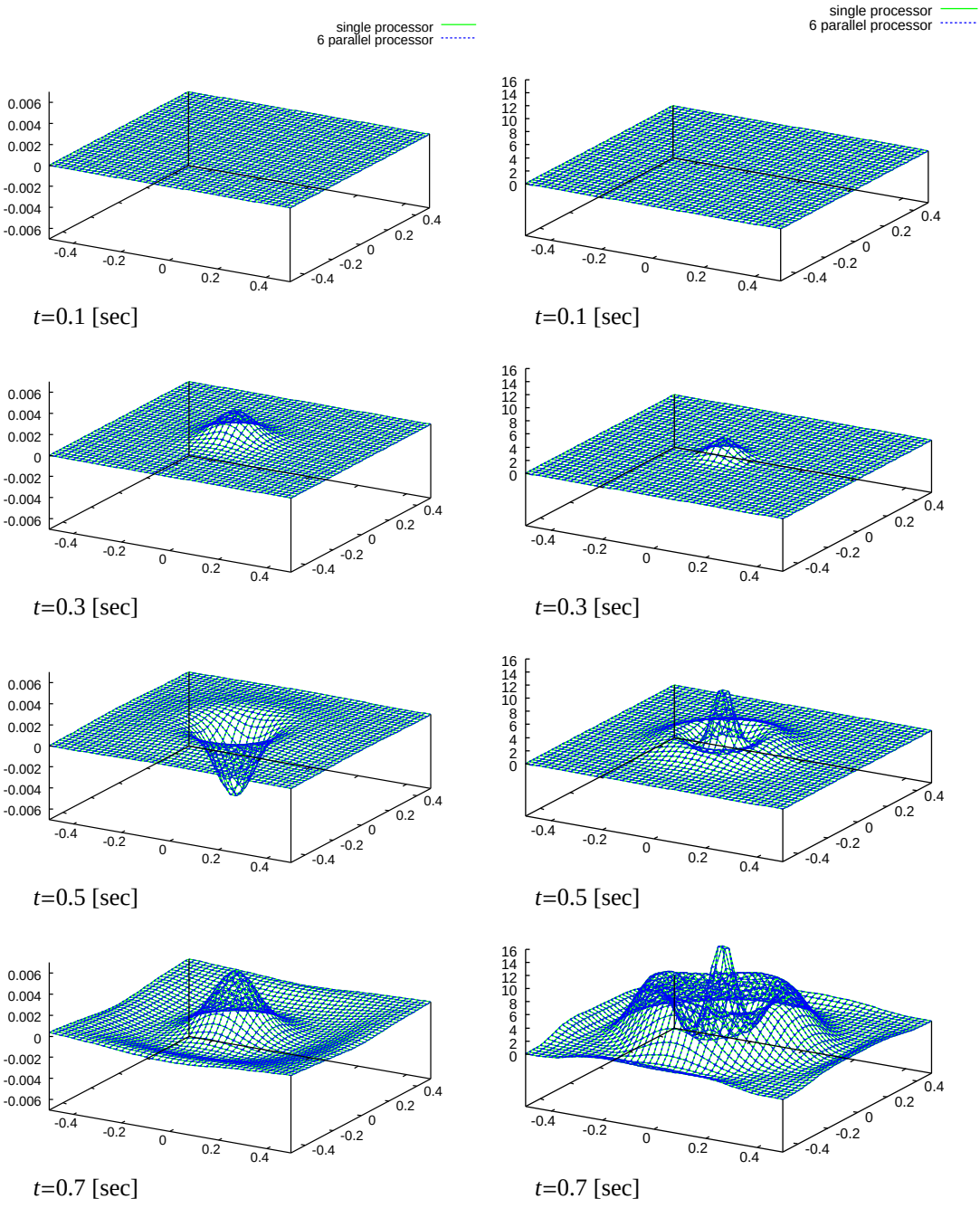
Computational Accuracy

Figure 4 compares the results obtaind by a single CPU and parallel computation. It plots the time histories of the expectation of z-component of displacement at observation point OP. It is oberved that they present good agreement.

Figure 5 shows the temporal variation of expectation and variation of z-component of displacement on the plane OS. They are plotted for four time levels : $t = 0.1, 0.3, 0.5$ and 0.7 [sec]. In the figure, computation results obtained by 6 CPU parallel computation using the proposed method is plotted with the results obtained by a single processor and there observes no difference.

To discuss the accuracy of the proposed method quantitatively, let us define an index to represent the "difference" as

$$(\text{difference}) = \left\{ \frac{\sum_1^N \{(\text{Results by 1 CPU}) - (\text{Results by 6 CPUs})\}^2}{\sum_1^N (\text{Results by 1 CPU})^2} \right\}^{1/2} \tag{38}$$



(a) Expectation (b) Variance

Figure 5: Temporal variation of the distributions of the expectation and variation of z-component of displacement on the plane $z = 0.1$. Comparison of the results obtained by a conventional single CPU scheme and 6 CPU parallel computation. They are in good agreement.

Table 1: Speedup Ratio in terms of Elapse Time

# of CPU	1	6
Elapse Time [min]	257.2	69.7
Speedup Ratio	1	3.69

Table 2: Speedup Ratio in terms of CPU Time

# of CPU	1	6
CPU time [min]	257.2	45.0
Speedup Ratio	1	5.71

where N denotes the total number of nodes. The "difference" of variance between the results by a single CPU and by 6 CPUs are of the order of 10^{-7}. It should be noticed that this "difference" index does not mean the computational error due to the parallelization of computation process. Both conventional and parallel schemes have accuracy of second order and error of that order should be considered as inherent. The difference presented above are of the smaller order and our results indicate that the proposed parallel computation algorithm for SSFEM assures sufficient accuracy for practical purposes.

Computational Efficiency

Improvement of computation efficiency is discussed by comparing the computation time. All computations are conducted on Pentium 4 processor with the clock frequency of 2.66 [GHz].

Table 1 compares the execution (elapse) time and speed-up ratio, which is defined as a ratio of the elapse time of the computation by a single CPU and that by plural CPUs. The speed-up ratio by using 6 processors is 3.06, which is a fairly good score but still considerably lower than the number of processors, 6. It should be naturally attributed to the overhead loss such as time for data communication among the processors. In our algorithm, since computation load can be fairly distributed among the processors, load imbalance is not a major reason of poor speed-up ratio.

Let us also estimate the efficiency of the proposed algorithm according to the Amdahl's law. Most of the computation process of the proposed algorithm can be parallelized and it can be observed in **Table 2** which lists CPU time for those computed cases. The table shows that speed-up ratio in terms of CPU time is 5.71 which is close to the number of CPUs. This indicates that computation (elapse) time can be further shortened by employing faster hardware for data communication and improving the program codes.

CONCLUSION

We proposed an algorithm for the parallel computation of the dynamic analysis of spectral stochastic finite element method (SSFEM) applied to the wave propagation analysis in the uncertain media.

In SSFEM, system equation is projected to a finite number of polynomial chaos which consist an orthogonal basis of a finitely truncated homogeneous chaos space. This generates a set of projected equations to be solved. The proposed algorithm treats each of these projected equations separately on different processors. The interaction among them is taken into consideration as an external force, which enables efficient computation by utilizing a non-iterative time integration (NITI) scheme [14, 15].

Accuracy and efficiency of the proposed algorithm are verified by the results of numerical simulations. Three dimensional wave propagation problem in the uncertain media is analyzed by a conventional single-

CPU scheme and the proposed parallel computation scheme. The difference of these computation results are sufficiently small and the accuracy of the prposed parallel computation algorithm was verified.

Computational efficiency is also discussed by comparing the computation time required by a single CPU computation and that by a parallel computation. Computation time is considerably reduced in terms of the elapsed time. Parallel computation with six processors required less than 30% of that required by an conventional algorithm with a single processor. Improvement of efficiency is clearly observed when they are compared in terms of CPU time. The simulation shows that six-processor computation requires the CPU time of 1/5.71 of that required by a single processor case. It indicates the possibility of further reduction of computation time.

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