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ADAPTIVE LMS METHODS FOR REAL-TIME STRUCTURAL HEALTH MONITORING OF STEEL FRAMED STRUCTURES

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SUMMARY

Structural health can be monitored by identifying changes in structural or modal parameters. However, modal parameters can be less sensitive to (localised) damage than directly identifying the changes in physical parameters of a structure. This research directly identifies changes in structural stiffness due to modeling error or damage using adaptive Least Mean Squares (LMS) filtering theory. The focus is on computational simplicity to enable real-time implementation. Several adaptive LMS filtering based approaches are used to analyse the data from the IASC-ASCE Structural Health Monitoring Task Group Benchmark Problem. Results are compared with those from the task group and other published results. The proposed methods are shown to accurately identify damage to within 1%, with convergence times of 0.4 – 13.0 seconds for the 4 and 12 degree of freedom Benchmark Problems. They are also able to identify the time at which damage occurs. The resulting modal parameters match to within 1% those from the Benchmark Problem definition. The methods require 0.14-1.40 Mcycles of computation at a 100Hz sampling rate and can therefore be readily implemented in real time.

INTRODUCTION

Structural Health Monitoring (SHM) is the process of comparing the current state of a structure to a baseline state and determining the existence, location, and degree of damage that may exist, particularly after a damaging input, such as an earthquake or other large load. Current vibration-based SHM methods are based on modal or frequency domain damage detection where changes in modal parameters, such as frequencies, mode shapes and modal damping, are a result of changes in the physical mass, damping and stiffness properties of the structure [1]. SHM can also simplify typical procedures of visual or localized experimental methods, such as acoustic or ultrasonic methods, magnetic field methods, radiography, eddy-current methods or thermal field methods [2], as it does not require visual inspection of the structure and its connections or components. Doebling et al [3] has an excellent review of the numerous different

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approaches for vibration-based damage detection methods. However, these studies apply different methods to different structures, rendering comparison difficult.

In 1999, the International Association for Structural Control (IASC) and the Dynamics Committee of the American Society of Civil Engineers (ASCE) Engineering Mechanics Division formed the SHM Task Group to study the efficacy of various SHM methods. The IASC-ASCE SHM Task Group developed a series of Benchmark SHM Problems and established a set of specific benchmark results for a specially designed test structure in the Earthquake Engineering Research Laboratory at the University of British Columbia [4]. After the Benchmark Problem was established, SHM research for civil structure concentrated on applying different techniques to this problem to examine the relative and absolute effectiveness of different algorithms.

SHM in civil structures is useful for determining the damage state of a structure. In particular, the ability to assess damage in real-time or immediately after a catastrophic event would allow Civil Defence authorities to determine which structures were safe. Current vibration-based methods are more applicable to steel frame or bridge structures where vibration response may be more linear. These problems typically have known, or reasonably estimated, input loads. However, the insensitivity of modal parameters to (localised) damage is potentially a major limitation for the methods that rely on identifying these parameters to assess and locate damage. This research uses adaptive filtering theory to assess the damage directly via changes in structural model parameters without relying on modal values.

A common method for identification of civil structural model parameters is the Eigensystem Realization Algorithm (ERA). The ERA method is based on knowledge of the time domain free response data. In ERA, a discrete Hankel matrix is formed, and the state and output matrices for the resulting discrete matrix are determined. The resulting modal parameters are found by determining the eigenvalues of this continuous time system. Dyke et al [5] use cross correlation functions in conjunction with the ERA method for identification of the modal parameters, which are used to identify frequency and damping parameters. Caicedo et al [6] introduces SHM methods based on changes in the component transfer functions of the structure, or transfer functions between the floors of a structure, and use the ERA to identify the natural frequencies of each component transfer function. Lus and Betti [7] also proposed a damage identification method based on ERA with a Data Correlation and Observer/Kalman Identification algorithm. Bernal and Gunes [8] also used the ERA with Observer/Kalman Identification for identifying modal characteristics when the input is known, and used a Subspace Identification algorithm when the input cannot be measured.

Wavelet analysis approaches for SHM and damage detection may found in Corbin et al [9] and Hou et al [10]. Damage, and the moment when the damage occurs, can be detected by a spike or an impulse in the plots of higher resolution details from wavelet decomposition of the acceleration response data. Wavelets offer the advantage of determining not only the extent of the damage but also the time of its occurrence.

A drawback of current approaches is their inability to be implemented in real-time, as the event occurs. The wavelet and ERA methods require the entire measured response to process and identify damage. Further, their reliance on modal properties, which can be subject to noise, has potential problems. In addition, modal properties have been shown in some cases, to be non-robust in the presence of strong noise and insensitive to small amounts of damage [10].

Adaptive identification methods were employed to identify modal parameters by Sato and Qi [11] and Loh et al [12]. Loh et al [12] used the adaptive fading Kalman filter technique, and Sato and Qi [11] an

Adaptive H_∞ Filter, to achieve real-time capable or near real-time capable results. What these approaches provide in real-time identification of modal parameters comes with significant computational complexity.

This paper presents the development of a much simpler and more computationally efficient algorithm than existing methods for continuously monitoring the status of a steel frame structure. Adaptive LMS filtering is employed for its computational simplicity to develop a SHM method that takes advantage of this filter's ability to adaptively model noisy signals to identify changes in structural parameters in comparison to a base structural model. The algorithms developed use a series of coupled adaptive LMS filters and are tested on the Benchmark Problem test cases.

DEFINITION OF THE SHM PROBLEM

A seismically excited structure is can be modelled using standard linear equations of motion, or a more complex computational model if desired:

$$\mathbf{M}\cdot\{\ddot{v}\}+\mathbf{C}\cdot\{\dot{v}\}+\mathbf{K}\cdot\{v\}=-\underline{\mathbf{M}}\cdot\ddot{x}_g \tag{1}$$

where **M**, **C** and **K** are the mass, damping and stiffness matrices of the model, respectively, $\{v\}$, $\{\dot{v}\}$ and $\{\ddot{v}\}$ are the displacement, velocity and acceleration, respectively, and $\ddot{x}_g$ is the ground motion acceleration. To develop this method these equations can be presented for a simple three-story shear building model, as a simple example:

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{bmatrix} \ddot{v}_1 \\ \ddot{v}_2 \\ \ddot{v}_3 \end{bmatrix}+\begin{bmatrix} c_1+c_2 & -c_2 & 0 \\ -c_2 & c_2+c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{bmatrix}\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \\ \dot{v}_3 \end{bmatrix}+\begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}=-\begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}\{\ddot{x}_g\} \tag{2}$$

where m_i, c_i and k_i are the mass, damping and stiffness coefficient for i^{th} story respectively, and v_i is relative displacement of the ith story to the ground. This system represents a linear and undamaged baseline model.

If damage occurred in the structure from an earthquake, or any other form of damaging excitation, structural properties such as stiffness will change, at least locally. These changes can be time varying or result without an input from simple modeling error. For the damaged, or mis-modeled, structure, the equations of motion can be re-defined:

$$\mathbf{M}\cdot\{\ddot{\bar{v}}\}+\mathbf{C}\cdot\{\dot{\bar{v}}\}+(\mathbf{K}+\Delta\mathbf{K})\cdot\{\bar{v}\}=-\underline{\mathbf{M}}\cdot\ddot{x}_g \tag{3}$$

where $\ddot{\bar{v}}$, $\dot{\bar{v}}$ and $\bar{v}$ are the responses of the damaged structure, and **ΔK** contains changes in the stiffness of the structure and can be a function of time. Identifying the **ΔK** matrix enables the structure's condition to be directly monitored without using modal parameters.

Per the Benchmark Problem definition this research examines changes in stiffness properties, which are the most likely to change significantly for steel frame structures. Damping changes, **ΔC**, could also be identified in a similar fashion and can occur due to hysteresis or other non-linear effects. Change in the mass matrix, **ΔM**, is not likely to be significant and is ignored.

To determine $\mathbf{\Delta K}$ with the filtering approach presented requires a new form of $\mathbf{\Delta K}$ to be defined in terms of time varying scalar parameters, α_i, to be identified using the LMS filter. For the three story example of Equations (1)–(3), the $\mathbf{\Delta K}$ matrix is sub-divided into three matrices to enable independent identification of changes in k_1, k_2 and k_3 with a 1 or -1 wherever that story stiffness appears in Equation (2).

$$\Delta\mathbf{K} = \alpha_1\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix} + \alpha_2\begin{bmatrix}1 & -1 & 0\\ -1 & 1 & 0\\ 0 & 0 & 0\end{bmatrix} + \alpha_3\begin{bmatrix}0 & 0 & 0\\ 0 & 1 & -1\\ 0 & -1 & 1\end{bmatrix} = \begin{bmatrix}\alpha_1+\alpha_2 & -\alpha_2 & 0\\ -\alpha_2 & \alpha_2+\alpha_3 & -\alpha_3\\ 0 & -\alpha_3 & \alpha_3\end{bmatrix} \quad (4)$$

where

$$\alpha_1 = \Delta k_1, \qquad \alpha_2 = \Delta k_2, \qquad \alpha_3 = \Delta k_3 \quad (5)$$

hence,

$$\mathbf{\Delta K} = \begin{bmatrix}\Delta k_1+\Delta k_2 & -\Delta k_2 & 0\\ -\Delta k_2 & \Delta k_2+\Delta k_3 & -\Delta k_3\\ 0 & -\Delta k_3 & \Delta k_3\end{bmatrix} = \sum_{i=1}^{3}\alpha_i\Delta K_i \quad (6)$$

The stiffness changes of the damaged structure can be determined by identifying the $\mathbf{\Delta K}$ matrix at every discrete time step. Rewriting Equation (3) using Equations (4)–(6) yields:

$$\mathbf{M}\cdot\{\ddot{\bar{v}}\} + \mathbf{C}\cdot\{\dot{\bar{v}}\} + \mathbf{K}\cdot\{\bar{v}\} + \sum_{i=1}^{n}\alpha_i\Delta K_i\bar{v} = \mathbf{F} \quad (7)$$

where n is the number of degrees-of-freedom (DOF) of the model and $\mathbf{F}$ is a known, or estimated, input load vector. Note that n is the maximum number of coefficients to identify in order to determine changes in each story stiffness. A lesser number can be used if some storys are assumed not to suffer damage. Similarly, a greater number could be used for a more complex structural model with more DOF per story to obtain greater resolution on the changes in structural parameters.

The varying stiffness term is simply the error between the, in this case, linear model and real measurements when actual measured values ($\bar{v}$, $\dot{\bar{v}}$ and $\ddot{\bar{v}}$) are used for $\{v\}$, $\{\dot{v}\}$ and $\{\ddot{v}\}$ in Equation (7). Hence, $\Delta\mathbf{K}\bar{v}$ is the error in the linear model.

$$\sum_{i=1}^{n}\alpha_i\Delta K_i\bar{v} = \mathbf{F} - \mathbf{M}\ddot{\bar{v}} - \mathbf{C}\dot{\bar{v}} - \mathbf{K}\bar{v} \quad (8)$$

where $\bar{v}$, $\dot{\bar{v}}$ and $\ddot{\bar{v}}$ are measured values of the structural displacement, velocity and acceleration that are obtained either directly and/or from a dynamic state estimator. Equation (8) is only valid at any point in time if the α_i have the correct values. At any discrete time, k, the difference between the linear model and actual measurements can be defined:

$$y_k = \mathbf{F}_k - \mathbf{M}\ddot{\bar{v}}_k - \mathbf{C}\dot{\bar{v}}_k - \mathbf{K}\bar{v}_k = \sum_{i=1}^{n}\alpha_i\Delta K_i\bar{v}_k \quad (9)$$

where $\mathbf{F}_k$ is the input at time k, and $\bar{v}_k$, $\dot{\bar{v}}_k$ and $\ddot{\bar{v}}_k$ are the measured displacement, velocity and acceleration at time k. The elements of the vector signal y_k can be modeled in real-time using an adaptive LMS filter so that the coefficients α_i can be determined from the resulting reduced noise modeled signal.

ADAPTIVE LMS FILTERING

Adaptive filters are digital filters with coefficients that can change over time. The general idea is to assess how well the existing coefficients are performing in modeling a noisy signal, and then adapt the coefficient values to improve performance. Because of their self-adjusting performance and built-in flexibility, adaptive filters have found use modeling signals in many real-time applications, particularly in advanced telecommunications such as cell phones. The Least Mean Squares (LMS) algorithm is one of the most widely used of all the adaptive filtering algorithms and is relatively simple to implement. It is an approximation of the Steepest Descent Method using an estimation of the gradient instead of the analytical value, considerably simplifying the calculations so it can be readily performed in real-time. The goal in this case is to model the individual, scalar elements of y_k using separate adaptive LMS filters.

In adaptive LMS filtering, the coefficients are adjusted from sample-to-sample to minimize the Mean Square Error (MSE), between a measured noisy scalar signal and its modeled value from the filter.

$$e_k = \hat{y}_k - W_k^T X_k = \hat{y}_k - \sum_{i=o}^{m-1} w_k(i) x_{k-i} = \hat{y}_k - \hat{n}_k \tag{10}$$

where W_k is the adjustable filter coefficient vector or weight vector at time k, $\hat{y}_k$ is the noisy measured signal to be modelled or approximated, X_k is vector the input to the filter model of current and previous filter outputs, x_{k-i}, so $W_k^T X_k$ is the vector dot product output from the filter to model a scalar signal $\hat{y}_k$, and m is the number of prior time steps or taps considered. Note that more taps makes the algorithm converge faster and improves accuracy, however it requires more computation. The Widrow-Hopf LMS algorithm for updating the weights to minimise the error, e_k is defined [13]:

$$W_{k+1} = W_k + 2\mu \cdot e_k \cdot X_k \tag{11}$$

where μ is a positive scalar that controls the stability and rate of convergence. The general computational procedure is summarized in Ifeachor and Jervis [13].

ADAPTIVE LMS FOR SHM

Two-Step Method:

The individual, scalar elements of the noisy signal vector, y_k, defined in Equation (9), can be modeled using multiple adaptive LMS filters.

$$y_k = \begin{bmatrix} (\hat{y}_k)^1 \\ \vdots \\ (\hat{y}_k)^n \end{bmatrix} = \begin{bmatrix} (W_k^T X_k)^1 \\ \vdots \\ (W_k^T X_k)^n \end{bmatrix} \tag{12}$$

where each different weight matrix, W_k^T, is updated individually for n different signals, and $\left(W_k^T X_k\right)^i$ is the output for the ith individual adaptive LMS filter. In the Two Step method, adaptive LMS filters model each element of the noisy signal and from Equation (9), the filter approximation is defined:

$$y_k = \sum_{i=1}^{n} \alpha_i \Delta K_i \bar{v}_k = \left[\Delta K_1 \bar{v}_k \quad \cdots \quad \Delta K_n \bar{v}_k\right] \alpha_k = A \alpha_k \tag{13}$$

where dimensions of the matrix A are $n \times n$ and α_k is a $n \times 1$ vector of coefficients α_i at time k. Therefore, values for α_i can be determined analytically by solving Equation (13) at each time step as long as the matrix, A, is full rank. For a moderate number of DOF the computation may not be too excessive.

The Two Step method is a fast and simple approach, and is also robust to noise because of its direct use of LMS filters. However, the computation required is more intense than desired due to the matrix solutions required each time step, particularly as the number of DOF rises. What is required is a method without the matrix solution.

One-Step Method:

The One Step method developed combines the two steps of noisy signal modeling and coefficient solution. The linear model error at the kth time step, between the measured noisy signal and its modeled value from the filter defined in Equation (10), can be expressed:

$$e_k = y_k - \sum_{j=0}^{m-1} \sum_{i=1}^{n} \alpha_{ij} \Delta K_i \bar{v}_k = y_k - Q_k \tag{14}$$

where $\bar{v}_k$ and y_k are noisy signals, Q_k is a $n \times 1$ vector, α_{ij} are weights where $i = 1, \ldots, n$ and $j = 1, \ldots, m$. Hence, the change in the ith story stiffness, k_i, will be the sum over j of α_{ij}. This averaged approach essentially low pass filters the signal $\bar{v}_k$ and further reduces the impact of noise. An exact unfiltered solution would simply use $m = 1$. Note that there are no prior time steps involved when estimating error at time k, because y_k is not stationary and adaptive LMS based algorithms are not effective in the presence of non-stationary signals [13]. In addition, the error, e_k, in Equation (14) is the error at this time step, which is a function of the response at time k only.

Hence, the mean square error (MSE) can be defined:

$$e_k^T e_k = y_k^T y_k + Q_k^T Q_k - 2 y_k^T Q_k \tag{15}$$

Since, adaptive LMS minimises the MSE with respect to the weights α_{ij}, the optimum solution occurs when the gradient of MSE is zero. The weight matrix of dimension $n \times m$ can therefore be updated using the derivation in Ifeachor and Jervis [13] and Equation (14) for the error.

$$w_{k+1} = w_k - \mu \nabla MSE = w_k + 2\mu\left[e_k^T \Delta K_i \bar{v}_k\right] \tag{16}$$

where the elements of the matrix in the second term, $e_k^T \Delta K_i \bar{v}_k$, is the same for all m elements in the ith row of the $n \times m$ matrix that defines the change in weight matrix values for that time step.

Variations of this algorithm include determining the gradient without the coupling of the ΔK_i matrices in Equation (16). Decoupling the gradient estimation by approximating ΔK_i as a zero matrix with a 1.0 for the (*i*,*i*) element results in the following weight update formula:

$$w_{k+1} = w_k + 2\mu\left[e_k^T(i)\bar{v}_k(i)\right]_{n\times m} \tag{17}$$

where the matrix elements, $e_k^T(i)\bar{v}_k(i)$, are the same for all *m* elements in row *i* of the matrix in the second term. Without these coupling terms the gradient is calculated based only on changes in diagonal elements of the stiffness matrix reducing the likelihood of coupling terms reducing the gradient near zero error values. Note that the error term defined in Equation (14) still includes the matrix ΔK_i, and Equation (17) only changes the means by which weights are updated.

APPLICATION TO THE BENCHMARK PROBLEM

The IASC-ASCE task group on SHM was established in 1999 and the group developed a series of benchmark SHM problems [4]. The benchmark structure is 4-story, 2-bay by 2-bay steel-frame scale structure. It has a 2.5m × 2.5m plan and is 3.6m tall. The task group analysed the structure using two models, one of 12 DOF and a second of 120 DOF. In this paper, only the 12 DOF model and a simplified one direction, 4 DOF model are considered. However, the methods presented are directly applicable to this more complex 120 DOF case. Per the algorithms presented, the displacement, velocity and acceleration at each DOF are assumed to be measured or estimated and include noise as defined in the Benchmark Problem. The input loads in the Benchmark Problems studied are assumed known.

Table 1 shows the cases and damage patterns of the Benchmark Problem considered in testing of the adaptive LMS-based methods developed. The four damage patterns are applied to each case in the table. Case 2 uses the 120 DOF model and is not considered here. The task group also provided the MATLAB® based data generation code for the model [14].

Table 1 Cases and Damage patterns considered in simulation

Case		Damage Pattern	
1	12 DOF, symmetric, loads on all floors	1	All braces in 1st story removed
3	12 DOF, symmetric, load at the roof	2	All braces in 1st & 3rd stories removed
4	12 DOF, asymmetric, load at the roof	3	One brace in 1st story removed
		4	One brace in 1st & 3rd stories removed

The following parameters are used in all simulations, unless otherwise stated: input load(s) = $1\times10^6 \sim 10^7 \sin(30t)$ N, sample rate = *100 Hz*, $\mu = 0.3$, number of taps, $m = 5$. The convergence rate of the weights in the algorithm depends on the LMS parameter μ and the number of taps used. Even though faster convergence for each different case of the Benchmark Problem can be achieved by varying those parameters, they would typically be fixed in a practical application. The values used here were developed by trial and error to illustrate the methods developed and may not be optimal.

The One Step method, using Equation (17) for uncoupled weight updating, was used throughout the tests presented because it is computationally the simplest. For cases 1 & 3 and damage patterns 1 & 2, the 12 DOF model can be approximated as a 4 DOF model (one DOF per floor), because it is symmetric and deforms only in the loading direction (y-direction). In the 4 DOF model, there are four α_i coefficients and four sub-matrices for ΔK_i, and each α_i represents the change in stiffness of each floor similar to the 3 DOF definition in Equation (5).

Figure 1 shows the results for the One Step method on the 4 DOF model for case 1 with damage pattern 1. The test was carried out for two different situations, a sudden failure in the structure at 5 seconds shown in Figure 1(a) in the left frame, and gradual damage that starts at 5 seconds and is fully damaged at 10 seconds as shown in Figure 1(b) in the right frame. The actual changes in stiffness due to changes in the weights are plotted in each case. The solid line represents the actual stiffness change during the simulation and the dashed line indicates the value, from the α_i, from applying this adaptive LMS based method. All braces at the first floor are removed in damage pattern 1, hence, only α_1 changes. The other values correctly remain at zero. The estimated α_1 value reached 90% of the actual change within 1 second after the damage occurred. Interestingly, the gradual damage case takes relatively longer to converge in Figure 1(b), which is likely due to the difficulty LMS-based methods have in tracking non-stationary changes and signals.

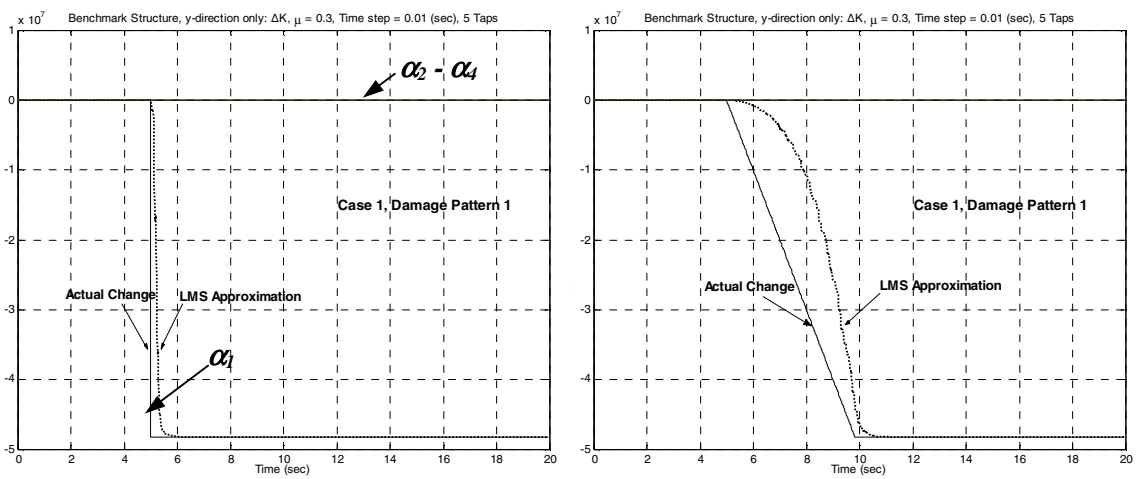

Figure 1: Identifying changes of α by adaptive LMS of 4 DOF model for Case 1 with Damage pattern 1 of the Benchmark Problem (a) step change (sudden failure) and (b) ramp change (gradual failure).

For case 3 and damage pattern 2 in Figure 2(a), the 4 DOF model was used as the stiffness in the 1st and 3rd floors are changed due to removing all of the braces in these floors, including changes in α_1 and α_3. For comparison, case 3 with damage pattern 4 was simulated using the 12 DOF model, as shown in Figure 2(b). Damage pattern 4 has partial damage in the 1st and 3rd floors and requires changes in four α_i coefficients to accurately model the similar damage in this larger model. The longest convergence time is about 10 seconds to reach 90% of the actual changes. Increasing μ and/or the number of taps will reduce this convergence time. Assigning μ_i individually to each DOF will result in even faster convergence rates. Note that during convergence, α_i values that are correctly zero valued are non-zero for a brief time while the correct solution is being found by the adaptive filter.

Figure 3 shows the results for case 4 with damage pattern 1 and case 1 with damage pattern 4 both of which were simulated with the 12 DOF model. As shown in Figure 3, because there are three DOF per floor in the 12 DOF model, the damage in the first floor requires changes in three α_i values. Hence, in Figure 3(a), two more α_i values are changing than when the 4 DOF model is used in Figure 1 for the same damage pattern. The results in Figures 1-3 show that the adaptive LMS algorithm can directly identify the stiffness changes in the structure for these selected, different Benchmark Problem cases.

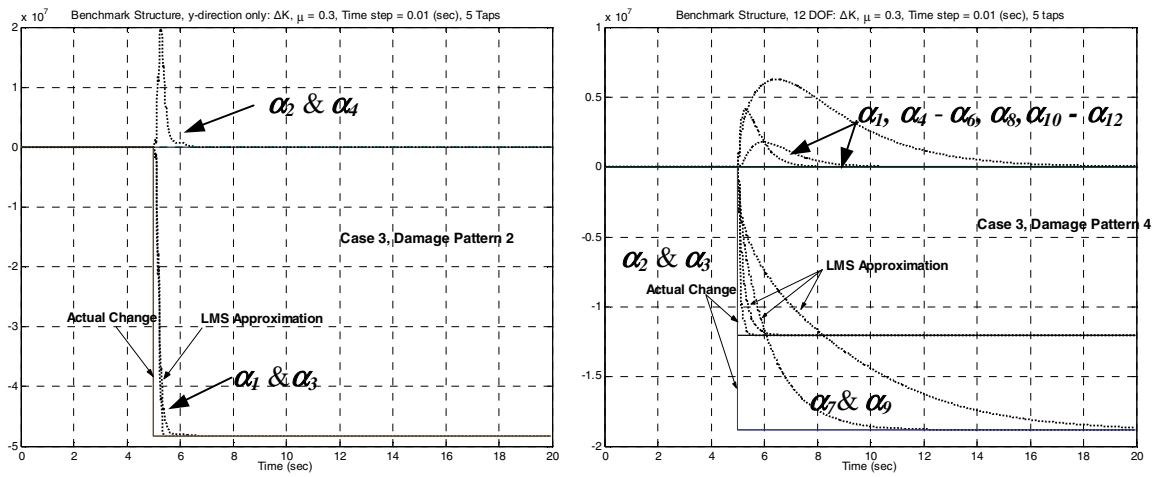

Figure 2: LMS approximations for case 3 with (a) damage pattern 2 and (b) damage pattern 4 of the Benchmark Problem.

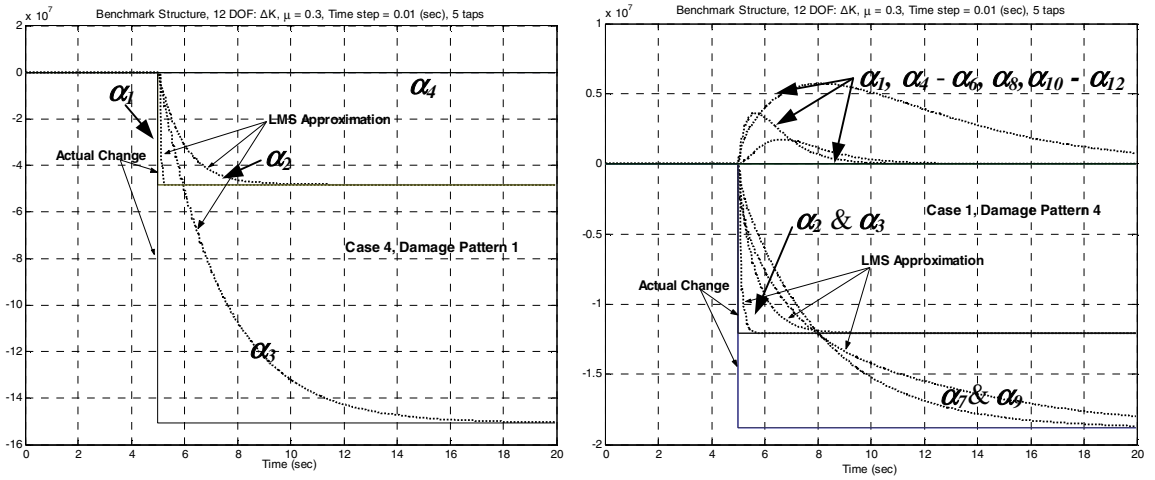

Figure 3: Adaptive LMS approximations for (a) case 4 with damage pattern 1 and (b) case 1 with damage pattern 4 of the Benchmark Problem

Table 2 lists the identified natural frequencies for all cases and damage patterns presented using the 4 DOF model compared to Benchmark Task group and published results. For the One Step adaptive LMS based method the resulting net stiffness and modeled mass matrices are used to find the natural frequencies. The Kalman method results in Table 2 are from Bernal and Gunes [8] and the Two-stage results are from Au et al [15]. The identified natural frequencies using the adaptive method presented are

within 1% of the Benchmark and as good as, or better, than the other published results. Finally, similar errors of less than 1% are obtained for all 12 modes of the 12 DOF model used for case 4 damage patterns 1-4, when compared to the results from Barroso and Rodriguez [16] and the SHM Task Group [14]. To the best of the authors' knowledge, none of the published articles have given results for cases 1 and 3 with damage patterns 3 and 4 except the SHM Task group [14], and in this case the resulting frequencies are also within 1% of the benchmark results for every mode.

Table 2: Identified natural frequencies (*Hz*) using the 4 DOF model

	Case 1							
	No Damage				Damage pattern 1			
Mode	Bench-mark	**LMS**	Kalman	Two-stage	Bench-mark	**LMS**	Kalman	Two-stage
1	9.41	9.41	9.41	9.41	6.24	6.24	6.24	6.24
2	25.54	25.54	25.54	25.6	21.53	21.53	21.53	21.6
3	38.66	38.66	38.67	38.9	37.37	37.37	37.58	37.6
4	48.01	48.01	48.01	48.4	47.83	47.83	47.83	48.2

	Case 1				Case 3			
	Damage pattern 2				No Damage			
Mode	Bench-mark	**LMS**	Kalman	Two-stage	Bench-mark	**LMS**	Kalman	Two-stage
1	5.82	5.82	5.82	5.83	9.41	9.41	9.41	9.42
2	14.89	14.89	14.89	14.9	25.54	25.54	25.53	25.6
3	36.06	36.06	36.06	36.3	38.66	38.66	38.66	38.9
4	41.35	41.35	41.35	41.6	48.01	48.01	48.09	48.5

	Case 3							
	Damage pattern 1				Damage pattern 2			
Mode	Bench-mark	**LMS**	Kalman	Two-stage	Bench-mark	**LMS**	Kalman	Two-stage
1	6.24	6.24	6.23	6.24	5.82	5.82	5.79	5.80
2	21.53	21.53	21.52	21.6	14.89	14.89	14.91	15.0
3	37.37	37.37	37.44	37.7	36.06	36.06	37.44	36.4
4	47.83	47.83	47.94	48.3	41.35	41.35	47.34	41.5

Table 3 compares the convergence time between the One Step method without coupling using Equation (17), the One Step method with coupling using Equation (16), and the Two Step method. Convergence time is measured as the time for α_1 (change in stiffness of the first floor in y-direction) to reach 90% and 95% of the actual change from when the damage first occurred. The Two Step method converges faster than the One Step method, because each element of the vector y_k is modeled individually and the individual filters converge more quickly than coupled filters.

The convergence times are faster when no coupling terms are involved in calculation of the gradient particularly for more complex cases, such as damage patterns 3 and 4. Using the One Step method without coupling, the stiffness change in the first floor, α_1 converges within 0.41 seconds for all cases and damage

patterns tested. However, with coupling the maximum time is 13.21 seconds. For damage patterns 1 and 2 the convergence times, particularly to 90%, are similar. The divergence of results between 90% and 95% for damage patterns 1 and 2 shows how the coupling in Equation (16) affects the gradients near zero error for the LMS-based One-Step methods based on approximate gradient estimates [13]. Hence, the method without coupling using Equation (17) might be more suitable for adaptive control applications given its much faster rate of convergence.

Table 3: Convergence (in seconds) to 90 and 95 % of the actual change of α_l, due to sudden failure

		One Step method with coupling		One Step method without coupling		Two Step method	
Case	Damage Pattern	90 %	95 %	90 %	95 %	90 %	95 %
1	1	0.39	0.59	0.33	0.41	0.20	0.21
	2	0.35	0.36	0.31	0.33	0.21	0.22
	3	3.23	9.11	0.11	0.12	0.20	0.21
	4	5.46	11.53	0.31	0.34	0.21	0.22
3	1	0.39	1.59	0.33	0.41	0.20	0.21
	2	0.35	3.26	0.31	0.33	0.21	0.22
	3	2.99	6.70	0.21	0.32	0.08	0.08
	4	5.14	7.37	0.21	0.32	0.08	0.08
4	1	0.33	0.35	0.21	0.22	0.09	0.01
	2	0.23	0.54	0.13	0.15	0.21	0.23
	3	4.44	12.91	0.29	0.32	0.22	0.28
	4	6.42	13.21	0.29	0.32	0.21	0.28

The convergence rate in adaptive LMS applications significantly depends on the sampling rate. Figure 4 shows the results for case 3 and damage pattern 1 with different sampling rates. Convergence time declines significantly with higher sampling rate for sampling rates that can be obtained using modern sensor and data acquisition equipment. The trade-off is that the computations must occur within a far smaller time step, placing a greater requirement on the computational components.

From an examination of the One Step method, there are, conservatively, 1400 single cycle operations per time step, including memory storage and retrieval for a 4 DOF model. If a sampling rate of 100 Hz is used, then 0.14 MHz (or mega-cycles) of computation is required and 1.4 MHz are required for a sampling rate of 1000 Hz. The 12 DOF model requires approximately 3 times more computational effort. A current Digital Signal Processing (DSP) chip operates at 300–1000 MHz. At a single operation per chip clock cycle, and many such chips have up to four or more operations per cycle [e.g. 17-19], computation of the One Step method is well within this range. The Two Step method would involve approximately ten times more computation due to the matrix solutions required. Therefore, SHM for civil structures using the adaptive LMS filtering based methods as presented could be readily implemented in real-time without significant computational simplifications or parallelization.

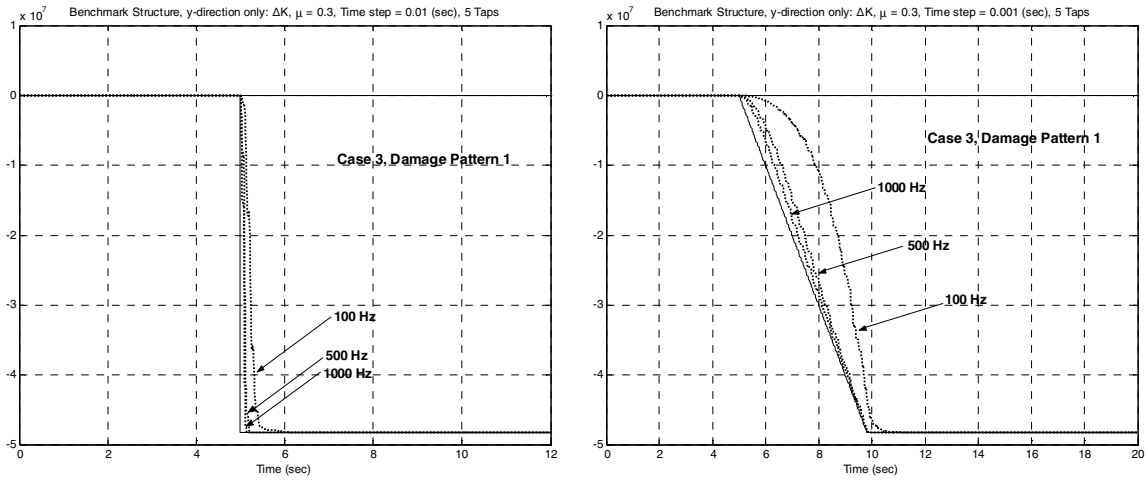

Figure 4: Adaptive LMS approximations for case 3 and damage pattern 1 of the Benchmark Problem (only α_l changes) with sampling rates of 100, 500 and 1000 Hz for (a) step change and (b) ramp change

CONCLUSIONS

This paper presents SHM methods for civil structures developed from adaptive LMS filtering theory. Damage that occurs in the structure is directly identified as changes in the stiffness matrix. One Step and Two Step adaptive LMS-based methods are developed and tested. All of the 4 and 12 DOF cases of the ASCE-IASC SHM Task Group's Benchmark Problems were tested using the proposed methods, and the results show that the adaptive LMS filtering is effective for identifying damage in real-time.

The method developed that ignores coupling terms in the gradient calculation is seen to converge the faster than those that include this term. The computationally more intense Two Step Method converges fastest as each individual element of the error signal is modeled with its own LMS filter. However, the final results for all methods converge to the desired final values. In each case, the changes in stiffness are determined directly and then the modal parameters are calculated for comparison with other methods, which primarily focus on determining these modal values. The resulting modal parameters are well within 1% of the IASC-ASCE Benchmark Problem results and all others in the literature.

The methods presented require 0.14–1.4 Mcycles of computation for a 4 DOF model and can operate on a sample to sample basis without requiring the entire sensor record. Hence, they are suitable for real-time implementation. The One Step method without coupling in the gradient calculation has convergence times for the Benchmark Problem under 0.41 seconds for the cases studied, making it suitable for adaptive control applications. Finally, the convergence times of the adaptive LMS methods presented improve as sampling rate increases from the 100Hz of the Benchmark Problem to a still practicable value of 1000Hz. Overall, these methods provide accurate, robust identification of damage with stability, little computational cost, and fast convergence.

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