



**IS THE PHASE OF THE ONE-SIDED AUTOCORRELOGRAM OF THE  
HORIZONTAL COMPONENTS OF AMBIENT VIBRATIONS  
(TOKESHI'S METHOD) ABLE TO REVEAL THE FUNDAMENTAL  
RESONANCE FREQUENCY OF A SITE ?**

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**SUMMARY**

Ambient vibration techniques for site effect studies are often limited to spectral amplitudes. Tokeshi [1] analysed the phase of one-sided autocorrelograms of horizontal components, to derive the fundamental frequency. Performing noise simulations, Tokeshi [2] concluded that this technique is more efficient in pointing out the fundamental frequency. Lelong [3] found it not reliable with recorded ambient vibrations. Tokeshi's method was checked on ambient vibrations recordings using comparisons with Nakamura's technique. Tokeshi's method does not always give the fundamental resonance frequency. It seems to work reliably only when ambient vibrations are white noise, and cannot be recommended for site effect studies.

**INTRODUCTION**

The European research project SESAME is aimed at testing, improving and standardising ambient vibration techniques that can be used as a tool for seismic site effect studies. The most common of these techniques is known as H/V technique, often referred to as Nakamura's technique (Nakamura [4]).

The H/V technique consists of the following procedure: The Fourier transform of the horizontal components of measured ambient vibrations is divided by the Fourier transform of the vertical component. Often, one or more peaks can be observed on the resulting H/V ratio. The majority of the seismological community agrees today that the peak at the lowest frequency gives a more or less reliable estimation of the fundamental eigen-frequency of the site under consideration (Bard [5]). In many cases of practice, however, there is no clear peak identifiable on the H/V ratio even where a clear site effect is expected. It is not well known under which conditions this happens, but it seems that this situation occurs particularly in the cases of weak impedance contrasts or pronounced lateral heterogeneity of soil conditions.

Tokeshi [1] proposed another method to determine fundamental eigen-frequencies from the same ambient vibration recordings as those used by Nakamura's method. Tokeshi calls this new technique "Fourier phase spectral method". It is henceforth referred to as "Tokeshi's method" (see paragraph 2), too. Tokeshi

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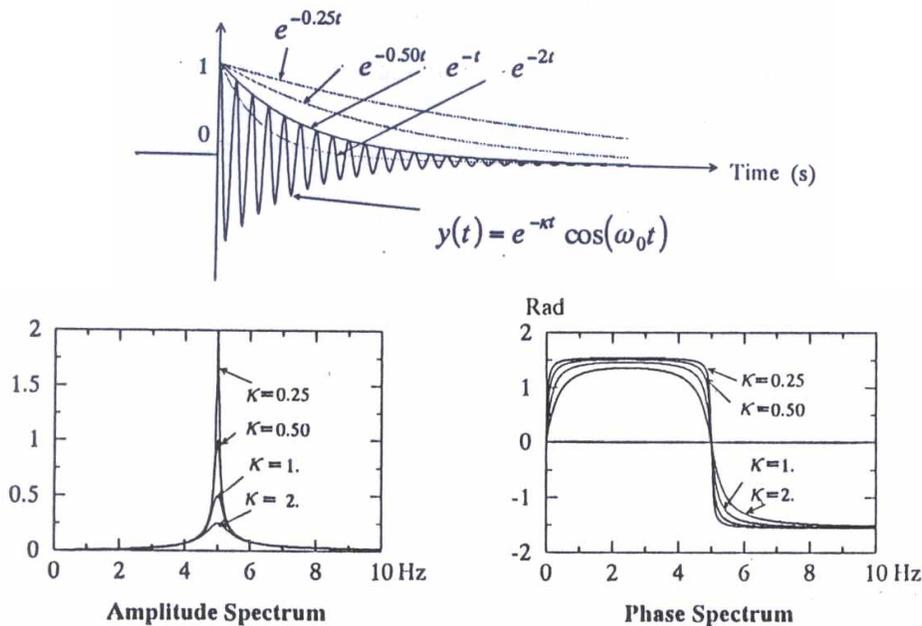
has succeeded in determining fundamental eigen-frequencies of simulated (Tokeshi [6]) and recorded ambient vibrations (Tokeshi [1]). According to Tokeshi, this method is more stable and gives results clearer than Nakamura's technique. However, Lelong [3] checked Tokeshi's method and concluded that it does not give reliable estimations of the fundamental eigen-frequencies with recorded ambient vibrations.

The aim of this study was to check Tokeshi's method with real recordings of ambient vibrations. These recordings proceed from several sites situated in Switzerland, Greece and France, where a site effect is expected. They were chosen in order to perform systematic comparisons between results found with Nakamura's technique, or other seismological and geophysical methods, and Tokeshi's technique. In particular, some examples were chosen in order to determine whether it was possible to find fundamental eigen-frequencies with Tokeshi's method where no clear peak was noticeable on the H/V ratio.

### THE FOURIER PHASE SPECTRAL METHOD (TOKESHI'S METHOD)

#### Methodology

This method uses the phase spectrum of a part of the autocorrelation of the signal. Indeed, the autocorrelogram of ambient vibration records is the power spectrum of the signal. Because this quantity is a real function, its phase is zero. Moreover, the autocorrelogram is an even function (symmetrical function) and so it is possible to study just half of this autocorrelogram, for instance for the range of positive abscissas, applying zero to negative abscissas ("one-side autocorrelogram"). Because only half of this quantity is taken into account, an imaginary part appears in its Fourier transform (see Figure 1, for the case of a stationary time function). According to Tokeshi, there is a concordance between the first zero of the phase spectrum of this one-side autocorrelogram, which corresponds to the maximum of the Fourier transform of the signal, and the fundamental eigen-frequency of the site. And so, the first intersection of the phase spectral curve with the abscissa axis should indicate the fundamental eigen-frequency of the site under consideration.



**Figure 1: Top: Right-side autocorrelogram for the case of a stationary time function. Bottom: Amplitude spectra and phase spectra of the right-side autocorrelogram (after Tokeshi [1]).**

In practice, Tokeshi's program (Tokeshi [2]) uses a corrected horizontal signal of ambient vibrations with a zero amplitude average in order to avoid some errors, especially for low frequencies. Then, the autocorrelogram of this record is performed and only the "positive" part ("right part") of the signal is taken into account. The spectrum phase of this "one-side autocorrelogram" is obtained thanks to a Fast Fourier Transform (FFT). After a smoothing operation the first intersection of the phase spectrum with the abscissa axis is looked for. Figure 2 shows the data processing procedure.

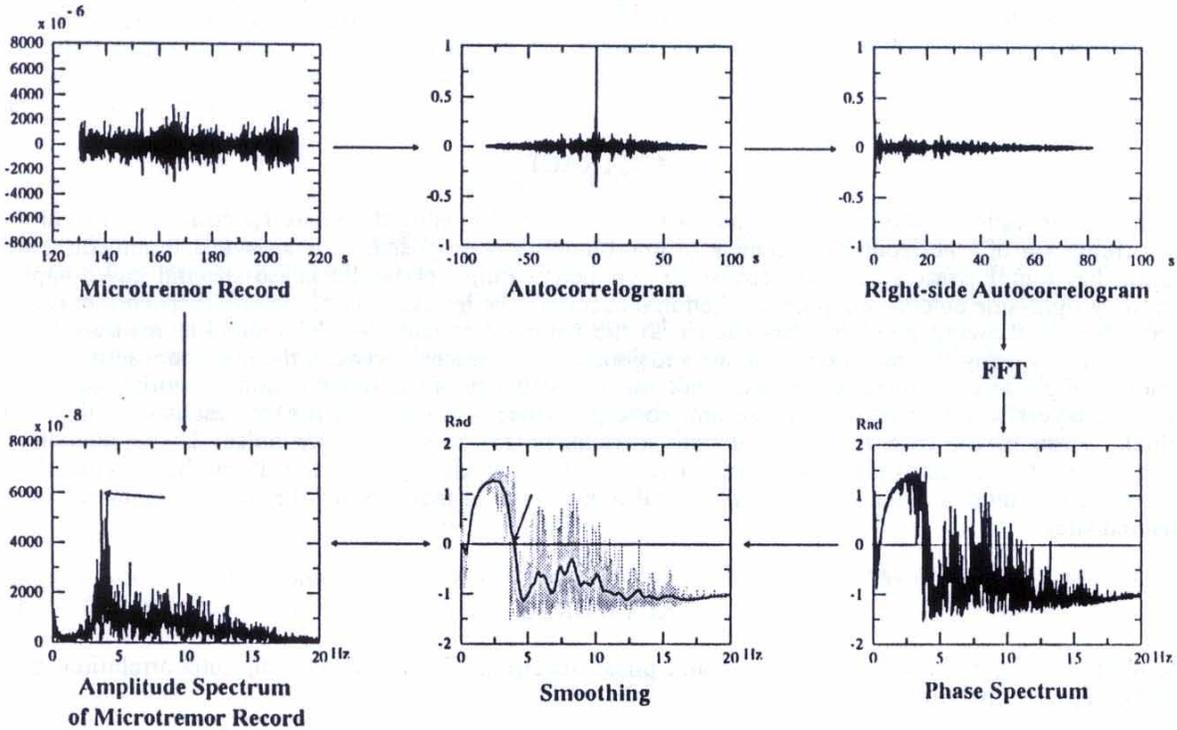


Figure 2: Data processing procedure for the Fourier phase spectral method (after Tokeshi [1]).

### Data processing

Several records of ambient vibrations in different format and with different time sampling were collected. These records last from 12 minutes to one hour. The two horizontal components (East-West and North-South) and the vertical component were used. For all the records, the same data processing was applied with the following stages:

- transformation of the original ambient vibration recordings in ASCII format,
- selection, for the three components, of 12 files of one minute each,
- calculation of the Fourier phase spectrum and the H/V ratio using Tokeshi's program,
- representation of the results in SAC format.

### Data sets used in this study

The entire data set consists of 22 recordings from Switzerland, 6 recordings from Greece and 14 recordings from France. The composition of this data set is summarised in Table 1.

**Table 1: Summary of the data set.**

Country	Site names	Station names	Number of records	Sampling rate [s]	Comments
Switzerland	Belp	S02	1	0.02	Weak rain
Switzerland	Saint-Aubin	S04, S05	2	0.02	Wind
Switzerland	Weinfelden	S03, S04	2	0.02	Weak wind
Switzerland	Chachberg	S01, S04	2	0.02	-
Switzerland	Bülach	S02, S04	2	0.02	-
Switzerland	Salez	S04	1	0.02	-
Switzerland	Eglisau	S04, S05	2	0.02	-
Switzerland	Blickensdorf	S03, S05	2	0.02	-
Switzerland	Wannenholz	S02	1	0.02	-
Switzerland	Mels	S05	1	0.02	-
Switzerland	Wichelsee	S02, S04	2	0.02	-
Switzerland	Zug	S01, S02	2	0.02	In town
Switzerland	Viège	S02	1	0.02	In town
Switzerland	Genève	S02	1	0.02	In town
Greece	Volvi	STC, TST	6	0.01	TST in the valley STC on the edge (weathered rock)
France	Grenoble	ARGO, ECO, MARA, MINI, PARO, ROBE, WASH	11	0.048 (day 105) 0.064 (day 90)	Day 105 : Stormy Day 90 : Anticyclonic
France	Grenoble	Mistral, Campus, Taillat	3	0.032 0.016	-

**Switzerland**

As part of other projects, for 23 sites in Switzerland where site effects were expected, ambient vibrations were recorded at 5 to 6 measurement points. From all these records, 21 were selected according to criteria explained later. Originally these measurements were performed to determine the fundamental eigen-frequencies of the sites using Nakamura's technique. Looking at the H/V ratios obtained for this Swiss data set, a few cases where it was possible to see a clear peak were kept. For all the other recordings it was not possible to determine with certitude a fundamental eigen-frequency with Nakamura's technique.

**Greece**

Data from the EUROSEISTEST strong-motion array at Volvi were used. The data set was provided by N. Theodulidis from ITSAK and was composed by three ambient vibration measurements and by two dozens of weak and strong motions for the same two sites at the centre and at the edge of the Volvi valley. Spectral ratios and H/V ratios were calculated with strong motion records in order to determine the fundamental eigen-frequency of the valley, taken as a reference. Then, H/V ratios and Tokeshi's Fourier phase spectra were performed from ambient vibration recordings. The resulting fundamental eigen-frequencies were compared with the reference frequency for the two sites.

**France**

The recordings stem from an ambient vibrations network and from the own database of the LGIT Grenoble. The geology is well known in this zone, so it was possible to determine the fundamental eigen-frequencies of the soil at different sites with geophysical techniques and compare them with the frequencies found with methods using ambient vibrations. 11 recordings from 7 stations of the network

installed near the LGIT were used. From these 11 ambient vibration recordings, 6 were performed during a day with anticyclonical weather conditions and 5 were recorded at the same stations during a stormy day. The frequency content of the ambient vibrations is very different during these two days. Finally, 3 recordings stemming from sites in the Grenoble valley (Campus, La Taillat, Mistral) were also used.

## RESULTS

### Classification of the results

The results have been sorted into 5 categories:

- Category 1: Nakamura's technique and Tokeshi's one give essentially the same result (a difference of less than 10% can be noted between the fundamental eigen-frequencies found with the two techniques),
- Category 2: the two methods give different results (more than 10% of difference),
- Category 3: only Tokeshi's technique gives a clear result,
- Category 4: only Nakamura's technique gives a result,
- Category 5: neither of the methods gives a result.

A summary of the results obtained with this classification is shown in Table 2. A question mark indicates that the eigen-frequency is uncertain or could not be determined.

**Table 2: Summary of the results.**

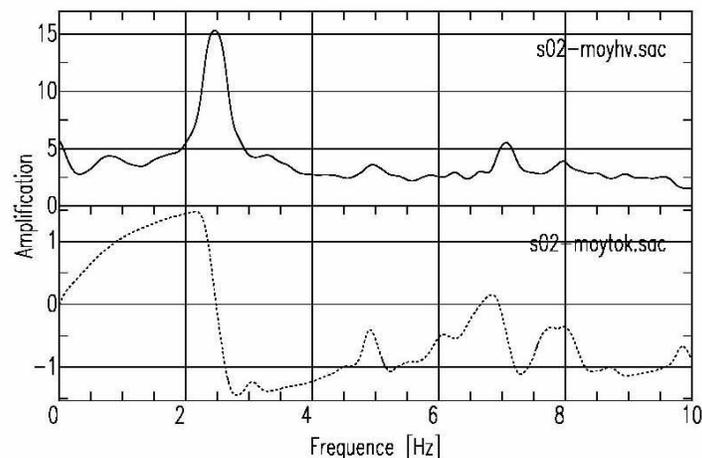
Country	Sites names	Stations names	$f_{0 \text{ Nakamura}}$ [Hz]	$f_{0 \text{ Tokeshi}}$ [Hz]		Category
Switzerland	Belp	S02	?	1.9	2.7	3
Switzerland	Saint-Aubin	S04	1.4	1.5	2.2	1
Switzerland	Saint-Aubin	S05	0.7	0.7		1
Switzerland	Weinfelden	S03	?	?		5
Switzerland	Weinfelden	S04	2.8	7		2
Switzerland	Chachberg	S01	?	2.8		3
Switzerland	Chachberg	S04	?	3.7		3
Switzerland	Bülach	S02	3.2	2.9		1
Switzerland	Bülach	S04	2.8	2.8		1
Switzerland	Salez	S04	3.9-4.9	4.7		1
Switzerland	Eglisau	S04	?	2.9		3
Switzerland	Eglisau	S05	?	14		3
Switzerland	Blickensdorf	S03	0.5	2.8		2
Switzerland	Blickensdorf	S05	?	3		3
Switzerland	Wannenholz	S02	6-7.3	6.1	7.1	1
Switzerland	Mels	S05	?	?		5
Switzerland	Wichelsee	S02	?	?		5
Switzerland	Wichelsee	S04	?	2.8		3
Switzerland	Zug	S01	3.6	3.9		1
Switzerland	Zug	S02	1.1	2.7		2
Switzerland	Viège	S02	2.5	2.5		1
Switzerland	Genève	S02	1.5	2.3		2
Greece	Volvi	STC	?	0.3		3
Greece	Volvi	STC	3	0.3	3.3	1-2
Greece	Volvi	STC	3	0.3	3.2	1-2
Greece	Volvi	TST	0.75	0.5		2

Greece	Volvi	TST	0.75	0.3	2
Greece	Volvi	TST	0.75	0.5	2
France	Grenoble	AR1	0.3	0.3	1
France	Grenoble	AR2	?	0.3	3
France	Grenoble	EC1	0.4	0.3	1-2
France	Grenoble	EC2	0.4	0.3	1-2
France	Grenoble	MR1	0.3	0.3	1
France	Grenoble	MR2	0.3	0.3	1
France	Grenoble	MN2	0.3	0.3	1
France	Grenoble	PR1	0.3	0.3	1
France	Grenoble	RB1	0.3	0.3	1
France	Grenoble	RB2	0.3	0.3	1
France	Grenoble	WS2	2 ?	0.3	2-3
France	Grenoble	Campus	0.3	0.3	1
France	Grenoble	Mistral	2.2	2.2	1
France	Grenoble	Taillat	0.3	0.3	1

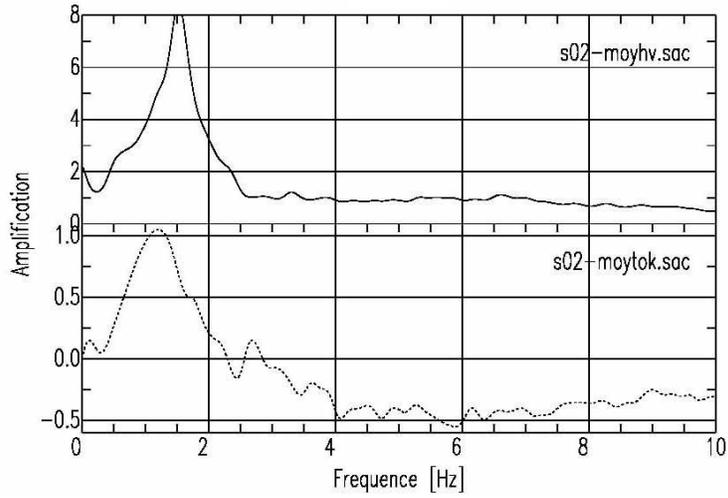
### Switzerland

No determination of the fundamental eigen-frequencies of the sites with geophysical investigations or with seismological techniques was available for this data set. Therefore, it was only possible to compare frequencies found with the two techniques without any “reference frequency” determined with other methods. Following the 5 categories defined previously, different situations were observed:

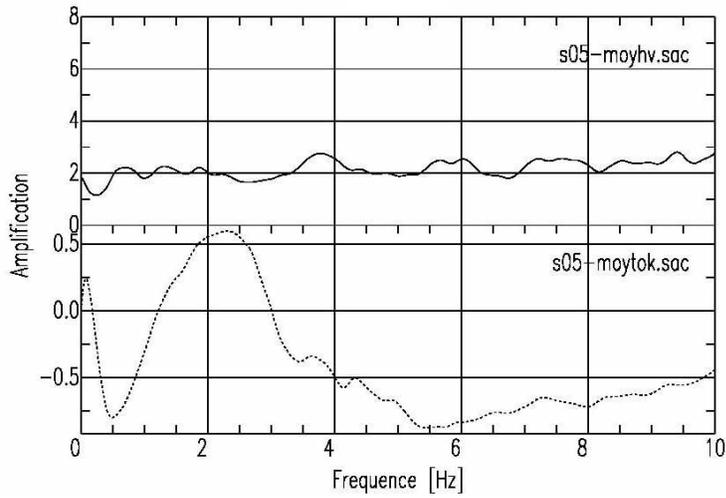
- For 8 cases, the two techniques gave essentially the same result (category 1). One example is shown in Figure 3 (point s02 at Visp).
- For 4 cases, the two techniques gave different results (category 2). One example is shown in Figure 4 (point s02 at Geneva).
- For 7 cases, no peak could be observed with Nakamura’s technique and the Tokeshi’s method gave a result (category 3). One example is shown in Figure 5 (point s05 at Blickensdorf).
- For 4 cases, it was not possible to find any result using the two techniques (category 5).
- No example has been found for the category 4: each time that Nakamura’s technique gave a result, Tokeshi’s one gave also a result (that could be, however, different from the first one).



**Figure 3: An example for Category 1 (point s02 at Visp). Top curve: H/V ratio, bottom curve: Tokeshi's method.**



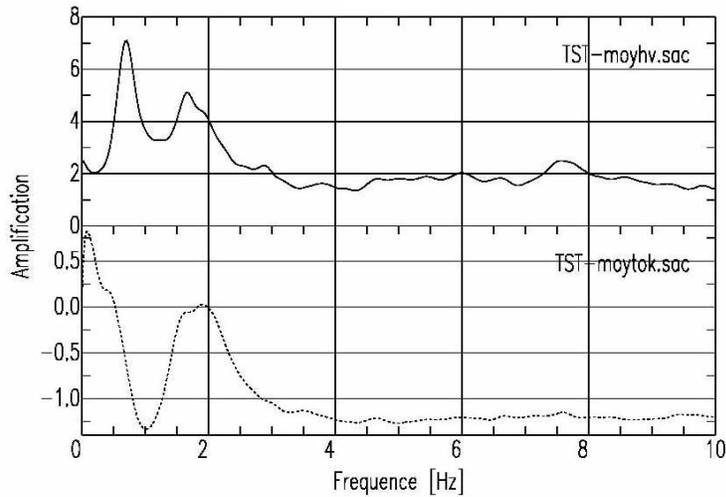
**Figure 4: An example for Category 2 (point s02 at Geneva). Top curve: H/V ratio, bottom curve: Tokeshi's method.**



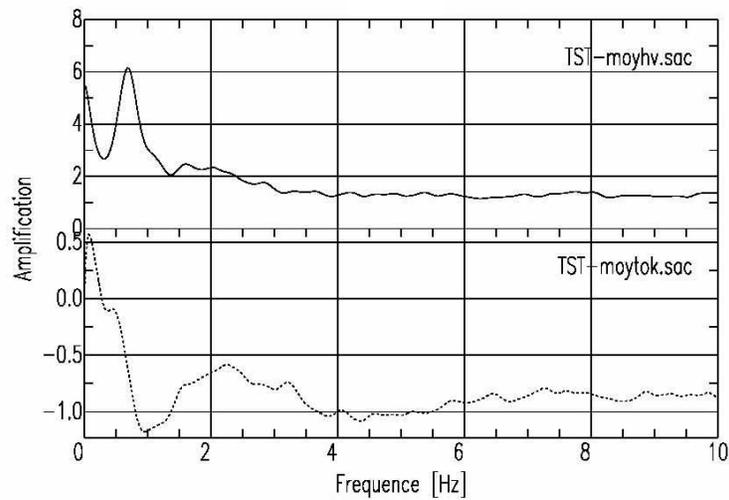
**Figure 5: An example for Category 3 (point s05 at Blickensdorf). Top curve: H/V ratio, bottom curve: Tokeshi's method.**

## Greece

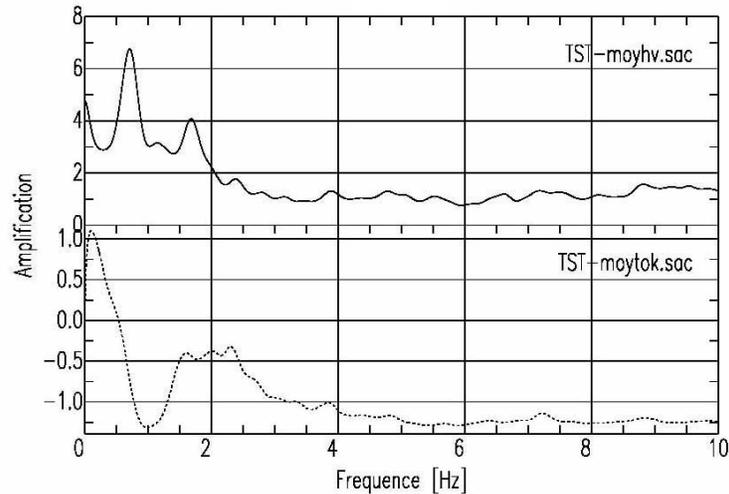
For this area, it was possible to estimate the fundamental eigen-frequency of the soil profile just below the station TST with a classical site/reference spectral ratio for a great number of earthquakes. This technique was performed with East-West and then North-South records and frequencies of 0.7 and 0.8 Hz were found. Moreover, it is possible to evaluate this frequency using the simplified formula  $f_0 = \beta_{\text{mean}}/4h$  ( $\beta_{\text{mean}}$  is the mean S wave velocity of the soil column and  $h$  is the thickness of this column). Using the velocity profile provided by ISTAK it is easy to estimate  $\beta_{\text{mean}} = \sum(\beta_i \cdot h_i)/h$  (with  $\beta_i$  S wave velocity of the layers and  $h_i$  thickness of these layers) and then  $f_0$ . In this case, the values  $\beta_{\text{mean}} = 580$  m/s and  $f_0 = 580/(4 \cdot 175) = 0.82$  Hz were determined, which agrees well with the value obtained with site/reference ratio technique.



**Figure 6: First record for the TST site (centre of the Volvi valley)**



**Figure 7: Second record for the TST site (centre of the Volvi valley)**



**Figure 8: Third record for the TST site (centre of the Volvi valley)**

With the H/V ratio technique using ambient vibrations, for the 3 records at the TST site, a frequency of 0.75 Hz (+/- 0.1 Hz) could be determined. As it can be seen on Figures 6, 7 and 8, Tokeshi's technique indicates frequencies of 0.5, 0.3 and 0.5 Hz for these 3 records.

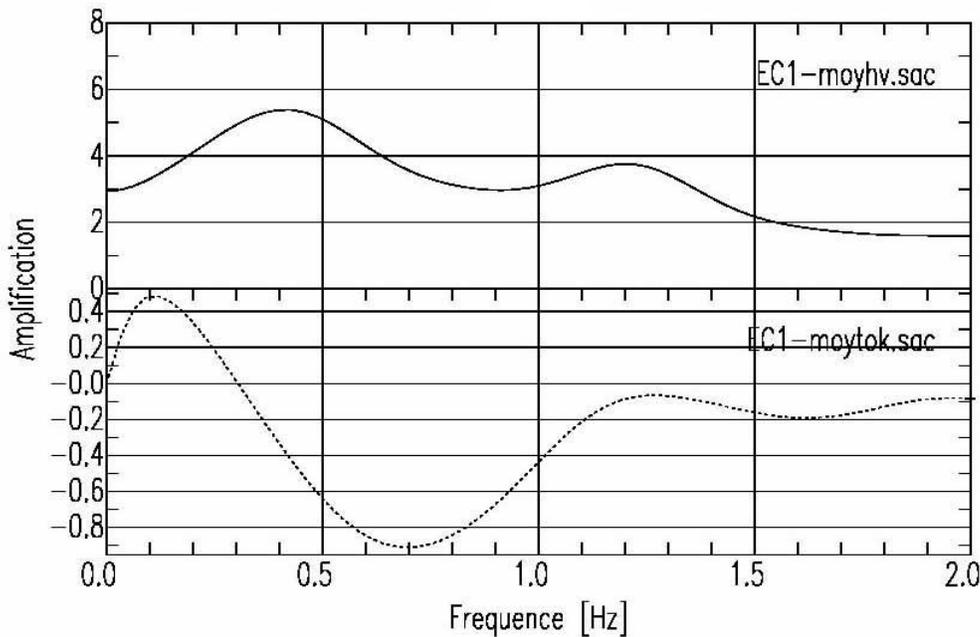
In order to understand why Tokeshi's technique resulted in a value that is half of the value determined with site/reference technique (0.3-0.5 Hz for 0.7-0.8 Hz), the Fourier spectra for a record of ambient vibrations were calculated. The maxima of the spectra for horizontal components occur at 0.3 and 0.7-0.8 Hz. For the vertical component only the maximum at 0.3 Hz is visible. The first peak at 0.3 Hz may be a characteristic of the ambient vibrations themselves. This peak is not visible on Nakamura's diagram because with this technique the ratio is calculated between horizontal and vertical components, but a zero of the phase spectrum is noticeable on the Tokeshi's diagram because only horizontal components are used with this method. This may indicate that the value found with Tokeshi's method (0.3-0.5 Hz) is not the fundamental eigen-frequency of the soil but only a characteristic of the spectral content of the recorded ambient vibrations. The value found with Nakamura's method (0.75 Hz) agrees well with values determined by means of other methods and indicates the real fundamental eigen-frequency.

The STC site is situated outside the valley on weathered rock. Therefore, no or at most a high frequency site effect is expected. This example was chosen in order to check if Tokeshi's method could give outcomes in this specific case where no noticeable amplification can be observed. Indeed Tokeshi's method gives a frequency of 0.3-0.4 Hz for the three records and a second frequency of 3.2-3.3 Hz for two of these records. However, the H/V ratios are flat for two records, as expected, and one record presents a

peak at 3 Hz. This confirms that the apparent fundamental eigen-frequency of 0.3 Hz found with Tokeshi's method is a characteristic of the signal and not of the soil profile.

### France

A lot of ambient vibration measurements have been done for 10 years in the Grenoble valley. It is now well known that the fundamental frequency of the basin ranges from about 0.2 Hz (where the substratum is very deep) to 0.5 Hz (where it is less deep), with sometimes a second frequency at about 3 or 4 Hz with strong amplifications (Le Brun [7]). Near the LGIT, where the network was installed, the fundamental eigen-frequency is between 0.28 and 0.35 Hz.



**Figure 9: Nakamura's method (up) and Tokeshi's method (down) for the ECO station (Grenoble) between 0 and 2.0 Hz.**

With Nakamura's and Tokeshi's methods, for 9 records, the eigen-frequency obtained is 0.3 to 0.4 Hz. For the last two records, Nakamura's technique gave no results. These two signals are very interesting because they have been recorded during a stormy day. In this case, the frequency content of the ambient vibration records is different from that of a "quiet" day. For the ARGO station, there are a lot of low frequencies (0.2-0.4 Hz) for the 3 components during the stormy day. No peak is noticeable for frequencies lower than 0.4 Hz on the H/V ratio. But Tokeshi's technique uses only horizontal components and so it is possible to obtain a result. As was already noticed for the Greek data, the frequency determined with Tokeshi's technique seems to indicate the maximum of the ambient vibration spectrum and not the eigen-frequency of the soil, which is in the same frequency range around 0.3 Hz. Therefore, there might be a coincidence between the eigen-frequency of the soil and the main frequency of the ambient vibration spectrum. For the ECO station for instance, Nakamura's technique shows a peak at 0.4 Hz, whereas Tokeshi's technique gives always the same frequency of 0.3 Hz (see Figure 9). This may signify that Nakamura's technique determines, for this example, the "real" eigen-frequency of the soil that can change from 0.3 to 0.4 Hz for the different stations of the network whereas Tokeshi's method only shows the maximum of the ambient vibrations Fourier spectra that doesn't change for the different stations.

For the 3 other stations in the Grenoble valley, the two techniques give the same results, which agree well with the expected values (a first peak near 0.3 Hz and a second peak at about 2-2.5 Hz, Le Brun [7]).

## CONCLUSIONS

The aim of this study was to check Tokeshi's method with real recordings of ambient vibrations, from several sites in Switzerland, Greece and France, where a site effect is expected. Systematic comparisons were conducted between results found with Nakamura's technique and Tokeshi's technique.

For about half of the examples that were tested in this study, the two techniques gave the same result. Nevertheless, many differences have been found for the other half of the examined cases:

- Tokeshi's method doesn't take into account the vertical component of records. In some cases, this may cause problems: when there is an amplification for a specific frequency on the Fourier spectra for the three components, Tokeshi's technique indicates this amplification as the fundamental eigen-frequency, which is a characteristic of the soil profile, even if it is actually a characteristic of the ambient vibrations. This problem was also described in the study conducted at Grenoble by Lelong [3].
- For the examples used in this study, Nakamura's method didn't give a result for every site where a site effect was expected. However, where the method gave something and where reference eigen-frequencies determined with other seismological techniques were available, frequencies estimated with Nakamura's method always agreed with these reference frequencies. In a lot of cases where Nakamura's method did not give an outcome, it was possible to obtain a result with Tokeshi's one. No example of the inverse situation has been found. It seems that Tokeshi's method gives almost always a result, even if no outcome is expected, since there is no significant site effect for the station STC outside the Volvi valley in Greece, for instance.
- It can be difficult to determine what is really the fundamental eigen-frequency of a site in the case of multiple zeros of the phase spectrum function whereas it is easier to estimate this frequency using the relative levels of amplification on H/V ratios.

It is accepted by the seismological community that Nakamura's technique does not give systematically a result. Nevertheless, when a peak is noticeable on the H/V ratios, the fundamental eigen-frequency estimated seems to be reasonably reliable. The results of the present study are coherent with this experience.

Tokeshi's method gives almost always a result, but the obtained frequency does not always correspond to the fundamental eigen-frequency of the site. In fact, Tokeshi's method seems to work reliably only in the case where the frequency characteristics of the sources of ambient vibrations correspond to white noise. Therefore, Tokeshi's method cannot be considered to be reliable and thus, its application cannot be recommended.

## ACKNOWLEDGEMENTS

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