



INITIAL STIFFNESS VERSUS SECANT STIFFNESS IN DISPLACEMENT BASED DESIGN

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SUMMARY

Displacement-based design (DBD) methods are emerging as a valuable tool for performance based seismic design. A distinguishing feature between the different DBD procedures proposed in recent years is the type of analysis used in the design process. This paper identifies various challenges associated with the application of both initial stiffness and secant stiffness based DBD methods and considers whether one form is more effective than the other. Four of the most recent DBD methods that utilise response spectra are reviewed, two of which are initial stiffness based and two of which are secant stiffness based. Through application of the procedures to various case studies some difficulties associated with their application are identified, and significant differences in design strength are observed. Aspects of the design process that are considered influential to the success of the methods are then examined. Finally, the performance of each procedure is assessed by means of non-linear time history analyses. Despite the differences in strength, the performance assessment indicates that each of the DBD methods ensure design limit states are not exceeded. The results of the study infer that DBD utilising response spectra with either initial stiffness or secant stiffness structural characteristics may be equally effective. The biggest difference between approaches may be related to the ease with which they can be accurately applied to various structural forms. It is emphasised that the key to a successful design will be an appreciation of the assumptions that exist within each method irrespective of the approach adopted.

INTRODUCTION

In the last decade, various displacement-based design (DBD) approaches have been proposed to better control the displacements of structures in earthquakes and thereby enable performance based seismic design. Of the many procedures that have been put forward, there are three principal forms of analysis adopted; (i) Response Spectra - Initial Stiffness Based, (ii) Response Spectra - Secant Stiffness Based, and (iii) Time History Analysis Based. Of these different approaches, those utilising response spectra based on either initial stiffness or secant stiffness are generally faster than methods incorporating time history analyses. The aim of this study is to consider how the two different forms of spectral analysis can affect the strength requirements and performance of a displacement based design procedure.

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APPROACHES TO DISPLACEMENT BASED DESIGN

Displacement based design might be described as a design procedure that considers the role of deformation during the design process. As such, DBD methods differ from traditional force-based design approaches that control force levels for an assumed level of inelastic deformation that is not typically appropriate. In the development of DBD procedures that are simple and effective, several different design strategies using response spectra analysis have been proposed, as listed in Table 1. This table groups the various contributions according to whether initial stiffness or secant stiffness based spectral analysis is used in the design process.

Table 1. Proposals for Displacement Based Design that are based on response spectra analysis

Initial Stiffness Based	Secant Stiffness Based
Moehle [1]	Gulkan [11]
FEMA [2]	Freeman [12]
UBC [3]	Priestley [13]
Panagiotakos [4]	Kowalsky [14]
SEAOC [5]	ATC [15]
Albanesi [6]	Paret [16]
Aschheim [7]	Freeman [17]
Fajfar [8]	Chopra [18]
Browning [9]	SEAOC [5]
Chopra [10]	Priestley [19]

Table 1 is not intended as an exhaustive list of contributions and rather should be considered as an indication of the many different procedures that have been proposed for DBD. Furthermore, it illustrates the considerable difference in opinion related to the appropriate form of analysis for DBD. Other criteria could also be used to distinguish between the methods however this is not within the scope of this paper. Sullivan [20] and fib [21] have proposed further sub-division of the contributions, with respect to the role that displacement plays in the design process. For the details of a particular method, refer to the individual contributions referenced or to the summary provided by fib [21] or Sullivan [22]. Subsequent paragraphs describe the differences between the two different types of analysis.

Initial stiffness based use of response spectra in DBD

DBD methods that incorporate initial stiffness based response spectrum analysis aim to control the dynamic response of a structure through knowledge of its elastic stiffness and using some approximate relation between the elastic and inelastic response. Figure 1 illustrates the concept of initial stiffness for a structure responding into the inelastic range to a displacement Δ_D and strength level V_B . A commonly adopted relation between the elastic and the inelastic response is the equal displacement approximation. This approximation argues that the displacement of the elastic system of initial stiffness, K_i , will be equal to that of the inelastic system. The performance of this and other R- μ -T relations are discussed in detail by Miranda [23].

Secant stiffness based use of response spectra in DBD

Secant stiffness based procedures utilise the secant stiffness to the design response level and the concept of equivalent viscous damping to characterise the non-linear response of structural systems. Figure 1 also illustrates how the secant or effective stiffness, K_{eff} , is defined as the ratio of the strength, V_B , to the maximum displacement Δ_D . To facilitate design using the linear secant stiffness, an equivalent viscous damping coefficient is used to account for the energy dissipated during the actual non-linear structural response.

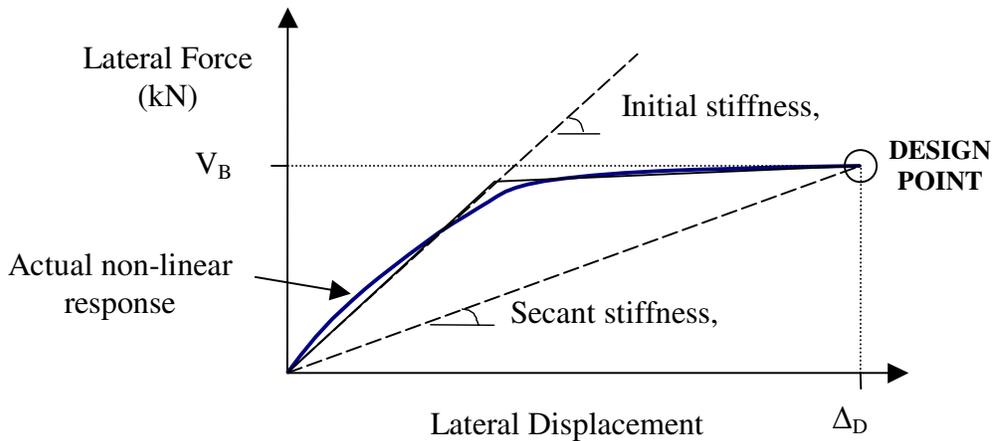


Figure 1. Illustration of initial-stiffness and secant stiffness concepts related to a structure's full non-linear response.

COMPARISON PROCEDURE

Comparison of initial stiffness and secant stiffness based DBD procedures is achieved through application of four of the different methods to five different case studies. In this way the study is able to compare design strength requirements and identify any application issues. Finally, an indicative performance assessment of the methods is carried out through time-history analyses. This section introduces each of the initial stiffness and secant stiffness based DBD procedures that are used for the investigation. Subsequently, the various case studies are described and the design criteria and general assumptions that were necessary are stated. Also presented is a brief description of the non-linear time-history models and the assumptions incorporated within the performance assessment.

Selected DBD methods

The two initial stiffness procedures to be used for the case studies are:

- **INSPEC** method = *Inelastic Spectra* method presented by Chopra [10].
- **YPS** method = *Yield Point Spectra* method presented by Aschheim [7].

The two secant stiffness procedures to be used for the case studies are:

- **CASPEC** method = *Capacity Spectrum* method presented by Freeman [17].
- **DDBD** method = *Direct Displacement Based Design* method presented by Priestley [19].

The chosen procedures are considered to be fairly representative of the design methods that adopt the two different forms of analysis. Note the abbreviated names assigned to each of the methods as these abbreviations will be used throughout the remainder of this paper. Brief descriptions of the methods follow, however, for details of the methods readers should refer to the contributions of the original authors referenced above.

INSPEC Method

The inelastic spectra DBD method proposed by Chopra [10] uses the design displacement to identify a target period from an inelastic displacement spectrum which is then used to obtain the design strength in proportion to the required initial stiffness and yield displacement. An iterative approach is proposed to ensure that the design strength, yield displacement and required initial stiffness values are compatible with each other. In definition of the design displacement, Chopra and Goel recommend the approach proposed by Priestley [24] whereby the yield displacement of the structure is added to a plastic displacement component. Inelastic spectra are developed in accordance with established procedures, for

the desired hysteretic nature, and at a level of ductility that corresponds to the yield and design displacement values set at the start of each design loop. It is suggested that the member sizes and detailing can be selected within each loop after the necessary initial stiffness and yield strength have been identified. The method designs structures to a target drift level and acceptable plastic rotation. The displacement ductility is not limited in the process, and instead is used to develop appropriate inelastic spectra.

YPS Method

The yield point spectra method presented by Aschheim [7] permits design to a number of performance criteria relatively quickly. The method involves development of yield point spectra, which are used to define a permissible design region considering target drift and ductility values. The required strength is then obtained by entering the YPS plot with the yield displacement of the structure and reading of the minimum strength value that lies within the permissible design region.

Yield point spectra (YPS) plot the yield points of oscillators having constant displacement ductility for a range of oscillator periods on axes of yield strength coefficient and yield displacement. It is suggested [7] that yield strength coefficients corresponding to specified displacement ductilities can be determined approximately from elastic spectra using smooth R- μ -T relationships such as those described by Miranda [23]. Yield displacement values of the YPS are obtained from the elastic periods of each oscillator and the inelastic pseudo-acceleration values in accordance with initial stiffness assumptions. Note that various curves corresponding to different ductility levels can be included within the YPS. To define the permissible design region, a drift control branch is required. To obtain this, one must first define a target displacement value that will satisfy a given drift limit for the desired risk event. The limiting drift branch is then formed by connecting points on the curves of the yield spectra, where the product of the yield displacement and the ductility give the target displacement. The permissible design region corresponds to the zone bounded by the drift control branch, and the YPS curve at the level of displacement ductility acceptable for the structure under consideration.

To permit design for various risk events simultaneously, the permissible design regions for the different earthquakes can be plotted on the same axes. Then, with knowledge of the structure's yield displacement, the strength required to satisfy all ductility and drift limits can be determined from the graph in one quick step. After obtaining the design base shear it is recommended [7] that conventional strength-based code approaches and software be used for proportioning the lateral force resisting system. The single design step means the method is relatively fast, however, designers may encounter difficulty when the procedure is applied to structures with flexible foundations as reported by Sullivan [20].

CASPEC Method

The capacity spectrum method proposed by Freeman [17] is best suited to checking the performance of existing structures for which the member sizes and strengths are known. The method proceeds by superimposing a capacity spectrum of the structure being considered, onto a suite of acceleration displacement response spectra (ADRS) at different ductility/damping levels. In order to relate the demand and capacity curves the approach requires use of an expression between ductility and damping for which Freeman references various papers. The intersection of the capacity curve with various demand curves at different levels of damping are considered until the point in which the capacity curve possesses the same equivalent damping value as a demand curve is identified. This intersection point indicates the structural response that is expected to develop during the design earthquake. In this way the approach is ideal as an assessment tool because it is relatively simple to obtain a prediction of the performance of a building under a given earthquake.

Recommendations for application of the procedure as a design tool were not located yet it appears that the method can relatively easily be adapted. The required design strength can be arrived at through consideration of the yield displacement and by assuming a post yield stiffness value. These parameters set the shape of the capacity curve of the structure, the strength of which might be estimated initially. Where the subsequent performance assessment indicates that ductility or drift demands are excessive, it is assumed that the best approach is to leave the dimensions unaltered and uniformly increase the strength of the structure. Because increasing the strength does not significantly affect the yield displacement, the new design can simply scale the forces up until the limiting value of the pushover curve reaches the demand curve at the appropriate level of damping.

DDBD Method

The Direct DBD method proposed by Priestley [19] is a relatively fast method that designs a structure to satisfy a pre-defined drift level. The code drift limit and the drift corresponding to the system's inelastic rotation capacity are considered in the design process, to establish a design displacement profile. With use of this displaced shape, the effective mass and target displacement of an equivalent single degree of freedom (SDOF) system are determined. The expected displacement ductility demand is established by dividing the target displacement by the yield displacement of the structure. This ductility value is used to calculate an equivalent damping value consistent with the expected hysteretic behaviour, and a displacement spectrum is then developed at this equivalent damping level. The target displacement is then employed to enter the displacement spectrum and read off the corresponding value of period, referred to as the effective period. The required effective (or tangent) stiffness is obtained for the equivalent SDOF system using this effective period value and the effective mass. Finally, the design base shear is obtained by multiplying the effective stiffness by the target displacement. To obtain other design actions, it is recommended [19] that the base shear is distributed up the structure in proportion to the displaced shape and mass distribution.

For reinforced concrete structures Priestley [19] suggests appropriate strain limits for two design states; serviceability and damage control. Ductility-damping relations that are consistent with the Takeda hysteretic model are also provided [19].

Buildings considered

Five different buildings of similar height but with significantly different characteristics were selected to assess the performance of the DBD methods. The five case studies considered include three wall structures and two frame structures as shown in Figure 2. Case Study 1 is an eight storey wall structure with regular layout on a rigid foundation. Only one earthquake direction is considered and the contribution of walls perpendicular to the earthquake direction is neglected. The second case study is identical to the first except that a flexible foundation beam is introduced. This case study was useful in identifying any methods that have difficulty incorporating foundation flexibility in design. The third case study is also a wall structure, however, the walls are arranged in an irregular layout as shown in the top part of Figure 2. The irregular layout causes the building to twist during an earthquake and therefore assesses each design method's ability to design for torsion problems. Case Study 4 is a seven-storey regular frame structure on a rigid foundation whereas the fifth case study examines an eight-storey frame building with a vertically irregular layout. This last case study considers the performance of design methods when applied to a vertically irregular but realistic structural shape. The geometry of these buildings, including beam and column dimensions are detailed elsewhere (refer [20]).

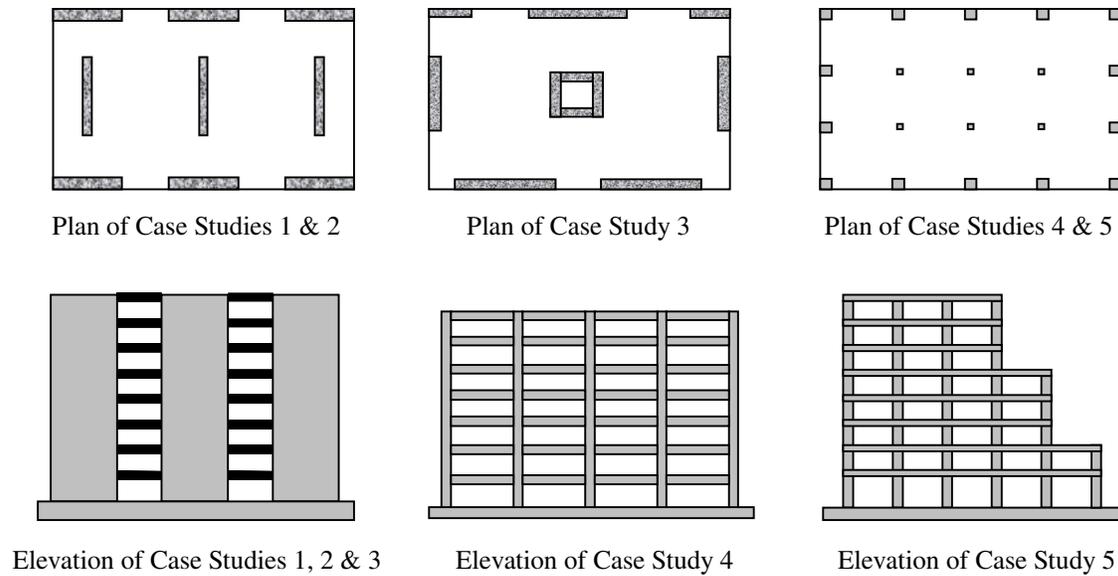


Figure 2. Schematic plans (top) and elevations (bottom) of the five case studies considered.

Design input

Demand spectra for the case studies were taken from the SEAOC blue book [5]. The decision to use spectra from the SEAOC blue book was made arbitrarily and does not indicate a limitation of the methods since any suite of spectra can be used. SEAOC provide displacement response spectra (DRS), acceleration response spectra (ARS) and acceleration-displacement response spectra (ADRS) for four different level earthquakes; EQ-I to EQ-IV. For design, the case studies utilise EQ-I, corresponding to a frequent earthquake and EQ-IV, corresponding to a maximum earthquake. Note that seventy percent of the SEAOC EQ-I ground motion has been used for all case studies except Case Study 2 for which the full EQ-I was used for reasons outlined by Sullivan [20].

Design drift limits and system displacement ductility values were also selected from the SEAOC blue book. The target values relevant to the case studies are shown in Table 2. Note that the parameters were set assuming that the longest wall for each of the case studies would be critical for the ductility limit.

Table 2. Drift and ductility limits adopted for the DBD case studies

	EQ Event	Case Study 1	Case Study 2	Case Study 3	Case Study 4	Case Study 5
Storey Drift Limit	EQI	1.0%	1.0%	1.0%	0.5%	0.5%
	EQIV	3.5%	3.5%	3.5%	4.0%	4.0%
Ductility Limit	EQI	1.0	1.0	1.0	1.0	1.0
	EQIV	5.0	5.0	5.0	8.0	8.0

The case studies consider two load combinations; (i) G+EQ-I and (ii) G+EQ-IV. The gravity loads (G) are only used as axial loads for the wall case studies and are applied as uniformly distributed loads along the beams of the frame case studies. Load cases other than earthquake combined with gravity are not considered.

General design assumptions

Various assumptions were necessary for the designs. Assumptions that could be considered as a limitation to a DBD method are described in the section on application issues. General assumptions that could reasonably be expected for any design are detailed below.

To enable clear comparison between methods, the case studies maintain the same dimensions and member sizes for each design method. For the INSPEC and DDBD methods, the inelastic rotations are directly limited by the design process. For these cases, concrete compressive strain limits of 0.004 and 0.018, and steel tensile strain limits of 0.015 and 0.06 were adopted for design to EQ-I and EQ-IV respectively, as recommended by Priestley [19]. It was assumed that these design limit strains would first be attained in the longest wall for the wall case studies and the first floor beam for the frame case studies. For the YPS and CASPEC methods, strain limits were not directly controlled in the design process and instead the displacement ductility values presented in Table 2 were used to restrict the inelastic deformations.

The concrete and steel material properties adopted for design are values that could typically be found in building practice. Values for the concrete include; (i) $f'c = 27.5$ MPa and (ii) $E_c = 28100$ MPa for Case Studies 1 & 2 and 32000 MPa for Case Studies 3, 4 and 5. Design values used for the reinforcing steel include: (i) $f_y = 400$ MPa and (ii) $E_s = 200\ 000$ MPa. Note that material strengths are not factored to dependable strength levels for design and instead, the expected strengths and stiffness values associated with the given material properties were adopted.

Time-history analysis assumptions

Time-history analyses are undertaken to evaluate the actual response of the case studies with strength as prescribed by the DBD methods. Results of the time history analyses are presented later in this paper to demonstrate the performance of the methods. As many simplifying assumptions are made in the modelling process for the time history analyses the assessment can only be considered as an indication of true performance. The main assumptions made for the time history analyses are outlined next.

Time-history records

Three spectrum-compatible time-histories were generated using SIMQKE that is included with the non-linear time-history analysis program, Ruaumoko (Carr [25]). The acceleration and displacement response spectra for the three time-histories generated to match EQ-I are shown in Figure 3. A time step of 0.01s and duration of 20s were chosen for the accelerograms. Because of the nature of these case studies, it was decided that artificial time-histories would best match the design spectra and would therefore most clearly demonstrate the performance of each method, despite their artificial characteristics.

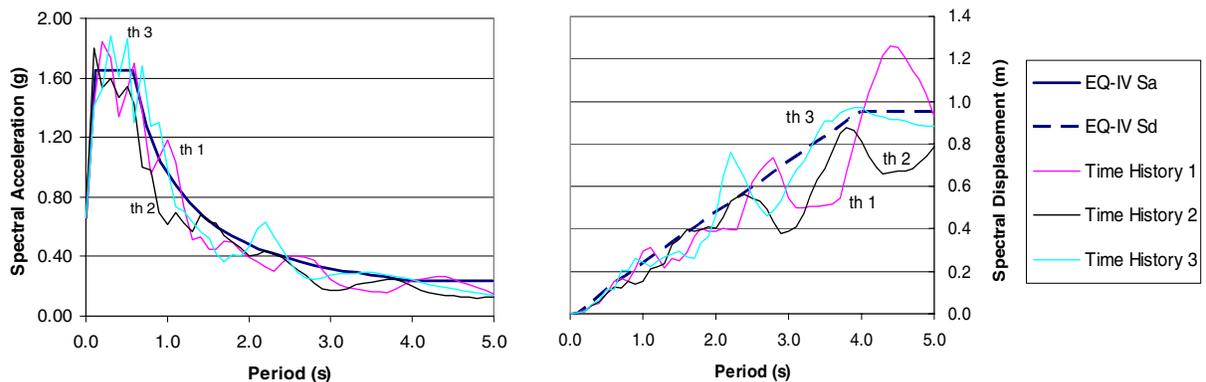


Figure 3. Comparison of 5% damped acceleration and displacement response spectra of the artificial time-histories with the design acceleration spectrum.

Modelling approximations

The Ruaumoko time history analysis program [25] is used to subject each of the structures to the three spectrum compatible accelerograms. Strengths obtained for each method are input into separate models, assuming that the actual strength provided in practice would exactly match the design strength required. The models use cracked section properties, obtained for the yielding elements by taking the design strength and dividing by the yield curvature ($EI = M_n/\phi_y$). Approximations for the yield curvatures were obtained from the expressions provided by Priestley [19].

The hysteretic behaviour of the structures was represented using the Takeda hysteresis model. This model included a 5% post-yield displacement stiffness and the unloading model according to Emori [26]. The plastic hinge lengths associated with the yielding elements were calculated using the recommendations from Paulay [27]. Elastic damping is modelled for the structures using tangent stiffness Rayleigh damping of 5% applied to the 1st and 2nd modes. It was assumed that the floor system is adequately connected to the structure and provides an efficient diaphragm action (rigid diaphragm) in order to introduce inertia forces to the structure at different levels. P-delta effects are not considered. Other modelling assumptions, particular to each case study are reported by Sullivan [22].

APPLICATION ISSUES

In general, the biggest difficulty associated with the application of the design methods relates to a lack of guidance. In many cases during design it was necessary to make significant assumptions in order to proceed. The main application issues with the four DBD methods are briefly described here. For a more comprehensive list of limitations associated with these and other DBD methods, refer to Sullivan [22].

To ensure clarity, the DBD methods being examined were presented by the original authors typically with reference to the design of SDOF structures. However, as a consequence, many of the methods do not provide recommendations for the correct representation of a multi degree of freedom (MDOF) system as an SDOF structure. The SDOF characteristics are required by all the DBD methods examined here for use with the response spectra. Accordingly, for the case studies it was sometimes necessary to assume values for the mass participating in the 1st mode and the effective height.

Another design step often overlooked, was how the design base shear should be distributed to the structure in order to obtain design strengths for all the elements within the structure. Where recommendations were not provided for the case studies, it was assumed that the load should be distributed vertically with respect to mass and height, in line with most modern code approaches. Having distributed the design base shear it was also noted that capacity design recommendations were rarely provided. Admittedly, it could be assumed that traditional capacity design guidelines are adopted however, it is considered that at the very least such a statement should be included amongst the description of each design method to render it complete as a design tool.

In performing the design of Case Study 2, the wall structure with flexible foundations, it was observed that few of the design procedures provide recommendations on how to account for foundation flexibility. It was recognised that the displacement due to foundation deformations was dependent on the strength assigned to the structure. Where models of the structure were used in the analysis, this affect could simply be accounted for through the introduction of springs at the base of the buildings. In other methods, adjustments of the system yield displacement and ductility demand were made through an iterative design process. Consequently, design for foundation flexibility was possible but became more time consuming and it is felt that additional recommendations would assist designers considerably.

No recommendations were found for any of the methods to take account of the twist-induced period lengthening that was anticipated for Case Study 3. This period lengthening occurs in structures such as Case Study 3 because the twist of the structure causes the centre of mass to displace further than the centre of rigidity. For methods that use a target displacement to obtain the required stiffness, it appears that an initial estimate of the twist could be used to increase the target displacement. This larger target displacement would then result in a longer period being designed for. However, neglecting this twist effect is unlikely to result in non-conservative design since the structure would essentially be given a shorter period and higher strength than what is necessary to maintain the target displacement. Initial stiffness methods that establish the initial period of the structure through construction of a 3D model would partially account for the building twist lengthening the period of the structure. However, depending on the strength distribution, this may not well represent the twist that occurs during the full inelastic response.

It was not always clear how the methods intended that the elastic stiffness and yield displacement be established. For the case studies, the effective EI was obtained in the same manner as for the time-history models, by considering the design strength over the yield curvature. Unless a method proposed otherwise, estimates for the yield displacement were computed using expressions provided by Priestley [19]. The importance of the yield displacement approximation is discussed in a subsequent section on the influential aspects of the design approaches.

DESIGN RESULTS

Values for the building base shear strength at development of the design yield strength for each of the methods and all case studies are shown in Table 3. The most striking result is the significant difference in design base shears between the methods, which can vary by a factor of two. While reviewing the design strengths it is worthwhile considering the parameter that governed the design for each method as presented in Table 4. It is apparent that design for EQ-I was often critical for the various design methods. The INSPEC and DDBD methods tend to require the lowest levels of strength because these methods do not require that ductility limits be maintained but instead design to drift and material strain limits associated with acceptable levels of damage. The section on influential design aspects elaborates on this point later in the paper.

Table 3. Building design base shear for each of the case studies.

Method	Building base shear at development of 1 st yield design strength (kN)				
	Case Study 1	Case Study 2	Case Study 3	Case Study 4	Case Study 5
YPS	3008	5755	4426	3732	4038
INSPEC	3416	3750	2434	3077	6307
CASPEC	4537	5419	5059	4499	4584
DDBD	2900	3494	3417	6136	7623

Table 4. Governing design parameter for each case study for each method.

Method	Governing Design Parameter				
	Case Study 1	Case Study 2	Case Study 3	Case Study 4	Case Study 5
YPS	EQ-I Ductility	EQ-IV Ductility	EQ-I Ductility	EQ-I Drift	EQ-I Drift
INSPEC	EQ-I Drift	EQ-I Drift	EQ-IV In. Rotn.	EQ-I Drift	EQ-I Drift
CASPEC	EQ-IV Ductility	EQ-I Ductility	EQ-IV Ductility	EQ-I Drift	EQ-I Drift
DDBD	EQ-IV In. Rotn.	EQ-IV In. Rotn.	EQ-IV In. Rotn.	EQ-I Drift	EQ-I Drift

1. The abbreviation "In. Rotn." signifies that the governing design parameter was the maximum Inelastic Rotation value..

Note that the design strengths obtained by the design methods for the frame structures of Case Studies 4 and 5 do not maintain the same proportions as for the wall case studies because of the unusual situation that the code drift limit for EQ1 was less than the drift associated with first yield of these structures. For the design procedures to correctly account for this it is necessary that the yield strength is greater than the design strength at the target displacement. In addition, in this situation the design strengths become more sensitive to assumptions related to yield displacement. For this reason, the INSPEC method, which arrives at a value of yield displacement through an iterative procedure, has a relatively high design strength for Case Study 4 and relatively low strength for Case Study 5.

Comparison of required strengths is limited to the design base shears in this paper. Design values of storey shear, flexural strength in the walls and beams, and longitudinal reinforcement ratios in the columns of the various case studies have also been determined and are presented by Sullivan [22].

INFLUENTIAL ASPECTS OF THE DESIGN APPROACHES

Within the DBD approaches examined, there are many assumptions and approximations made that can affect the outcome of the design. Logically, some of these design decisions are of greater significance than others. This section considers the cause of the observed strength differences between the methods and identifies various aspects of the approaches that are considered to be most influential on their performance.

Critical design parameter

The critical design parameter selected by the methods is an important cause of differences in required strength. Of the methods examined, the design using the INSPEC and DDBD methods is controlled by a code drift limit and the structure's inelastic rotation capacity. In contrast, the YPS and CASPEC methods control drift and displacement ductility limits. In these case studies where a displacement ductility limit was required by a method, the values presented in Table 2 were adopted, whereas inelastic rotation capacities were governed by the material strain limits. The consequence of this difference is that the target displacements associated with the inelastic rotation limits were typically greater than those associated with the ductility limits. Where a given structure is required to maintain smaller displacement limits, it must be given greater stiffness through the addition of strength. This observation accounts, at least in part, for the differences in required strength presented in Table 3.

Definition of initial stiffness

An important key to the success of the initial stiffness based DBD approaches is the correct definition of initial stiffness. This rather obvious point deserves some attention since it can not only lead to significant differences in required strength, but is also difficult to define for irregular structures.

For a simple structure, such as a cantilever wall, an acceptable initial stiffness definition is considered to be the flexural strength divided by the yield curvature of the wall. This ratio is equivalent to the product of the cracked section second moment of inertia and the modulus of elasticity. Examination of the moment-curvature behaviour of concrete sections reveals that there is typically a significant reduction in stiffness that occurs due to cracking, before the flexural strength of a member is attained. As such, the use of gross uncracked stiffness values for the calculation of the initial stiffness is incorrect, and would lead to significant errors in the design if adopted.

In the case of less regular structures, the correct definition of initial stiffness is not as obvious. One might consider the various possibilities shown in Figure 4, which presents a pushover curve for Case Study 3; the wall structure with irregular layout. It can be seen that because of different yield displacements associated with different length walls, the development of the full base shear is very gradual.

Consequently, the appropriate initial stiffness might be considered to be that associated with the yield of the 8m walls, or it might be claimed that the yield point of the 6m walls is more representative. An alternative argument yet again might be to say that the initial stiffness should be taken considering the initial stiffness of each wall weighted with proportion to the base shear it carries. This last approach does at least provide a more consistent basis for the initial stiffness definition. What is clear however, is that the initial stiffness is not an easily defined parameter. This point suggests that secant stiffness based DBD methods will offer more consistent results than initial stiffness methods for structures that are not highly regular since they are not dependent on the definition of initial stiffness adopted.

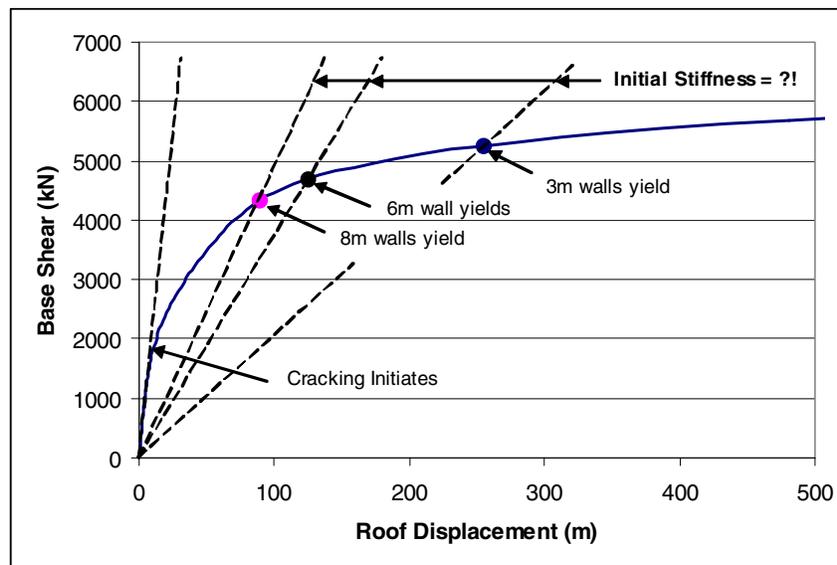


Figure 4. Pushover curve for Case Study 3, with various possibilities for the initial stiffness shown.

Equivalent viscous damping approximation

Secant stiffness based DBD procedures overcome the need to consider the initial stiffness of a system through use of an equivalent viscous damping relation. The equivalent viscous damping is related to the energy absorbed by the structure in deforming to the target displacement being considered. Various relations between displacement ductility and equivalent viscous damping have been developed for different forms of hysteretic behaviour (refer Priestley [28]). Furthermore, in the case where members of a structure undergo different ductility demands, such as Case Study 3, the equivalent damping approach is able to represent the total energy absorbed by considering the inelastic response of the individual elements weighted by the shear they carry.

Unfortunately however, even the equivalent damping approach is not likely to perform well for all types of structures. It is expected that when structures possess a hysteretic response that degrades quickly or is unstable and significant permanent displacements tend to develop, then the equivalent viscous damping approximation is not appropriate. A general weakness with the approximation is that viscous damping is greatest at high velocities, whereas a typical structure absorbs the greatest amount of energy at low velocities when the maximum displacements are approached.

These limitations with secant stiffness based DBD approaches are not debilitating however. In design situations, degrading and unstable hysteretic behaviour can and should be avoided through proper detailing. As for the fact that viscous damping does not dissipate energy in the same manner as the actual inelastic energy dissipation, current studies indicate that the approximation is sufficiently accurate for

design purposes. For these reasons the use of equivalent damping relations is considered reasonable for design, however, as with any simplified approach it is expected that results will be approximate.

Relationships between the elastic and inelastic response

An important design decision for the application of an initial stiffness method is the selection of an R- μ -T relation. There have been many R- μ -T relations proposed over the years owing to the fact that traditional force based design methods depended on their use and because none of them perform exceptionally well. Recall that the R- μ -T relations are required in the initial stiffness based DBD approaches in order to convert the response of the elastic system to that of the inelastic system. As mentioned earlier, the initial stiffness characteristics of a structure are not always easy to define. Furthermore, in a similar manner to the equivalent viscous damping expressions, the most appropriate R- μ -T relation is likely to be dependent on the hysteretic characteristics of the structure being considered. These limitations again suggest that, as with the secant stiffness approach, analysis using the initial stiffness with a relationship between the elastic and inelastic response will be of limited accuracy.

In some respects, the role of the R- μ -T relations in initial stiffness based DBD is much the same as that of the equivalent viscous damping relation in secant stiffness based DBD. This is because in both approaches the response of a non-linear system is being approximated using a linear system with some sort of adjustment for non-linear effects. With this in mind, one could argue that the decision to use one type of approach over another may be made considering only the ease with which the methods can be applied.

Estimation of the yield displacement

Especially important for the initial stiffness based methods, but also of significance for the secant stiffness methods, is the estimation of the yield displacement of the design structure. In the YPS method, the yield displacement is used to enter the yield point spectra and therefore directly controls the design strength. Also in the INSPEC method, the yield displacement is obtained iteratively and multiplied by the required initial stiffness to obtain the design strength. At the same time, the initial stiffness methods examined use the yield displacement when determining the ductility demands associated with the target displacement, for construction of the inelastic spectra. Secant stiffness based DBD methods also use the yield displacement to establish ductility demands, which are used to establish the design level of equivalent viscous damping. However, in this respect the role of the yield displacement in the design process is not as fundamental, since the damping levels are not very sensitive to the ductility, especially for large values.

For the reasons described earlier with reference to initial stiffness, definition of the yield displacement of a MDOF structure is not an easy task. However, the yield displacement is even more difficult to define than the initial stiffness because it requires knowledge of how curvatures develop within the structure as the seismic demand increases. Such knowledge may be especially limited for complex structures, in which case the yield displacements might be determined through development of an inelastic model that is subject to pushover analyses. However, significant assumptions must still be made in such a process, firstly related to the non-linear characteristics of the elements within the model, and then with relation to the appropriate loading or deformation pattern used for the pushover analyses. It is clear therefore, that there is an unpreventable uncertainty associated with the yield displacement of a structure. Given the role of the yield displacement within the various design approaches, this point suggests that secant stiffness based DBD methods will be more reliable than initial stiffness based methods.

Accounting for irregularity

In general, it is more difficult to apply the various DBD methods to structures that are irregular. This point relates to definition of the initial stiffness and the system yield displacement, and the best distribution of strength. The difficulties associated with definition of the yield displacement and initial stiffness, have

already been examined. The selected strength distribution is also of importance to response, as it can affect the resistance that is available at different displacement levels. At small displacements, members that have relatively large yield curvatures in comparison to the critical structural element may not be able to develop their full strength. Alternatively, at large displacement levels the full strength can be counted on and alternative motivations, such as control of torsion effects may dictate the optimal strength distribution. Further discussion of the effect of strength distribution is provided by Sullivan [22].

An additional difficulty with the design of irregular structures is that the simplified design procedures make the assumption that the 1st mode controls the structural response. For most buildings such an assumption is usually valid, however, where a structure possesses large irregularities then higher modes can become more influential.

RESULTS OF PERFORMANCE ASSESSMENT

Table 5 presents the maximum storey drifts and ductility demands obtained for each method from the time-history analyses for both the EQ-I and EQ-IV levels. It is clear that this indicative performance assessment shows that the methods have all performed satisfactorily, as they have maintained the limiting design parameters outlined earlier. Detailed findings of the performance assessment of these and other methods are reported by Sullivan [22]. Perhaps the most striking result provided by the time history analyses is the low influence the design strength has had on displacements. This is best seen by considering the range of design forces presented in Table 4 with the differences in drift and ductility shown in Table 5. It can be seen that despite ratios of strength as great as two between methods, ratios of drift and ductility demand never exceed the square root of two. In fact, the ratio of displacements between two given methods is always less than or equal to the square root of the ratio of the strengths for the same methods. This observation is in line with the relation between strength, stiffness and displacement as explained by Sullivan [22].

Other factors, particular to the case studies examined, are also partly responsible for the small differences in recorded displacements. These include the fact that the structures required relatively low design strengths to satisfy the large design storey drift values. Consequently, some of the case studies had such large periods that the maximum inelastic response lay within the constant displacement region of the displacement spectra. Another factor that brought the recorded displacements closer together, related to fulfillment of minimum strength requirements. In satisfying minimum strength levels the actual strengths provided to the time-history models were artificially increased and therefore the relative performance of the design methods was further obscured. Acknowledging these aspects of the case studies, it is emphasised that the performance assessment can only be considered as indicative. Further case studies that consider different height structures, perhaps using smaller design drift values, are expected to reveal more substantial differences in the performance of the design approaches.

CONCLUSIONS

This paper has explored the basis and performance of both initial stiffness and secant stiffness based DBD methods. The design methods have been used to obtain design strengths for five different case studies. In the process, various application issues have been identified and significant differences in strength have been recorded. Subsequently, several influential aspects of the design approaches have been highlighted with the aim of improving the reliability of the methods when used in actual design situations. Finally, an indicative performance assessment has shown that differences in design strength do not necessarily cause significant differences in drift and displacement, and that all of the design methods examined can be used

Table 5 Maximum storey drift and ductility values obtained from the time-history analyses of the case studies for each DBD method

	Method	EQ-I				EQ-IV			
		Storey Drift		Ductility Demand		Storey Drift		Ductility Demand	
		Peak	Average	Peak	Average	Peak	Average	Peak	Average
Case Study 1 Wall Structure	YPS	0.75%	0.65%	1.05	0.88	3.1%	2.8%	6.1	5.5
	INSPEC	0.63%	0.62%	0.83	0.80	2.7%	2.5%	5.3	5.0
	CASPEC	0.61%	0.49%	0.74	0.63	2.7%	2.4%	5.4	4.6
	DDBD	0.76%	0.65%	1.06	0.88	3.0%	2.8%	5.9	5.5
Case Study 2 Walls with Flexible Foundation	YPS	1.02%	0.79%	0.96	0.72	2.6%	2.4%	3.9	3.7
	INSPEC	0.82%	0.76%	0.81	0.76	3.1%	2.7%	5.8	5.0
	CASPEC	0.96%	0.78%	0.92	0.73	2.7%	2.6%	4.8	4.3
	DDBD	1.01%	0.73%	1.06	0.74	3.1%	2.7%	5.8	5.0
Case Study 3 6m Wall	YPS	0.47%	0.43%	0.70	0.66	2.1%	1.9%	3.7	3.4
	INSPEC	0.46%	0.43%	0.70	0.66	2.2%	2.1%	3.9	3.5
	CASPEC	0.47%	0.43%	0.70	0.66	2.0%	1.9%	3.5	3.3
	DDBD	0.46%	0.43%	0.70	0.66	2.2%	2.1%	3.9	3.6
Case Study 3 8m Walls	YPS	0.30%	0.27%	0.58	0.53	1.9%	1.7%	4.3	3.8
	INSPEC	0.30%	0.27%	0.58	0.53	2.0%	1.9%	4.6	4.1
	CASPEC	0.31%	0.27%	0.58	0.53	1.7%	1.6%	4.0	3.6
	DDBD	0.30%	0.27%	0.58	0.53	2.1%	1.9%	4.6	4.2
Case Study 4 Regular Frame Structure	YPS	0.42%	0.35%	0.77	0.65	2.7%	2.4%	5.1	4.7
	INSPEC	0.40%	0.40%	0.78	0.75	3.3%	2.8%	5.0	4.8
	CASPEC	0.41%	0.36%	0.77	0.66	2.8%	2.4%	5.0	4.5
	DDBD	0.40%	0.36%	0.78	0.67	2.4%	2.0%	4.2	3.6
Case Study 5 Irregular Frame Structure	YPS	0.58%	0.53%	0.54	0.49	3.5%	2.8%	3.7	3.1
	INSPEC	0.64%	0.53%	0.52	0.45	3.1%	2.8%	2.7	2.4
	CASPEC	0.60%	0.51%	0.58	0.51	3.1%	2.8%	3.2	2.9
	DDBD	0.62%	0.46%	0.64	0.49	3.2%	2.9%	2.9	2.4

1. Peak value refers to the largest of the maximum values obtained from the 3 time-history analyses.

2. Average value refers to the average of the maximum values obtained from the 3 time-history analyses.

3. Displacement ductility demands have been obtained using the maximum displacement and an assumed effective height.

4. Inter-storey drift values obtained from maximum displaced shape.

5. For Case Study 2 the ductility demand values have been determined accounting for foundation rotation

6. The critical 6m and 8m walls of Case Study 3 lie on opposite sides of the building and therefore experience different effects due to torsion.

to control the seismic response. The results of the study infer that DBD utilising response spectra with either initial stiffness or secant stiffness structural characteristics may be equally effective. It is proposed that the key to a successful design will be an appreciation of the assumptions that exist within each method irrespective of the approach adopted. The biggest difference between the DBD approaches may be related to the ease with which they can be reliably applied to various structural forms.

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