13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 3053

QUADRATIC FORMS AS FUNCTIONAL REPRESENTATIONS OF LOADING CASES FOR SEISMIC DESIGN

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SUMMARY

Buildings often require irregular structures, making the traditional design approaches for member design dubious. Codes have faced this difficulty increasing the number of prescribed loadings and groups of factored combinations ("shotgun approach"). This paper is an attempt to show the use of geometrical forms instead of sets of single numbers for member design. The material presented comes from a real project, done following the code and using tridimensional analyses. It was found that 8 repetitive ellipses are the geometrical loci of 8 of the 9 code groups of cases (Vertical loadings generate only vectors). Quadratic Forms theory deals with these ellipses, to enable us use them as design tools.

INTRODUCTION

It should be no surprise that if we consider structural behavior linear and independent of the number, kind and order of the static loads we use to analyze them, the structures themselves should also respond following the laws of linear algebra. Quadratic Forms are just a part of Linear Algebra so it is natural that many structural behaviors, like their response to "rotating loads", happens to follow quadratic forms. We also find Quadratic Forms as "surfaces of energy" when we study torsional stability of building frames or column buckling phenomena.

It is interesting to remember that in European academic media, it was common until the middle of the XX century, to study many structural properties with methods like the "elasticity ellipse" and many of the theorems of Strength of Materials were deducted using Projective Geometry, now a forgotten discipline. In modern books of Strength of Materials the Elasticity Ellipse is used, but not mentioned, when they treat the variations of the moment of inertia of plane figures according to axis orientation, or the response of beams to oblique loads. Then it is not strange that the same laws should apply to the complex cantilevers constituting the skeletons of buildings.

This paper, based on an undergraduate thesis (Chacón [1]), following the steps of three previous works (Colvee and Taucer [2], Paparoni [3]), tries to show that the 65 loading cases per section prescribed by modern seismic codes (Covenin [4]) can be represented by Cartesian or polar quadratic forms (Ellipses). The increase in the number of loading cases came recently into code use as a consequence of having postulated the existence of statistical correlations between two mutually orthogonal seismic accelerations. The chosen approach for design purposes was to mix 100% of the design force in one direction with ±30% of the design force in the other direction, that is, we use 8 differently oriented loads. This scheme is really an attempt of applying a variable magnitude rotating force only with 8 limited orientations. The locus of the magnitudes of the fully rotating force complying with the 100%±30% approach is also an

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ellipse. One must say that in any case, even with no correlation, the rotating force scheme is a better approach, because buildings may have some directional weaknesses, not easily detectable at first sight.
The results obtained in our work show that this "elliptical" scheme of loading is much simpler. All the design cases can be handled, for each column section, simply by translating and, eventually, by also rotating a single ellipse in the plane Mx; My. There exists one single ellipse per section, representing the moments induced solely by a "multidirectional earthquake", in all sectional directions. Different groups of factored vertical loadings and seismic torsions applied as pure moments to the building combined with the effects of seismic torsion do not change the ellipse size or shape. There is also a single plane in space, per section, containing all possible design axial loads, this plane is normally not parallel to the plane Mx; My. Torsions associated with rotating forces and fixed eccentricities do rotate the ellipses. Ellipse shapes and Plane slopes are structural properties.
Let us try to describe this in another way: we could say that 1) Vertical Loads and pure Seismic Moments generate VECTORS. 2) the "rotating earthquake" generates a DEFINITE AND UNCHANGING ELLIPSE of moments. 3) fixed eccentricities and rotating horizontal forces generate a finite ROTATION of the "ellipse of moments". This last case is a frequent scheme of quantifying seismic torsion for Code purposes, because it is simple, but may be deceiving.
"Conceptual objects" handled in this study: (geometrical loci): Ellipses of Forces; Ellipses of Deformations, Ellipses of Moments, Ellipses of Axial Loads, Framing Axes, Principal Axes of a Building Framework.

Experimental Procedures
A building Project, done by a professional structural engineering firm, was offered to the Authors for the purposes of this study. The building was designed according to the Venezuelan seismic code Covenin-Mindur 1753-2001, [4] very similar to recent A.C.I. Codes. The building is a combination of a commercial center at the bottom, with a superimposed tower placed on top of it at one of its short ends (Chacón [1]). This is a very frequent type of building which leads to moderately irregular configurations in plan and height. A space-frame program (ETABS™) was used to perform the modal dynamic structural analysis, using three modes of vibration per floor and the code spectrum.
Of all the columns of the building, three were selected for detailed study, a corner column, a façade column and an internal column. As usual with all commercial computer programs, the original basic loading cases for each column were all numerically mixed in the 130 prescribed combinations. So it was necessary to decouple these results using several systems of linear equations to be able to get the original raw data (basic loads before combining) (Chacón [1]). This required a very time consuming effort, but allowed us to understand why by mixing so many things one loses trace of the ingredients and one becomes unable to look for patterns in the results from the point of view of the original loading cases. At first sight all the results put together almost appear to be random, looking like clusters of points representing each loading combination. Once we look for specific load group related geometrical loci as patterns, all the results begin to show regularities, a combination of apparent straight lines (point translations) and ellipses (the result of the "rotating earthquakes"). One must say that unless we understand geometrically what the data represents and the nature of the data itself, the patterns are not easily visible. It has to be stressed that the original seismic loading used in the project *did not* include rotating forces, but only eight different seismic loadings, resulting from 2 directions * 4 angles = 8 different loadings. (Using 4 of them only, one could define the "Seismic Ellipses", halving in this way the needed numerical data).

Mathematical treatment of the obtained data
From the purely algebraic point of view, any ellipse needs no more than 5 points per loading group to be defined (four points are enough strictly speaking, when the ellipse is not degenerate, that is, any three points among the chosen ones should not be in a common straight line). *Each displaced or rotated ellipse pertains to each one of 8 loading groups, and contains in itself all possible values of moments and axial*

loads for all possible directions and magnitudes of the seismic forces used in the design. (Of course, vertical loading only does not generate genuine ellipses, but degenerate ones, that is straight lines) These ellipses merely displace parallel to themselves on a single plane located in the space Mx; My; N under the action of vertical loadings or pure torsion moments applied to the whole building. *Only seismic torsions originating in fixed eccentricities additionally produce rotations, but only of the ellipses including this loading case.*

Physical factors pertaining to the building configuration which influence the observed results

1) *Framing directions:* (orthogonal frames). It can be said that frames respond as we usually idealize them, as structures which deform through pure translations of nodes in their own planes, but that is true only when we take into account their response to pure shear. We are disregarding their rotations due to column axial deformations and the out of plane translations due to mutual frame interactions. So the old hypotheses of admitting that they act as an assemblage of planes which have principal directions coinciding with their planar directions is not true. Old pseudo-spatial programs using this approach might produce errors when applied to irregular structures.

2) *Column axes orientation (Moments of Inertia):* They determine, in a general sense, the orientation of the ellipses of moments, except in corner columns, where other influences are more important. Framing schemes also influence the orientations, in addition to column axes.

3) *Column areas* can also influence the principal directions of structural response. When the resultant force of the vertical loads at each floor does not pass through the center of gravity of the column areas, there is an induced moment, which displaces the structure horizontally. Generally the directions of these displacements do not coincide with the framing directions, because many times column areas tend to be influenced more by architectural factors and traditions rather than by logic. These "stray" displacements also happen with horizontal loads. The Framing acts as a flexural beam.

4) *Seismic Torsion:* There are important differences in the orientation of the moment ellipses depending on the way the torsion moments are entered as data. If seismic torsions are entered as pure moments, the ellipses are merely translated, if they enter trough seismic forces and prescribed directional eccentricities, then they can change the direction of ellipse axes. Pure moments are insensitive to force directions, single force-induced moment magnitudes depend on the mutual angles between eccentricities and applied forces, these moments become cyclic when a rotating force is used as a seismic input.

5) *Oblique Frames:* In any orthogonally framed building, any additional frames which are not set parallel to one of the main framing directions can change the direction of the principal axes of the horizontal deformation ellipses of each floor.

6) *Dimensional changes or omission of repetitive members* in any level will also change the principal directions of the deformation ellipses.

7) *Accidental Torsion:* If we consider accidental torsion as a random variable, it seems to be contradictory to assign 4 single lengths to quantify it. If it is a random moment it should be represented by geometrical loci, or by constant moments. Torsional Dynamic Analyses generally give their results in terms of moments, without regard to the eccentricities directions, In other words, If we use such moments as input data, there will be no tilting for the moment ellipses.

8) *Coincidence of the structural principal axes with the maximum column moments direction:* In the project analyzed by us it can be said that they never had exactly the same directions, Except perhaps at the ground floor level, where they were better correlated with the column axes orientations.

9) *Axial loads generated by horizontal (seismic) loads* do have preferential directions of response *for every individual column*. There is always a seismic direction relative to each column, which causes no seismic axial loads in it, and another one, its perpendicular, which causes the maximum axial loads. These directions vary from column to column, and do not necessarily coincide with the framing directions or principal directions of the building. This means that in irregular buildings and *in the corner columns of regular buildings* there should be a multiplying factor to account with axial loads increases if the horizontal load analyses are made only for two framing directions or two framing principal directions not

going through the affected corners. It must also be taken into account that if an irregular building changes principal framing directions with height, we surely have a complex problem.

Conclusions of the study regarding procedures or conceptual approaches for design purposes:

1) All the ellipses generated for this study can be handled algebraically by third order matrices. (Pettofrezzo [5])

2) Matrix Algebra (Pettofrezzo [5]) can handle all the necessary steps; the kind of data treatment necessary already exists in graphics programs. The load factors matrix, properly ordered, already contains the loading groups submatrices with the necessary data.

3) Algebraically speaking, the "objects" handled in this study conceptually include eigenvalues and eigenvectors, as when one deals with linear algebra (Lay [6]).

4) We can also look at the general problem of designing reinforced concrete column as a descriptive geometry classical problem: Intersecting an interaction volume with a plane containing certain geometrical loci.

5) Purely planar pseudo-spatial programs (assemblage of independent plane frames) cannot handle rigorously even moderately irregular space frames. There are many local disturbances they cannot account for.

6) The seismic loads themselves could be represented by elliptical loci rather easily, if we want to go into mathematics, they could also be handled by complex number functions, like we do in electrical engineering with alternating currents (rotating vectors).

7) The most convenient representation of Torsion should be reexamined. Is seismic torsion independent from seismic forces or not? Are there separate spectra for seismic torsion?

8) An important effort should be invested in informing developers, architects and engineers about the hidden costs generated by the architectural or structural decisions capable of generating unexpected behaviors of building frames. Absolutely creative freedom has a price, and one should decide if the results are worth the price we must inevitably pay for it.

9) Ideally, the seismic analyses should be performed relative to the principal axes (elastic axes) of a building. It is difficult to deal with locally diverse directions of principal axes

10) The new code (Covenin-Mindur [4]) loading schemes, using two seismic loads at the same time, even if this originated in the spatial correlation of seismic movements, could be easily extended. One could span a 360° angle, instead of the limited 17°+56°+17° sweep per quadrant, resulting from the combination 100%Sx,y ±30%Sy,x. (8 directions for 8 different horizontal loads, for orthotropic framings; because only double symmetrical framings, elastically speaking, can produce equal loads, with equal angles). The resultant force magnitude increase is only of the maximum order of 104% of the maximum directional load. The real problem does not lie in the force magnitudes, but on the directions of the seismic loads, if we look in detail for weaknesses in our traditional loading schemes and analytical procedures.

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Table 1. The 65 Loading Combinations. Code prescribed factored combinations, per column section, giving a total of 130 values for each floor and each column to be reckoned with if we ignore their fixed patterns.

	Gravity Loads	Torsion X	Torsion Y	Earth-quake X	Earth-quake Y
1	1.475	0	0	0	0
2	1.106	1.000	0,3	1.000	0,3
3	0,675	1.000	0,3	1.000	0,3
4	1.106	1.000	0,3	-1.000	0,3
5	0,675	1.000	0,3	-1.000	0,3
6	1.106	-1.000	0,3	1.000	0,3
7	0,675	-1.000	0,3	1.000	0,3
8	1.106	-1.000	0,3	-1.000	0,3
9	0,675	-1.000	0,3	-1.000	0,3
10	1.106	1.000	-0,3	1.000	0,3
11	0,675	1.000	-0,3	1.000	0,3
12	1.106	1.000	-0,3	-1.000	0,3
13	0,675	1.000	-0,3	-1.000	0,3
14	1.106	-1.000	-0,3	1.000	0,3
15	0,675	-1.000	-0,3	1.000	0,3
16	1.106	-1.000	-0,3	-1.000	0,3
17	0,675	-1.000	-0,3	-1.000	0,3
18	1.106	1.000	0,3	1.000	-0,3
19	0,675	1.000	0,3	1.000	-0,3
20	1.106	1.000	0,3	-1.000	-0,3
21	0,675	1.000	0,3	-1.000	-0,3
22	1.106	-1.000	0,3	1.000	-0,3
23	0,675	-1.000	0,3	1.000	-0,3
24	1.106	-1.000	0,3	-1.000	-0,3
25	0,675	-1.000	0,3	-1.000	-0,3
26	1.106	1.000	-0,3	1.000	-0,3
27	0,675	1.000	-0,3	1.000	-0,3
28	1.106	1.000	-0,3	-1.000	-0,3
29	0,675	1.000	-0,3	-1.000	-0,3
30	1.106	-1.000	-0,3	1.000	-0,3
31	0,675	-1.000	-0,3	1.000	-0,3
32	1.106	-1.000	-0,3	-1.000	-0,3
33	0,675	-1.000	-0,3	-1.000	-0,3
34	1.106	0,3	1.000	0,3	1.000
35	0,675	0,3	1.000	0,3	1.000
36	1.106	0,3	1.000	0,3	-1.000

37	0,675	0,3	1.000	0,3	-1.000
38	1.106	0,3	-1.000	0,3	1.000
39	0,675	0,3	-1.000	0,3	1.000
40	1.106	0,3	-1.000	0,3	-1.000
41	0,675	0,3	-1.000	0,3	-1.000
42	1.106	-0,3	1.000	0,3	1.000
43	0,675	-0,3	1.000	0,3	1.000
44	1.106	-0,3	1.000	0,3	-1.000
45	0,675	-0,3	1.000	0,3	-1.000
46	1.106	-0,3	-1.000	0,3	1.000
47	0,675	-0,3	-1.000	0,3	1.000
48	1.106	-0,3	-1.000	0,3	-1.000
49	0,675	-0,3	-1.000	0,3	-1.000
50	1.106	0,3	1.000	-0,3	1.000
51	0,675	0,3	1.000	-0,3	1.000
52	1.106	0,3	1.000	-0,3	-1.000
53	0,675	0,3	1.000	-0,3	-1.000
54	1.106	0,3	-1.000	-0,3	1.000
55	0,675	0,3	-1.000	-0,3	1.000
56	1.106	0,3	-1.000	-0,3	-1.000
57	0,675	0,3	-1.000	-0,3	-1.000
58	1.106	-0,3	1.000	-0,3	1.000
59	0,675	-0,3	1.000	-0,3	1.000
60	1.106	-0,3	1.000	-0,3	-1.000
61	0,675	-0,3	1.000	-0,3	-1.000
62	1.106	-0,3	-1.000	-0,3	1.000
63	0,675	-0,3	-1.000	-0,3	1.000
64	1.106	-0,3	-1.000	-0,3	-1.000
65	0,675	-0,3	-1.000	-0,3	-1.000

Table 2. Eight combinations dealing with earthquake loads and vertical loads only, no torsion, that show a pattern. This pattern can be seen by examining the numbers shown. Notice that the two column vectors M_x and M_y together have permutative patterns. The vertical loads column vector has constant values. There are eight combinations of constituting octuples of factors, they are the origins of all the eight observed ellipses per section found in this research (64÷8=8), giving only nine different octuples (eight plus the vertical load alone). They can be grouped in rectangular matrices with three columns and eight rows. Each of these matrices defines an ellipse, with constant shape and size, which can translate and rotate.

Combination	Vertical Loads	M_X	M_Y
2	1.106	1	0.3
4	1.106	1	-0.3
6	1.106	-1	0.3
8	1.106	-1	-0.3
10	1.106	0.3	1
12	1.106	0.3	-1
14	1.106	-0.3	1
16	1.106	-0.3	-1

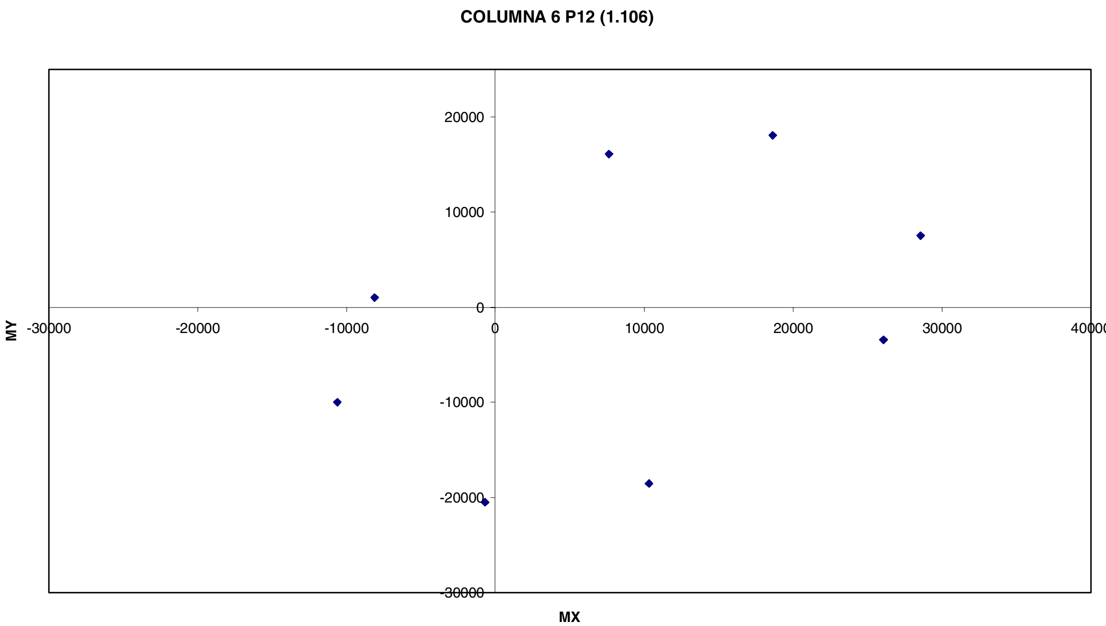

Figure 3. The first octet of results. Notice the elliptical pattern of the eight points. This ellipse still contains the origin of coordinates (zero moments and zero axial loads), but the center of the ellipse was displaced, together with all the plotted results, by the vertical loading vector.

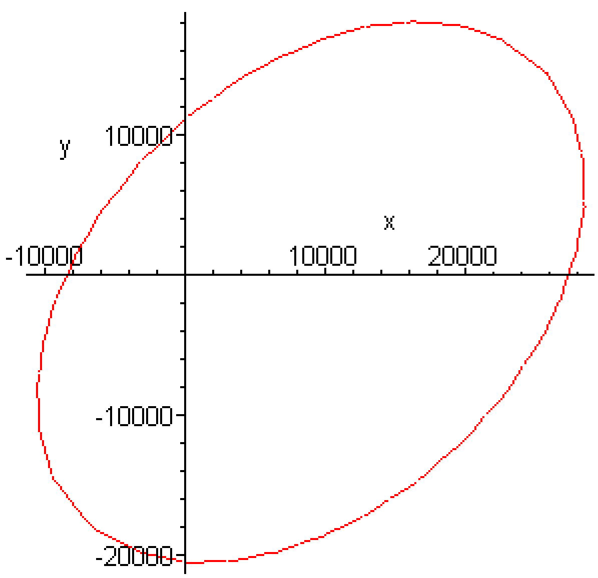

Figure 4. An ellipse, analytically fitted to the plotted results shown in Figure 3.

Table 3. Second Octet of results.

Combinación	CV	MX	MY	Tx	Ty
2	1,106	1	0,3	1	0,3
4	1,106	1	-0,3	1	0,3
6	1,106	-1	0,3	1	0,3
8	1,106	-1	-0,3	1	0,3
10	1,106	0,3	1	1	0,3
12	1,106	0,3	-1	1	0,3
14	1,106	-0,3	1	1	0,3
16	1,106	-0,3	-1	1	0,3

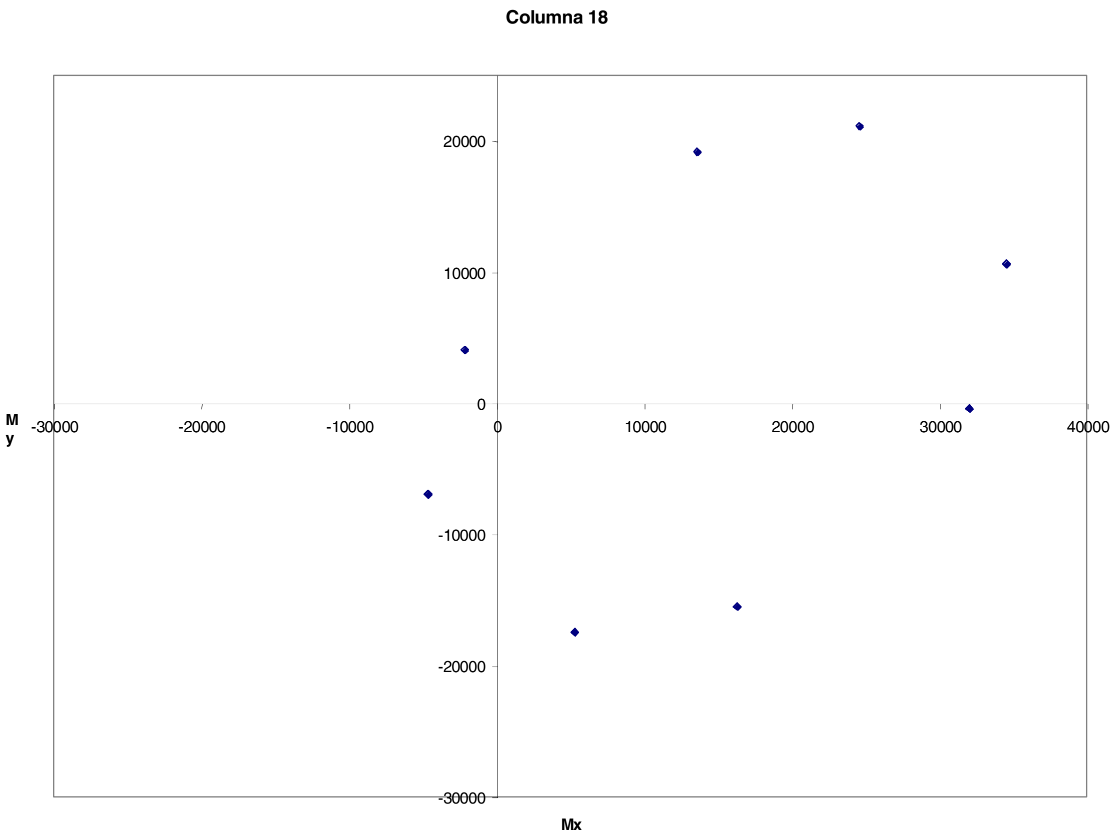

Figure 5. Plot of the second octet of results, including the Seismic Torsion T.

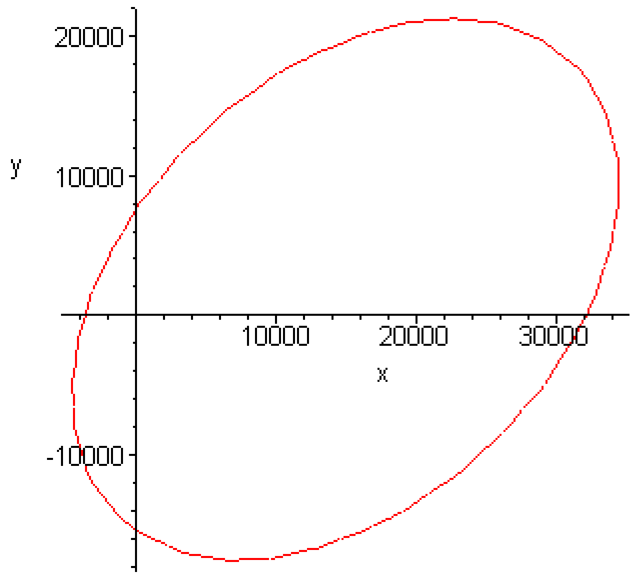

Figure 6. An ellipse, analytically fitted to the plotted results shown in Figure 5. This ellipse is translated relative to Figure 4 (torsion was applied as pure Moments). If we rotate and displace the coordinate axes of reference of this ellipse (vertical loading+earthquake loading+torsion as a moment) to make them coincide with the ellipse principal axes, we get A = 23023.19325 and B = 15265.30896. The rotation angle is –44.03891904°. If we do the same things to the first ellipse, (vertical loading + Earthquake loading, without torsion) we get A = 23017,18081 and B = 15261,32247. And the rotation angle was –44.03891904°. Apart from computational rounding errors, one can say that both ellipses have identical principal axes and identical angles of tilting. If torsion is introduced as a pure moment, the original ellipse is only displaced, not rotated.

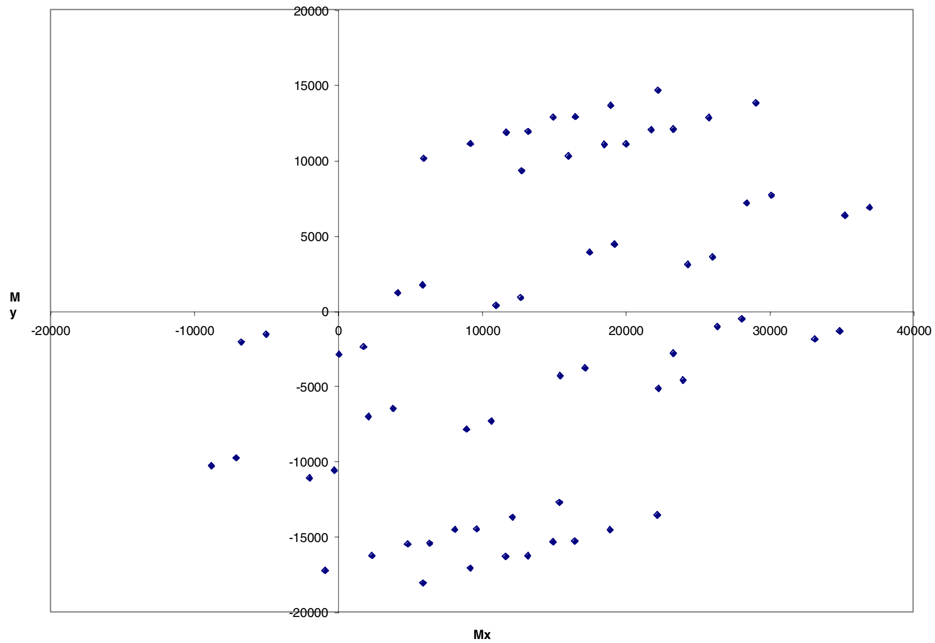

Figure 7. It is difficult to perceive the ellipses which join each set of eight results, if we plot the 65 combinations prescribed in the code.

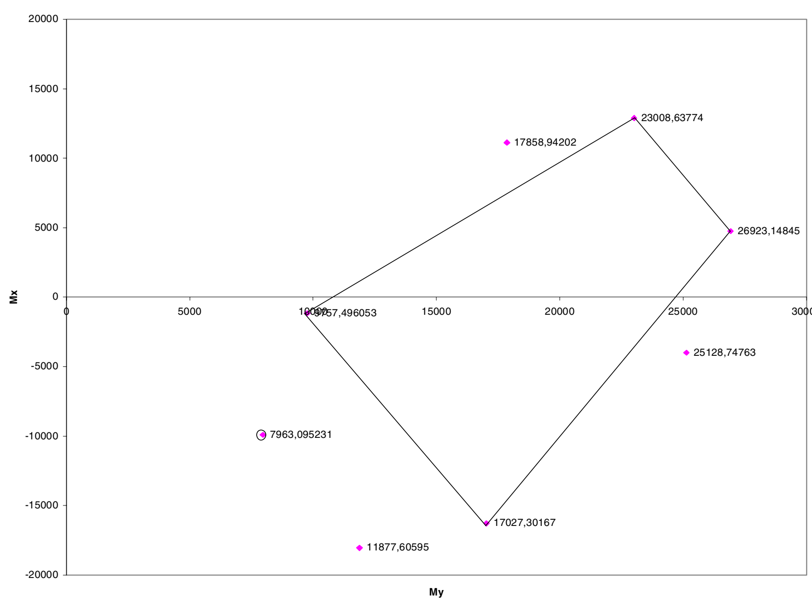

Figure 8. Selecting four non-aligned points to determine each ellipse analytically.

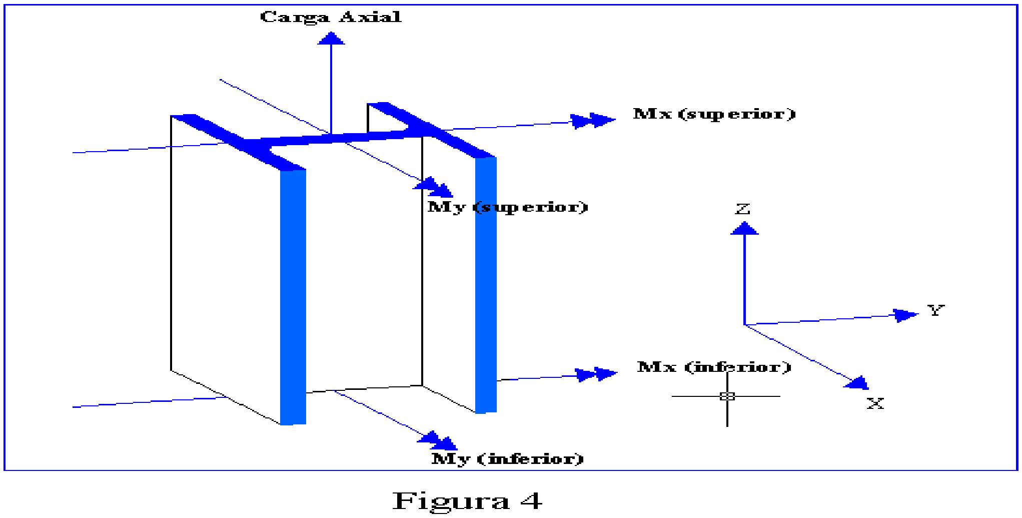

Figure 9. Coordinate axes and sign conventions used in this paper.

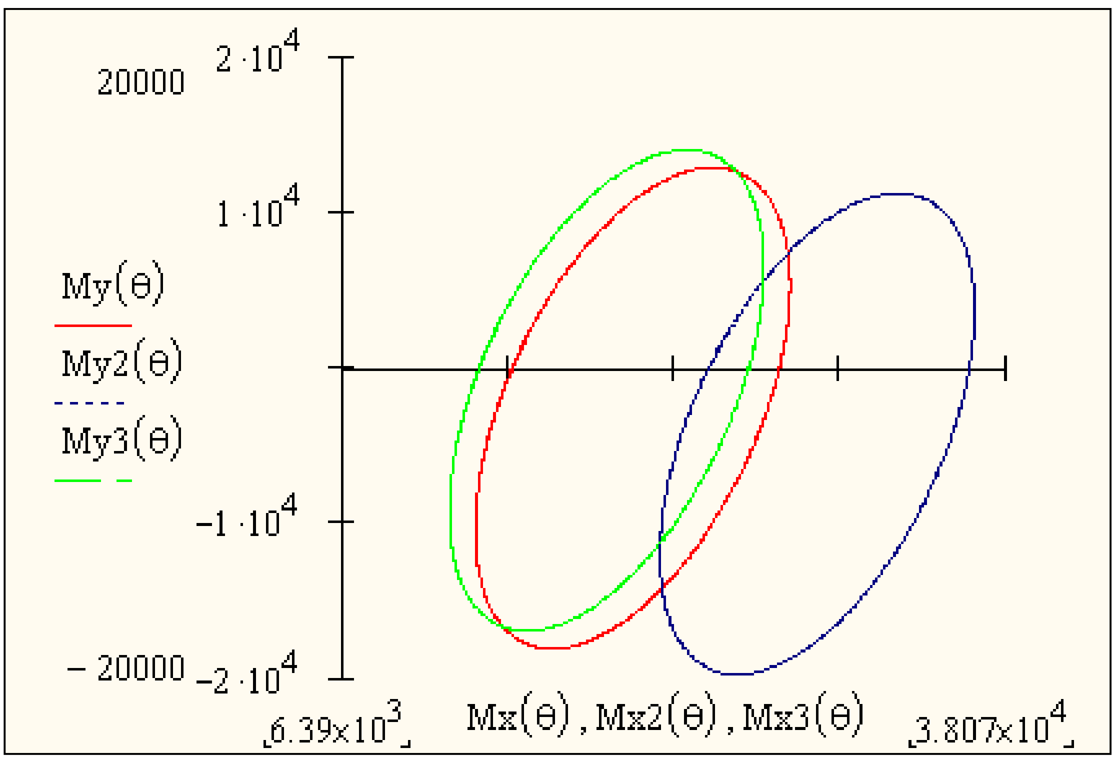

Figure 10. Relative displacements of three cases of ellipses. 1) My(θ): Vertical loads * 0,675+ Earthquake Forces. 2) My2(θ): Vertical Loads *1.106. + Earthquake Forces. 3) My3(θ): My2(θ) + Seismic Torsion as a pure Moment.

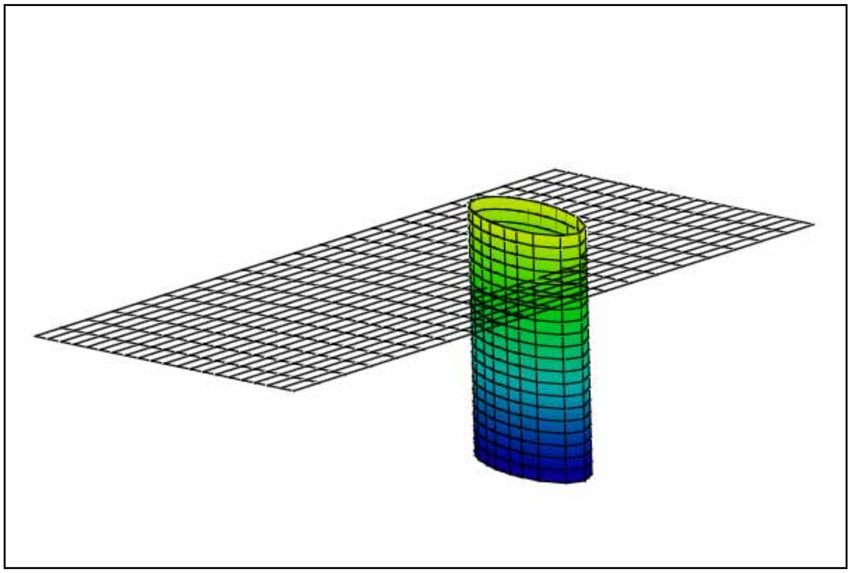

Figure 11. The design ellipse results from the intersection of an elliptical cylinder (Moments) with a plane (Axial loads). Purely vertical loads correspond to a horizontal plane.

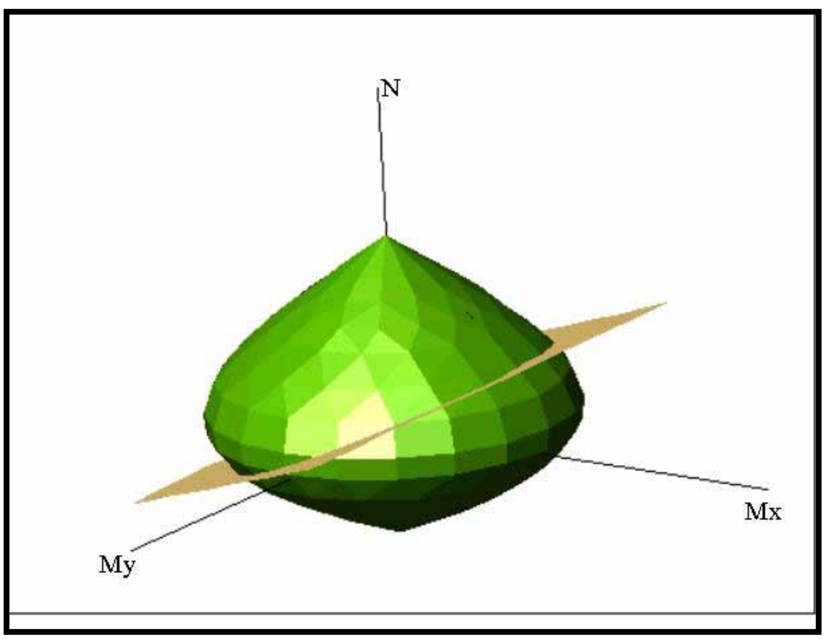

Figure 12. The three-dimensional representation of the design of a structural column. The tilted plane contains all possible points representing the axial loads and moments. The interaction volume represents all the allowable possibilities.

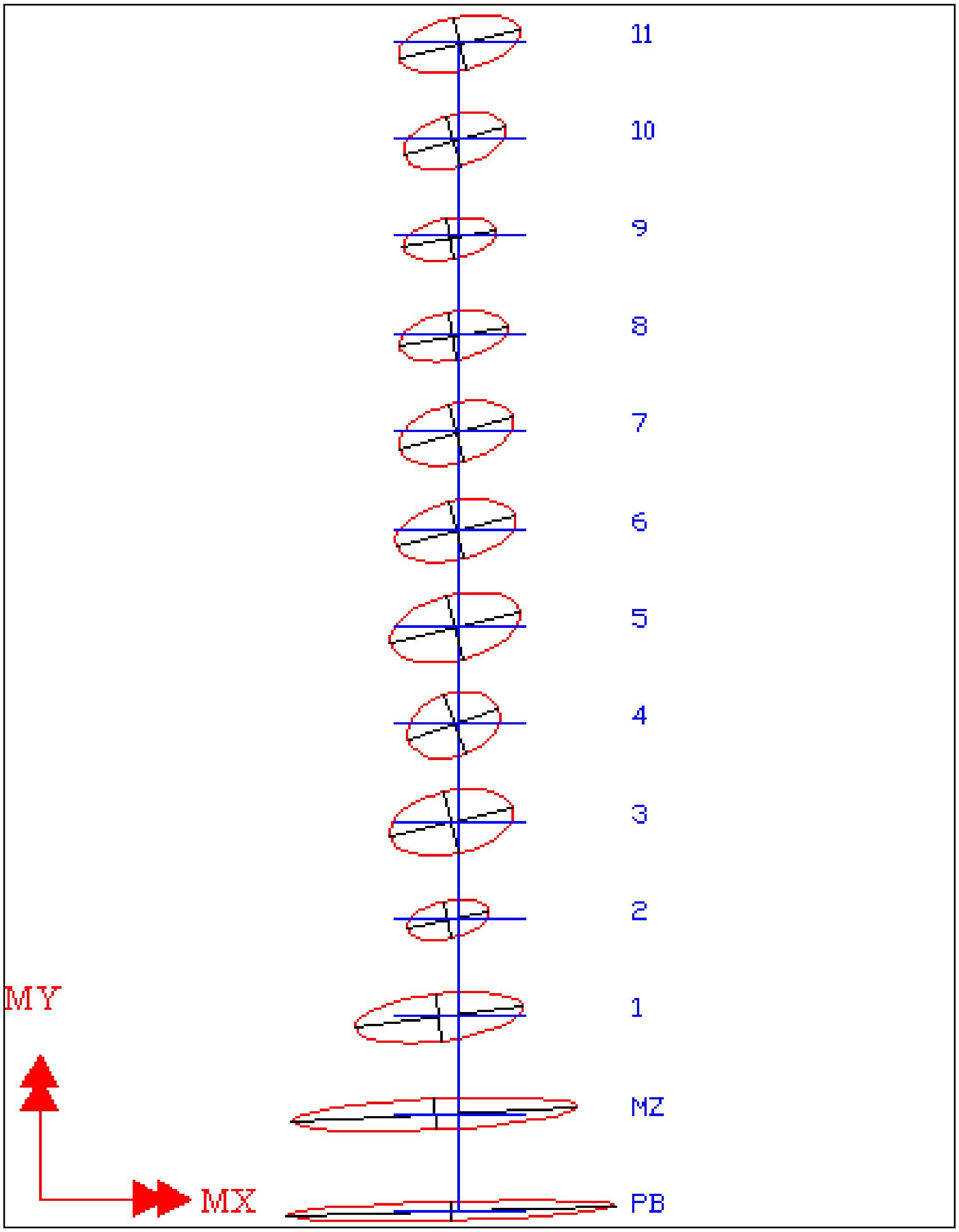

Figure 13. Distribution in size, shape and orientation of the t-sectional ellipses along the height of a column.

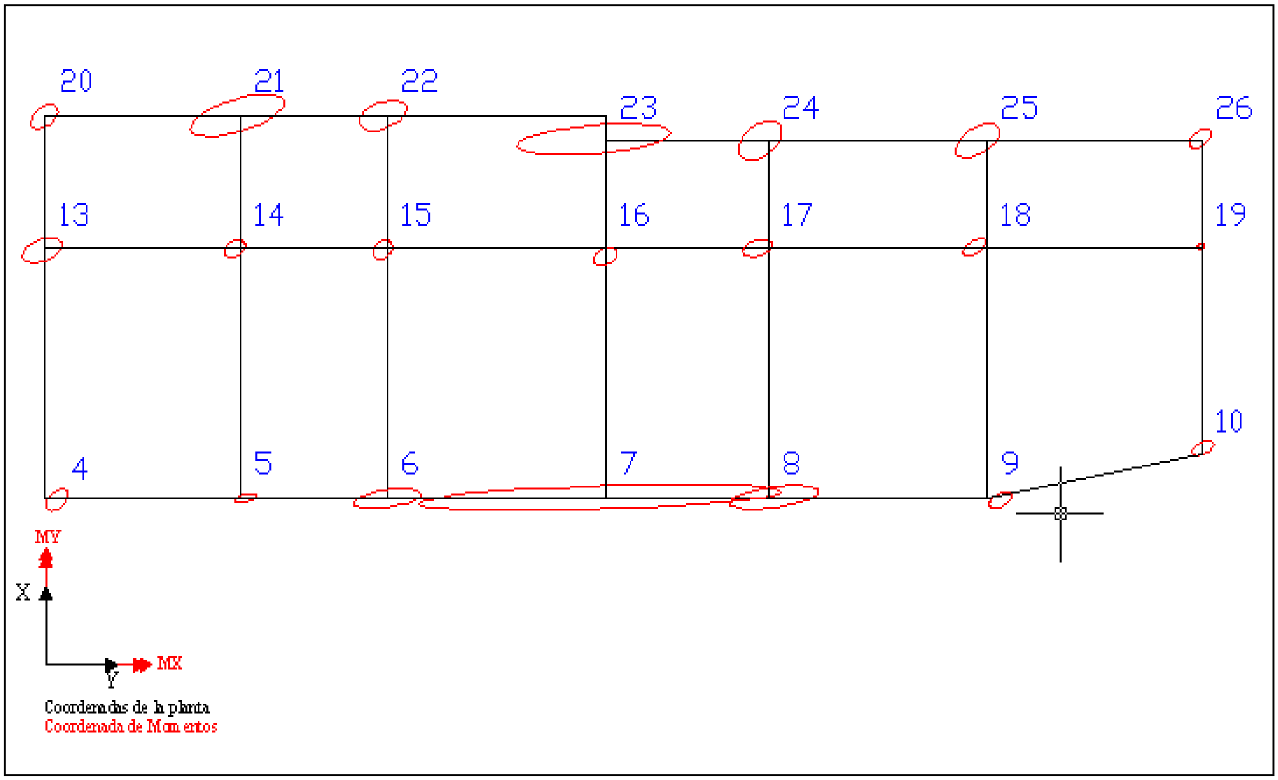

Figure 14. Example of moment ellipses at the ground level of a building.

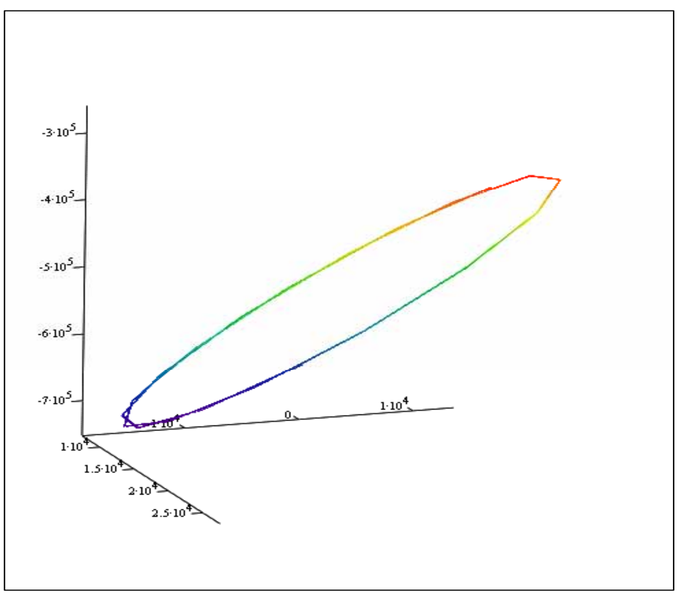

Figure 15. Moments and axial forces described as an ellipse in space (Geometrical Locus).