13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 3081

DYNAMIC RESPONSE OF CIRCULAR FOOTINGS ON SATURATED POROELASTIC HALFSPACE

Bo JIN[1]

SUMMARY

The dynamic responses of footings on a saturated poroelastic half space and subjected to horizontal and rocking harmonic excitation are studied. By treating the footing as a rigid disk, the problems for vibrations of disk on poroelastic half space are formulated to Fredholm integral equations of the second kind which are solved numerically. The compliance coefficients for the horizontal and rocking vibration of disk are presented to illustrate the dynamic characteristics of footings on a poroelastic half space.

INTRODUCTION

In soil-structure interaction problems, the dynamic compliance of circular footings on a halfspace play a key role in determining the response of surface structures to dynamic loadings, in particular, seismic excitation and machine vibration [1-7]. Most formulations of dynamic interaction problems represented the material of halfspace as being either an elastic or viscoelastic solid. However, in certain cases, it may be more appropriate to represent the halfspace as a two-phase medium. Recently, much interest has been shown in reexamining this problem by considering the supporting medium to be a fluid-saturated poroelastic [8-11]. This paper analytically studies the horizontal and rocking vibrations of circular footings on a saturated poroelastic halfspace. Firstly the pressure-solid displacement form of the harmonic equations of motion for a poroelastic solid are developed from the form of the equations originally presented by Biot [12]. Then Fredhlom integral equations for the horizontal and rocking vibrations of footings on a saturated poroelastic halfspace are established according to the mixed boundary value condition. Numerical results for the horizontal and rocking compliance are presented and compared with the corresponding ideal elastic solutions to demonstrate the influence of poroelastic material parameters.

GOVERNING EQUATIONS

At this stage, it is convenient to non-dimensionalize all quantities with respect to length and stress by selecting the radius of the disk "a" as a unit of length and the shear modulus of the halfspace as a unit of stress. Biot's wave equations [12] can be written for the present case in terms of dimensionless variables as

[1] Department of Engineering Mechanics, Tongji University, Shanghai, China. Email: jinliuyk@online.sh.cn

$$u_{i,jj} + (\lambda^* + 1)u_{j,ji} + a_0^2(1-\rho^*\vartheta)u_i - (\alpha - \vartheta)p_{,i} = 0 \tag{1}$$

$$p_{,ii} + \frac{\rho^* a_0^2}{M^*\vartheta}p + \frac{\rho^* a_0^2(\alpha-\vartheta)}{\vartheta}e = 0 \tag{2}$$

in which $e = u_{i,i}$ is solid strain and u_i (i=1,2,3) are the dimensionless solid displacements; p is the dimensionless pore fluid pressure; α, $M^* = M/\mu$, $\lambda^* = \lambda/\mu$, $\rho^* = \rho_f/\rho$, $m^* = m/\rho$ and $b^* = ab\big/\sqrt{\rho\mu}$ are the dimensionless material parameters, where λ and μ = Lame constants; α and M = Biot's parameters accounting for compressibility of the two-phase material; ρ and ρ_f = mass densities of the bulk material and the pore fluid, respectively; m = a density-like parameter that depends on ρ_f and the geometry of the pores; b = a parameter accounting for the internal friction due to the relative motion between the solid matrix and the pore fluid. The parameter b is equal to the ratio between the fluid viscosity and the intrinsic permeability of the medium. If internal friction is neglected then $b = 0$. $\vartheta = \rho^* a_0^2 \big/ (m^* a_0^2 - ib^* a_0)$ is a dimensionless parameter. $a_0 = \omega a\sqrt{\rho/\mu}$ is a dimensionless frequency where ω is the frequency of the motion. For brevity, the time factor, $e^{ia_0 t}$, where t is a dimensionless time has been omitted from Eqs.(1) and (2) and also from the sequel.

The constitutive relations can be expressed as

$$\sigma_{ij} = \lambda^*\delta_{ij}e + \mu(u_{i,j} + u_{j,i}) - \alpha\delta_{ij}p \tag{3}$$

$$p = -\alpha M^* e + M^*\zeta \tag{4}$$

where

$$\zeta = -w_{i,i} \tag{5}$$

and σ_{ij} = total stress component of the bulk material.

INTEGRAL EQUATIONS FOR HORIZONTAL AND ROCKING VIBRATIONS

The footing is modeled as a rigid circular disk resting on a poroelastic half space. A cylindrical coordinates system r, θ, z is employed; r, θ plane coincides with the half-space surface with z axis directed into the half-space. The origin of the coordinate system is located at the center of the circular disk. For horizontal vibration the relationship between the force (H) and the displacement (D_h) is:

$$aD_h = \frac{(2-\nu)H}{8\mu a}C_{hh} \tag{6}$$

he dynamic compliance coefficient can be expressed as follows:

$$c_{hh} = \frac{-l_1}{(2-\nu)\int_0^1 \Phi_1(t)dt} \tag{7}$$

in which ν is Poisson's ratio and $\Phi_1(t)$ satisfies the Fredholm integral equations of the second kind:

$$\Phi_1(r) + \tfrac{l_2}{l_1}\left[\int_r^1 \tfrac{1}{t}\Phi_2(t)dt - \Phi_2(t)\right] + \int_0^1 K_{11}(r,t)\Phi_1(t)dt$$

$$+\frac{l_2}{l_1}\int_0^1 K_{12}(r,t)\Phi_2(t)dt = 1 \qquad 0 \le r \le 1 \tag{8}$$

$$\frac{l_4}{l_3}\Phi_1(r)+[\frac{1}{r}\int_0^r \Phi_1(t)dt - \Phi_1(t)] + \int_0^1 K_{21}(r,t)\Phi_1(t)dt$$
$$+\frac{l_4}{l_3}\int_0^1 K_{22}(r,t)\Phi_2(t)dt = 0 \qquad 0 \le r \le 1 \tag{9}$$

where

$$K_{11}(r,t) = \sqrt{rt}\int_0^\infty \xi[\frac{Q_{11}(\xi)}{l_1} - 1]J_{-1/2}(\xi r)J_{-1/2}(\xi t)d\xi \tag{10}$$

$$K_{12}(r,t) = \sqrt{rt}\int_0^\infty \xi[\frac{Q_{12}(\xi)}{l_2} - 1]J_{-1/2}(\xi r)J_{3/2}(\xi t)d\xi \tag{11}$$

$$K_{21}(r,t) = \sqrt{rt}\int_0^\infty \xi[\frac{Q_{21}(\xi)}{l_3} - 1]J_{3/2}(\xi r)J_{-1/2}(\xi t)d\xi \tag{13}$$

$$K_{22}(r,t) = \sqrt{rt}\int_0^\infty \xi[\frac{Q_{22}(\xi)}{l_4} - 1]J_{3/2}(\xi r)J_{3/2}(\xi t)d\xi \tag{14}$$

In the interest of brevity the functions $Q_{ij}(\xi)$ ($i = 1,2$; $j = 1,2$) and l_i ($i = 1,4$) have been omitted.

For rocking vibration the relationship between the rocking moment (M) and the rocking angle of the disk (ϕ) is:

$$\phi = \frac{3(1-\nu)M}{8\mu a^3}C_{mm} \tag{15}$$

The dynamic compliance coefficient are given as follows:

$$C_{mm} = \frac{-l_1}{3(1-\nu)\int_0^1 t\Phi(t)dt} \tag{15}$$

in which $\Phi(t)$ satisfies the Fredholm integral equation of the second kind:

$$\Phi(r) + \int_0^1 K(r,t)\Phi(t)dt = r \qquad 0 \le r \le 1 \tag{16}$$

where

$$K(r,t) = \sqrt{rt}\int_0^\infty \xi[\frac{\xi\Delta_1(\xi)}{l_0\Delta(\xi)} - 1]J_{1/2}(\xi r)J_{1/2}(\xi t)d\xi \tag{17}$$

The functions $\Delta(\xi)$, $\Delta_1(\xi)$ and l_0 have been omitted.

NUMERICAL RESULTS

After obtaining $\Phi_1(t)$ from Eqs.(8) and (9), and $\Phi(t)$ from Eq.(16) we can calculate the horizontal and rocking compliance coefficient C_{hh} and C_{mm} respectively. The selected poroelastic materials are considered in the numerical study. The dimensionless parameters of the poroelastic materials are $\lambda^* = 1.5$, $M^* = 10$, $\rho^* = 0.53$, $m^* = 1.1$ and $\alpha = 0.97$. In addition, $b^* = 0.1$ or 10. The larger the

permeability, the smaller the b^* is. Figs.1-4 show the horizontal and rocking compliance coefficients varying with dimensionless frequency a_0. In the figures the solid line and dotted line represent $b^* = 0.1$ and $b^* = 10$ respectively, and "O" represents the solution for an elastic half-space obtained by Luco and Westmann [6]. It is found from Figs.1-4 that real part and imaginary part of the horizontal or rocking compliance increase with increasing the permeability of the fluid in poroelastic medium (or the decrease of b^*). The real part of the horizontal compliance is increased about 10%, while the real part rocking compliance is increased about 14%. The imaginary part is increased about 17% for horizontal vibration and 8% for rocking vibration when b^* is decreased from 10.0 to 0.1. It should be noted that the findings in Figs.1-2 are valid for elastic response only. If the structure of the soil changes during vibration excess pore pressures are developed and the material properties will change.

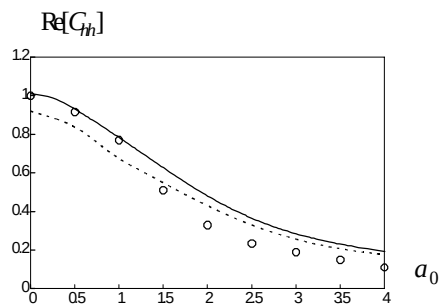

FIG.1 Real part of horizontal compliance

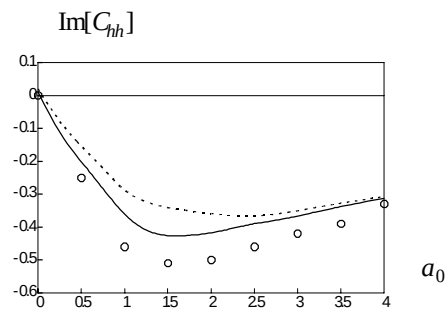

FIG.2 Imaginary part of horizontal compliance

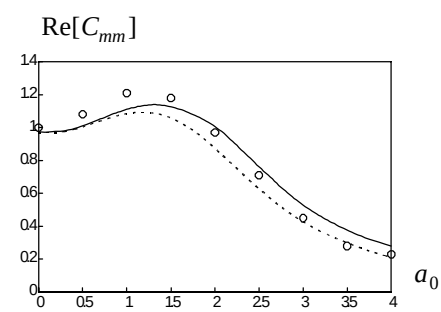

FIG.3 Real part of rocking compliance

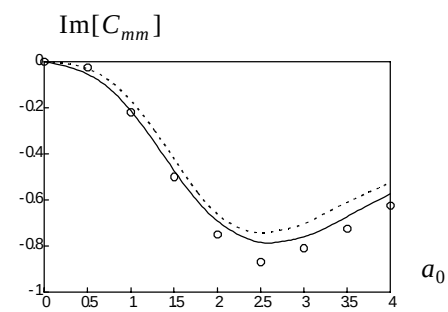

FIG.4 Imaginary part of rocking compliance

CONCLUSIONS

This paper analytically examines the horizontal and rocking vibration of a rigid disk on a poroelastic half space. With aid of integral transforms, the horizontal and rocking vibrations of a disk on a poroelastic half space is formulated as Fredholm integral equations of the second kind. The numerical results presented in the paper indicated that 8~20% differences exist for the horzontal and rocking compliance when permeability varies from $b^* = 0.1$ to $b^* = 10$. The differences between the rocking compliance for poroelastic medium and for elastic medium are smaller than 18% which can be neglected in the practical engineering, while differences for the horizontal compliance are small than 40%.

REFERENCES

1. Sung, T. Y., "Vibrations in semi-infinite solids due to periodic surface loadings." ASTM Spec. Techn. Publ., Symposium on Soil Dynamics, 1953; 156: 35-6
2. Bycroft, G. N., "Forced vibrations of a rigid circular footing on a semi-infinite elastic space and on a elastic stratum." Phil. Trans. Roy. Soc. Lond. A, 1956; 248; 327-368
3. Awajobi, A. O., and Grootenhuis, P., "Vibration of rigid bodies on semi-infinite elastic media." Proc. Roy. Soc. A, 1965; 287: 27-63
4. Robertson, I. A., "Forced vertical vibration of a rigid circular disc on a semi-infinite elastic solid." Proc. Camb. Phil. Soc. A, 1966; 62: 547-553
5. Gladwell, G. M. L., "Forced tangential and rotatory vibration of a rigid circular disc on a semi-infinite solid." Int. J. Engrg. Sci., 1968; 6: 591-607
6. Luco, J. E., and Westmann, R. A., "Dynamic response of circular footings." J. Engrg. Mech., ASCE, 1971; 97(5): 1381-1395
7. Veletsos, A. S., and Wei, Y. T., "Lateral and rocking vibrations of footings." Journal of Soil Mechanics and Foundations Division, ASCE, 1971; 97(9): 1227-1248
8. Philippacopoulos, A. J., "Axisymmetric vibration of disk resting on saturated layered half space." J. Engrg. Mech., ASCE, 1989;115(10): 2301-2322
9. Jin, B., and Liu, H., "Vertical dynamic response of a disk on a saturated poroelastic half space." Soil Dynamics and Earthquake Engineering, 1999; 18(6): 437-443
10. Jin, B., The vertical vibration of an elastic circular plate on a fluid-saturated porous half space, International Journal of Engineering Science, 1999; 37(3): 379-393
11. Zeng, X., Rajapakse, R. K. N. D., "Vertical vibrations of a rigid disk embedded in a poroelastic medium." Int. J. Numer. Anal. Meth. Geomech., 1999; 23(15): 2075-2095
12. Biot, M. A., Mechanics of deformation and acoustic propagation in porous media, J. Appl. Phys., 1962; 33(4): 1482-1498