



## **USE OF STOCHASTIC SUBSPACE IDENTIFICATION MEHODS FOR POST-DISASTER CONDITION ASSESSMENT OF HIGHWAY BRIDGES**

**Nicolás A. LONDOÑO<sup>1</sup>, Serge L. DESJARDINS<sup>2</sup>, David T. LAU<sup>3</sup>**

### **SUMMARY**

After the occurrence of a major earthquake, reliable methods are required for post-disaster condition assessment of highway bridge structures. Structural health monitoring techniques based on field vibration monitoring can be applied to post-earthquake investigations of structural integrity. The system identification techniques applied to structural testing in laboratory environment are typically based on impact-load or other forced vibration excitation techniques to obtain information on the dynamic properties of the structure. In this case, the characteristics of the applied forces are known, and thus the responses of the test structure can be correlated with the input excitations. For large-scale structures, such as major bridges, the use of a forced vibration testing technique in the field is very often impractical and expensive to undertake. Ambient vibration responses due to transient dynamic load effects, such as, traffic loads, wind, and earthquakes, may be used instead in this case. Special numerical techniques of system identification are needed to analyze the ambient vibration data obtained by field monitoring systems because of the lack of information on the input excitation. Previous studies have shown that the stochastic subspace identification (SSI) methods are one of the most robust output-only identification techniques for civil engineering applications. In this paper, the SSI method is presented in detail along with case study results using field-monitoring data from the Confederation Bridge in Canada. The results of the case studies show that there is significant variability in the dynamic properties of the structure extracted from different datasets collected at different times under different loading scenarios and/or different environmental conditions. The variability in the extracted structural parameters represents a challenge for structural condition assessment algorithms based on detecting changes in the structural static or dynamic properties. Observed changes in the structural properties may be due to one or a combination of the following effects: (1) stiffness degradation due to deterioration or damage; (2) environmental effects such as temperature variation; (3) differences in the loading scenarios; (4) computational inaccuracies and modeling assumptions. In order to perform accurate and reliable post-disaster condition assessments a thorough understanding of these effects is needed. Studies have been conducted to evaluate the influences

---

<sup>1</sup> Graduate Student, Department of Civil and Environmental Engineering, Carleton University, Ottawa, Canada. Email: nlondono@ccs.carleton.ca

<sup>2</sup> Graduate Student, Department of Civil and Environmental Engineering, Carleton University, Ottawa, Canada. Email: sdesjard@ccs.carleton.ca

<sup>3</sup> Professor, Department of Civil and Environmental Engineering, Carleton University, Ottawa, Canada.

of these factors on the variability of the extracted dynamic properties of highway bridges. The results and findings of these studies are presented in this paper.

## INTRODUCTION

After the occurrence of major earthquake there is the obvious need to determine how vital infrastructures, such as highway bridges, have survived the event. Significant damage may occur in the internal parts of the structure where it is not directly observable by visual inspection. In such cases, the existence, severity and location of the damage need to be determined by other non-visual methods. Vibration based structural condition assessment or structural health monitoring is a relatively new field of research in Civil Engineering which has seen many rapid developments in recent years. Monitoring systems have been developed and installed in the field to collect information on the material properties and structural behaviour of civil engineering structures and systems, such as buildings and bridges. In structural evaluation, the challenges and research needs of using vibration monitoring data for structural condition assessment are recognized and much research has been carried out to address the problems, such as the development of numerical system identification techniques for accurate determination of dynamic properties from ambient vibration measurement data of which the sources of excitation are not known or measured (Peeters [1-2]). The development of a continuous monitoring system to provide rapid and accurate condition assessment of the monitored structure either on a continuous basis or after the occurrence of an extreme event, such as an earthquake, is the objective of the current research. The research presents numerous challenges on the development of intelligent automatic data processing and analysis, reliable condition assessment algorithms that take into account the stochastic variations in data behavior, and the development of graphical user interface (GUI) and visualization tools to facilitate engineering interpretation of the monitoring information.

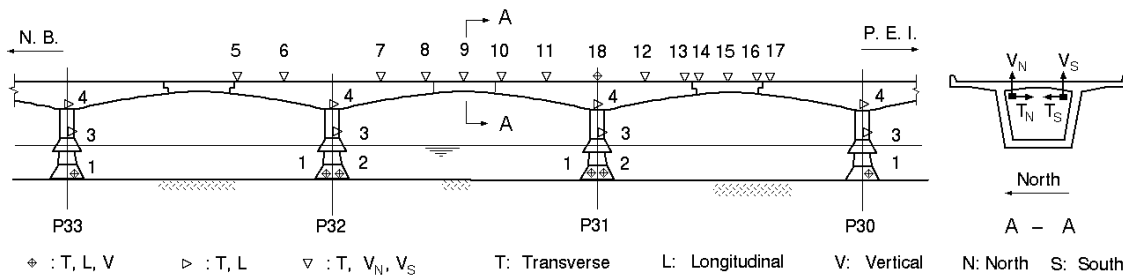
The Confederation Bridge Monitoring Project in Canada presents a unique opportunity to address and explore these challenges. The structural monitoring system of the Confederation Bridge, in operation since 1997, consists of a sophisticated network of sensor instrumentation and multiple continuous data acquisition systems to capture the dynamic responses of the bridge under wind, traffic, ice impact and earthquake loads. The monitoring system is designed with advanced automatic data collection algorithms to record bridge responses under both normal ambient vibration conditions on a continuous basis and triggered extreme loading scenarios, including earthquakes. Monitoring the ambient structural responses on a continuous basis, as opposed to capturing triggered data of events, provides a unique perspective which allows detailed studies of long-term changes of structural properties, behaviour and performance over time. An understanding of the signature characteristics and variations of the observed structural properties determined from the ambient dynamic monitoring data by system identification, and factors affecting these variations under different loading scenarios and environmental conditions, is essential for the establishment and use of a baseline on the behaviour and performance of the monitored structure for structural condition assessments that must distinguish between changes in structural properties and behaviour due to damage/deterioration of the structure and changes due to normally occurring variations.

### **Confederation Bridge and Monitoring System**

The 12.9 km Confederation Bridge crosses the Northumberland Strait in Eastern Canada, linking the provinces of New-Brunswick and Prince Edward Island. It is the world's longest bridge constructed over ice covered water and one of the longest continuous multi-span bridges. The Confederation Bridge superstructure is a pre-stressed haunched concrete box-girder bridge. It has 21 approach spans of 93 m

each, 2 transition spans of 165 m and 43 main spans of 250 m each at a typical height of 40 m above the mean sea level. The main span portion of the bridge is constructed of rigid frames linked by simply supported drop-in spans. Each rigid frame is composed of two piers and two 192 m double cantilever box girders linked by a continuous drop-in span. The depth of the box girders varies from 14.5 m at the pier supports to 4.5 m at mid-span.

A comprehensive long-term monitoring system of the Confederation Bridge has been in operation since the bridge opening in 1997 to collect information about its behaviour and performance. The monitoring system measures and records both environmental and bridge response data related to: ice forces, short and long-term deflections, thermal effects, traffic loading, corrosion and dynamic responses. The dynamic part of the monitoring system is designed and configured to measure the vibration responses of the bridge caused by significant sources of dynamic excitations, including wind, heavy traffic, ice loads and earthquakes. The vibration instrumentation comprises 76 accelerometers distributed mainly along a typical structural unit of one rigid frame unit and a simply supported drop-in expansion span. Vibrations of the girders are measured in the vertical and transverse directions, as shown in Figure 1.



**Figure 1: Locations of accelerometers in Confederation Bridge continuous monitoring system**

This setup permits the recovery of vertical bending, transverse bending and torsional vibration modes of the bridge superstructure. The vibration sensors used in the monitoring system include piezo-electric accelerometers and servo accelerometers. The measured analog accelerometer signals are conditioned and filtered for anti-aliasing by an 8-pole 50 Hz low-pass Bessel filter. Signals are then sampled and digitized by a network of distributed high-speed data loggers. The data-loggers may be programmed to either collect data on a continuous or triggered basis. The data acquisition can either be manually triggered or automatically triggered upon detection of specific dynamic events. Data are collected at user specified sampling rates that typically vary between 100 Hz and 167 Hz. In ‘triggered’ acquisition mode, the time interval of data recording usually varies between 90 sec and 15 min and includes a 30 sec pre-trigger buffer. Details of the monitoring system setup have been described in the references (Montreuil [3], Cheung [4]).

## DATA PROCESSING AND ANALYSIS

### Preprocessing

Preprocessing of the raw monitoring data is performed before any data analysis. The preprocessing tasks include:

- i. Baseline adjustment of the accelerometer time-history signals, by de-trending or high-pass filtering (high-pass 6<sup>th</sup> order Chebyshev Type II filter with a 0.1 Hz stop-band edge frequency);
- ii. Data error corrections for small segments of data where duplication of data recording is detected;

- iii. Data patches of missing sample gaps. Gaps are patched by cubic polynomial interpolation of the recorded data.
- iv. Re-sampling to bring all records to a common sampling rate and duration. This step is needed only when different data loggers operate at different sampling rates.
- v. Low-pass filtering to reduce noise in the band of interest (0-15 Hz). An eighth order Chebyshev Type I low-pass filter is applied to the monitoring data with a cut-off frequency of 16.67 Hz.
- vi. Down-sampling to one-third of the original sampling rate (125 Hz) is performed, resulting in a reduced sampling frequency of 41.67 Hz. The result is a substantial reduction in the number of samples without significant loss of resolution in both time-domain and frequency-domain.

### Stochastic Subspace Identification (SSI)

System Identification techniques are employed to analyze the data signatures and to extract structural dynamic properties of the bridge from the collected monitoring data. The extracted properties can be used for condition evaluation of the structure. For large scale complex structures like the Confederation Bridge, it is impractical or the cost is prohibitive to carry out forced vibration measurements. Consequently, ambient loading, without detailed knowledge of the sources, is relied upon as the input excitations. In the system identification analysis of ambient vibration data, output-only system identification techniques are required. Studies have shown that among the numerous system identification techniques proposed for civil engineering structural monitoring applications the stochastic subspace identification (SSI) methods are considered as a robust output-only identification technique compared to other available methodologies (Peeters [2]).

SSI algorithms identify a stochastic state-space model of the structure. The resulting model can then be translated into a more convenient structural model form for engineering interpretation of the results. The state-space model can be related to both modal model and Finite Element (FE) model formulations. The dynamic behaviour of civil engineering structural systems is traditionally modeled through discrete Finite Element approximations, which may be represented by the following matrix equation of motion

$$\mathbf{m}\ddot{\mathbf{u}}(t) + \mathbf{c}\dot{\mathbf{u}}(t) + \mathbf{k}\mathbf{u}(t) = \mathbf{f}(t) = \mathbf{r}\mathbf{p}(t) \quad (1)$$

where  $\mathbf{u}$  is the  $nx1$  displacement vector of the  $n$  degree of freedom (DOF) FE model,  $\mathbf{m}$ ,  $\mathbf{c}$ , and  $\mathbf{k}$  are the  $nxn$  mass, viscous damping, and stiffness matrices respectively;  $\mathbf{f}$  is an  $nx1$  load vector;  $\mathbf{r}$  is an  $nxm$  location coefficient that relates the locations of the  $m$  inputs with the response degrees of freedom of the system model, and  $\mathbf{p}$  is an  $mx1$  vector describing the  $m$  inputs or excitations. For system identification of discretely sampled responses, it is more convenient to reformulate the FE model into a discrete-time state-space model form, which can be expressed as follows

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{p}_k \\ \mathbf{y}_k &= \mathbf{C}\mathbf{x}_k + \mathbf{D}\mathbf{p}_k \end{aligned} \quad (2)$$

In this formulation,  $\mathbf{x}$  is a state vector which describes the system displacement and velocity at the instant  $k\Delta t$ , where  $\Delta t$  is the sampling interval;  $\mathbf{p}_k$  and  $\mathbf{y}_k$  are the sampled excitation and measured output vectors respectively. The state-space model formulation of Equation 2 is entirely deterministic. However, in civil engineering applications, especially in the context of continuous field monitoring, it is usually impractical or impossible to measure the input excitations,  $\mathbf{p}_k$ . Thus, the identification algorithms must rely on the measured outputs  $\mathbf{y}_k$  only. In the SSI algorithm the input terms  $\mathbf{B}\mathbf{p}_k$  and  $\mathbf{D}\mathbf{p}_k$  are considered as random white noise. Thus, the discrete-time state-space model becomes

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{A}\mathbf{x}_k + \mathbf{w}_k \\ \mathbf{y}_k &= \mathbf{C}\mathbf{x}_k + \mathbf{v}_k \end{aligned} \quad (3)$$

Both  $\mathbf{w}_k$  and  $\mathbf{v}_k$  are assumed to be white, zero-mean stochastic processes, independent of the state vector. Based on these assumptions, from the stochastic state-space model formulation of Equation 3, it can be shown that the correlations of the outputs  $\mathbf{R}_i$  can be factorized into a triplet containing the state matrices (Peeters [1-2])

$$\mathbf{R}_i = \mathbf{C}\mathbf{A}^{i-1}\mathbf{G} \quad (4)$$

where output correlation matrices  $\mathbf{R}_i$  are defined as  $\mathbf{R}_i = \mathbf{E}[\mathbf{y}_{k+i}\mathbf{y}_k^T]$ , where  $\mathbf{E}$  denotes the expectation operator. The subscript  $i$  indicates a time lag of  $i\Delta t$ , and  $\mathbf{G} = \mathbf{E}[\mathbf{x}_{k+1}\mathbf{y}_k^T]$  is a “next-state-output” correlation matrix.

Equation 4 represents the basis of the correlation driven SSI algorithm. It indicates that suitable decomposition of the correlations can yield the state-space matrices, which in turn contain the structural parameters of mass, stiffness and damping. Data correlations also offer the advantages of eliminating uncorrelated noise and compressing the data while preserving the modal information.

Correlations of the data with respect to a subset of reference sensor channels computed at a sequence of different time lags may be assembled into a block Toeplitz matrix as follows

$$\mathbf{T}_{ll}^{ref} = \begin{bmatrix} \mathbf{R}_i^{ref} & \mathbf{R}_{i-1}^{ref} & \dots & \mathbf{R}_1^{ref} \\ \mathbf{R}_{i+1}^{ref} & \mathbf{R}_i^{ref} & \dots & \mathbf{R}_2^{ref} \\ \dots & \dots & \dots & \dots \\ \mathbf{R}_{2i-1}^{ref} & \mathbf{R}_{2i-2}^{ref} & \dots & \mathbf{R}_i^{ref} \end{bmatrix} \quad (5)$$

The factorization property of Equation 4 allows the block Toeplitz matrix to be factorized as follows

$$\mathbf{T}_{ll}^{ref} = \begin{bmatrix} \mathbf{C} \\ \mathbf{C}\mathbf{A} \\ \dots \\ \mathbf{C}\mathbf{A}^{i-1} \end{bmatrix} \begin{bmatrix} \mathbf{A}^{i-1}\mathbf{G}^{ref} & \dots & \mathbf{A}\mathbf{G}^{ref} & \mathbf{G}^{ref} \end{bmatrix} = \mathbf{O}_i\mathbf{\Gamma}_i^{ref} \quad (6)$$

where  $\mathbf{O}_i$  and  $\mathbf{\Gamma}_i^{ref}$  are rank  $n$  matrices known as the *extended observability* and *reversed extended stochastic controllability* matrices, respectively. The Singular Value Decomposition (SVD) technique is then used to reduce the block Toeplitz matrix into suitable factors as follows

$$\mathbf{T}_{ll}^{ref} = \mathbf{U}\mathbf{S}\mathbf{V}^T = \begin{bmatrix} \mathbf{U}_1 & \mathbf{U}_2 \end{bmatrix} \begin{bmatrix} \mathbf{S}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1^T \\ \mathbf{V}_2^T \end{bmatrix} = \mathbf{U}_1\mathbf{S}_1\mathbf{V}_1^T \quad (7)$$

where  $\mathbf{S}_1$  is a diagonal matrix containing singular values in descending order. Using Equations 6 and 7, the following so-called internally balanced realization may be obtained

$$\mathbf{O}_i = \mathbf{U}_1\mathbf{S}_1^{1/2} \quad (8)$$

$$\mathbf{\Gamma}_i^{ref} = \mathbf{S}_1^{1/2}\mathbf{V}_1^T \quad (9)$$

The state-space matrix  $\mathbf{A}$ , which contains the mode shapes and eigenvalues, can be extracted from the extended observability matrix as follows

$$\mathbf{A} = \mathbf{O}_i(1:l(i-1),:)^+ \mathbf{O}_i(l+1:li,:) \quad (10)$$

where  $l$  is the number of output channels,  $i$  is the number of block rows of the block Toeplitz matrix,  $\mathbf{O}_i(1:l(i-1),:)$  and  $\mathbf{O}_i(l+1:li,:)$  denote rows 1 to  $l(i-1)$  and  $(l+1)$  to  $li$  of the extended observability matrix respectively, and  $+$  denotes the Moore-Penrose pseudo-inverse. The eigenvalue decomposition of the discrete-time state matrix  $\mathbf{A}$  yields

$$\mathbf{A} = \mathbf{\Psi} \mathbf{\Lambda} \mathbf{\Psi}^{-1} \quad (11)$$

where  $\mathbf{\Psi}$  is a complex eigenvector matrix and  $\mathbf{\Lambda}$  is diagonal matrix containing the discrete-time eigenvalues  $\mu_j$  directly related to the system poles  $\lambda_j$  or eigenvalues of the original second order system described by Equation 1. The system poles contain the modal frequencies  $\omega_j$  and damping ratios  $\xi_j$

$$\lambda_j = \frac{\ln(\mu_j)}{\Delta t} = -\xi_j \omega_j + i \sqrt{1 - \xi_j^2} \omega_j \quad (12)$$

The observed mode shapes at sensor locations are computed as follows

$$\mathbf{V} = \mathbf{C} \mathbf{\Psi} \quad (13)$$

where  $\mathbf{C}$  can be obtained directly as the first  $l$  rows of  $\mathbf{O}_i$ , as shown in Equation 6.

In practice, the SSI algorithm involves some approximations due to:

- i. Modeling approximations and measurement inaccuracies
- ii. The use of finite datasets to compute estimates of the correlations
- iii. Non-stationarity and non-linearity behaviour in the data

As a result, the true model order of the system is typically masked by the appearance of spurious system poles which result from all the above effects. True or stable system poles can be distinguished from the spurious poles using stabilization diagrams, which allow the analyst to discriminate between them. A typical stabilization diagram is shown in Figure 2. Details of the SSI algorithm are given in the references (Peeters [2], Bogunović Jakobsen [5]).

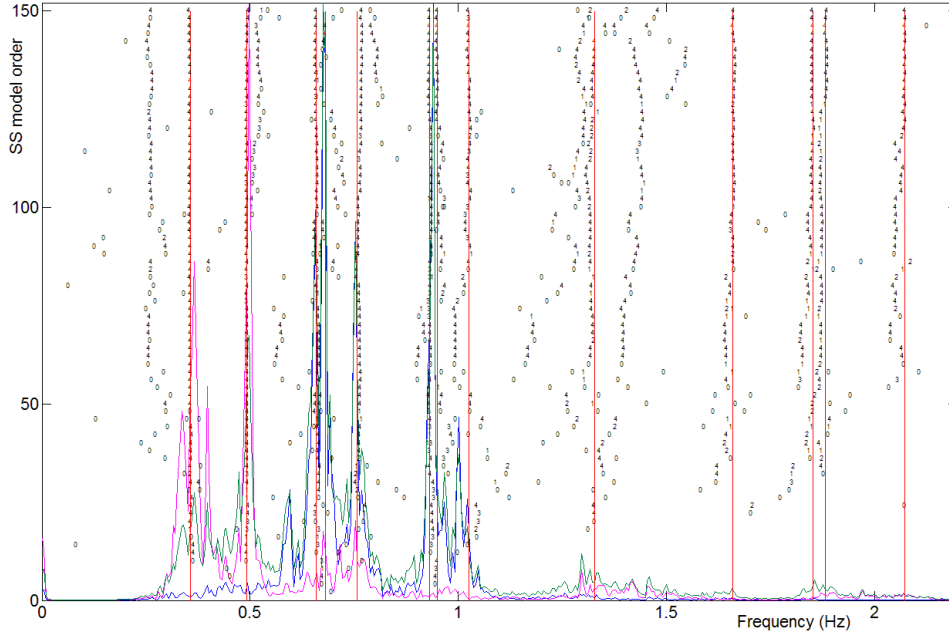


Figure 2: Stabilization Diagram

## CASE STUDIES USING FIELD MONITORING DATA OF CONFEDERATION BRIDGE

A series of case studies have been conducted involving the application of SSI to analyze Confederation Bridge dynamic field monitoring data. The first case study is aimed at verifying the design values of dynamic properties versus the actual measured properties retrieved by SSI (Londoño [6]). This case study is based on the analysis of four different monitoring datasets collected at different times and different loading scenarios. The results of the verification study are summarized in Table 1.

**Table 1: Verification of design values of dynamic properties, summary of results.**

Analytical Frequency (Hz)		Experimental Frequency (Hz)	Mode Type	Damping Ratio (%)	Modal Assurance Criterion value (MAC)	Excitation
Beam model	Shell model					
0.28	0.30	0.33 - 0.36	T	0.15 - 0.79	0.94	WS, W, Tr
0.33	0.35	0.39 - 0.40	T	0.35 - 1.02	0.93	WS, W
0.46	0.45	0.47 - 0.48	T	0.24 - 0.82	0.55	W, WS
0.58	----	0.65	V	0.10 - 0.76	0.86	W
0.62	0.62	0.65 - 0.71	V	0.24 - 1.48	0.96	WS, W
0.79	0.88	0.91	V	0.07	0.94	W
0.82	0.88	0.94 - 1.03	V	0.03 - 0.60	0.72	WS, W, Tr
1.08	----	1.17 - 1.21	V-L	0.39 - 0.81	0.78	Tr
1.60	1.64	1.62	V-L	0.04	0.83	W
2.34	----	2.69 - 2.83	V	0.05 - 0.25	0.94	Tr
3.15	----	3.30-3.50	V-L	0.01 - 0.40	0.57	Tr
----	----	3.38	To	0.1	----	Tr
----	----	4.63 - 4.81	V	0.01 - 0.38	----	W
4.43	----	4.96 - 5.20	V	0.04 - 1.05	0.85	Tr

The results of the verification study show a reasonable agreement between the expected design values of dynamic properties and the measured values retrieved from different monitoring datasets. However, as can be observed from the experimental frequencies and damping ratios, there are significant variations in the results identified. For example, for the first mode there is a 10% range of variation in the frequency retrieved from the four different datasets considered. This variability is a serious obstacle for the use of ambient field vibration monitoring data for structural condition assessment that relies upon the detection of changes in the dynamic properties because changes caused by damage or deterioration of the structure can be masked by normal variability of the data.

The potential causes of variation in the dynamic properties can be grouped as follows:

- (a) Environmental Effects (e.g. temperature, wind, moisture)
- (b) Differences in loading scenarios (traffic, wind, ice, earthquake)
- (c) Measurement inaccuracies, computational inaccuracies & deviations from modeling assumptions
- (d) Non-linearity & non-stationarity in the measured data
- (e) Stiffness degradation reflecting deterioration or damage of the structure

A thorough understanding of the normal variability of the identified dynamic properties from each of these causes is essential before vibration based structural health monitoring techniques can be applied reliably in the field. A second case study is conducted to evaluate the baseline level of variability attributable to the measurement, computational and modeling inaccuracies (Londoño [7]). In this variability study, ten ambient vibration datasets collected under similar loading scenario and similar environmental conditions over a four day period are processed and analyzed. The determination of this baseline level of variability is

necessary before the variability attributable to the other potential causes of variation can be investigated. The results of this baseline variability case study are summarized in Table 2.

**Table 2: Summary of Results of Baseline Variability Study**

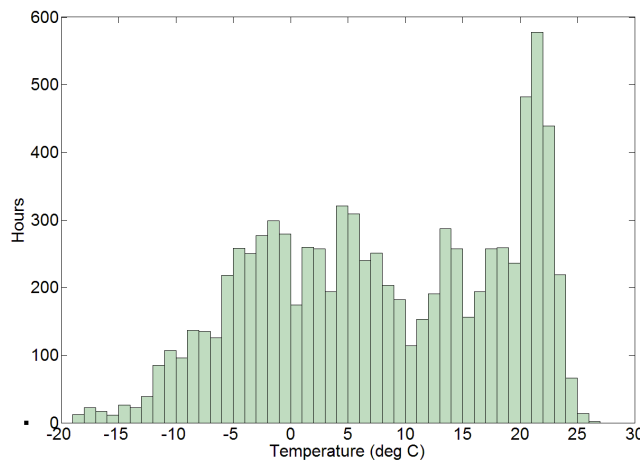
Mode Type	Mean Frequency (Hz)	$\sigma_f$ (Hz)	$\sigma_f$ (% of mean)	$\xi$ (%)	$\sigma_\xi$ (%)	$\sigma_\xi$ (% of $\xi$ )	MAC average
Transverse	0.474	0.003	0.6	1.53	0.61	40	98.7
Vertical	1.641	0.009	0.5	1.34	0.56	42	98.4
Vertical	1.828	0.008	0.4	1.56	0.53	34	99
Vertical	2.774	0.017	0.6	0.91	0.5	55	97.4

The results of this case study indicate that it is possible to retrieve highly consistent modal frequencies and mode shapes from monitoring datasets of similar excitation scenario and environmental conditions. Under the prescribed circumstances, the estimated frequencies of well-excited modes, retrieved from multiple datasets, show a relatively low level of variability. Standard deviations of the frequency estimates are below 0.6% of the mean. This finding indicates that the accuracy of the measured dynamic data and the identification algorithm is highly consistent.

The third and last case study addresses some of the other potential causes of variability, particularly the effects of the changes in the environmental conditions on the identified dynamic properties. The results of this case study are presented in details in the following section.

### Case Study of Environmental Effects on Measured Dynamic Properties

Forty-two datasets collected over a period of ten months of bridge operation under varying environmental conditions and loading scenarios have been analyzed to evaluate the potential effect of temperature and cross-section temperature differentials on the identified vibration properties of the Confederation Bridge. It is assumed here that during this period of time the structural condition of the bridge remains unchanged. The datasets used in the study are selected from the dynamic monitoring database based on the average concrete temperatures at the time of acquisition. A histogram showing the annual distribution of average concrete temperature for the Confederation Bridge is shown in Figure 3.

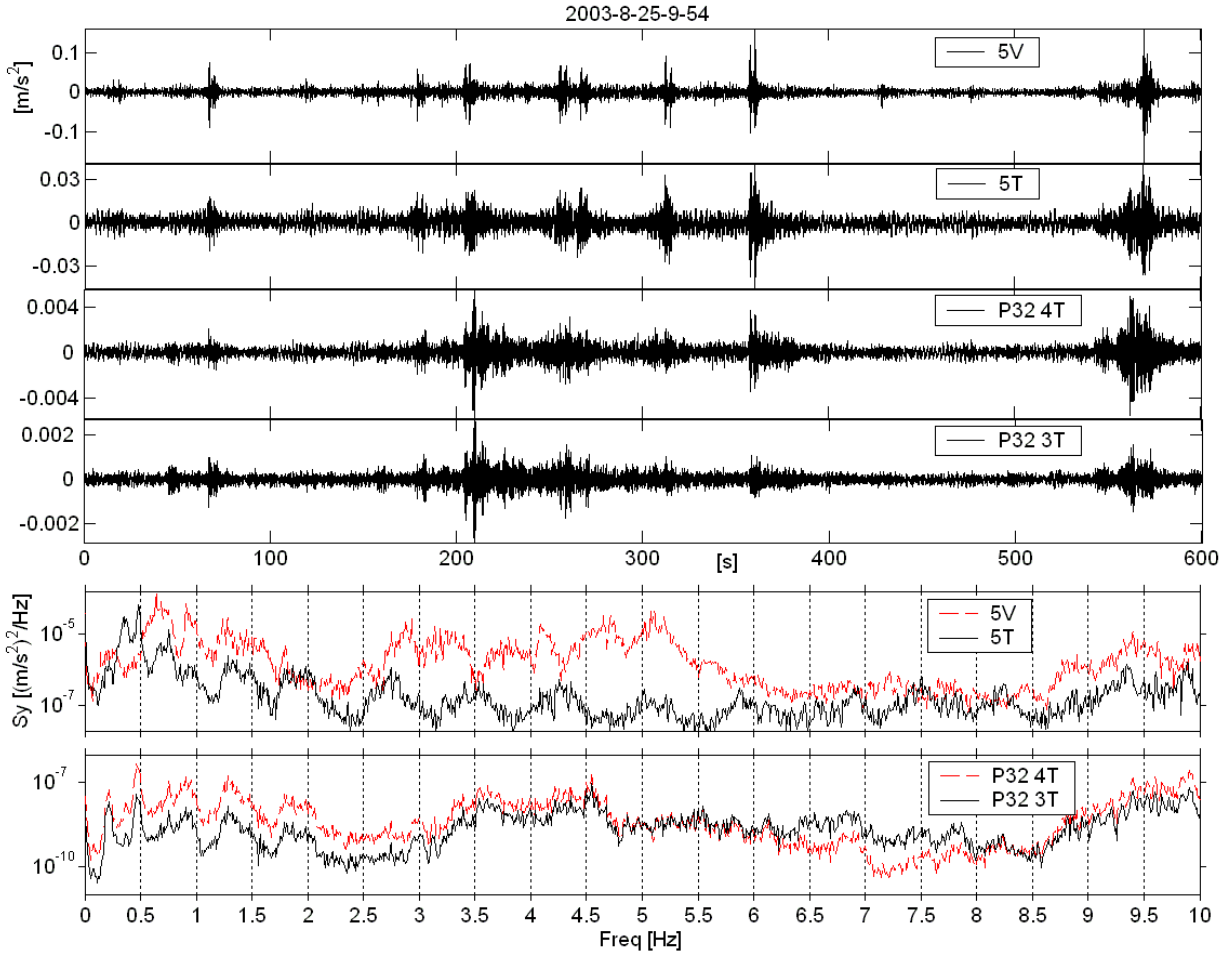


**Figure 3: Histogram of hourly average concrete temperatures of the Confederation Bridge for 2003**

To avoid bias and give equal weight to all temperatures, one dataset is selected per degree wide temperature range between -18°C to 25°C of average concrete temperatures. Each of the 10 minute duration datasets used in the study consists of the sampled acceleration responses from 19 sensors at locations 3 to 9 of Figure 1.



Figure 4 shows an acceleration time history from a typical monitoring dataset. Datasets are pre-processed as described earlier and analyzed using the SSI method to identify the structural vibration parameters of frequencies, mode-shapes and damping ratios in the 0 to 5 Hz range, which includes the most important global structural vibration modes of the bridge. Data from thermocouples embedded in the concrete at three different cross-sections are used to obtain average concrete temperatures and cross-section differentials. Details of the thermal monitoring of the Confederation Bridge are given in the reference (Li [8]).



**Figure 4: Typical ambient vibration acceleration time histories in vertical and transverse directions at locations 5, Pier 32-4 and Pier 32-3 and corresponding power spectral density estimates.**

Temperature averages and gradients are calculated as follows

$$\bar{T} = \frac{1}{A} \iint_A T(x, y) dA \quad (14)$$

$$\Delta T_x = \left( \frac{1}{I_y} \iint_A T(x, y) \cdot x dA \right) \cdot w \quad \text{or} \quad \Delta T_y = \left( \frac{1}{I_x} \iint_A T(x, y) \cdot y dA \right) \cdot h \quad (15)$$

where  $w$  and  $h$  are the girder cross-section width and depth respectively.

The transverse component of the wind speed and the mean of the Root Mean Square (RMS) amplitude values of all channels are also calculated and recorded for each event to study the potential effect of wind speed and amplitude on the variability of the identified structural parameters.

In the post-processing of the system identification results the Modal Assurance Criterion (MAC) is applied to distinguish between the different identified structural vibration modes. Structural vibration modes of the Confederation Bridge often occur at closely spaced frequencies and have similar vibration mode shapes in the bridge portion considered for the study. In this study it is desired to determine the variation in the properties of individual modes, thus it is necessary to distinguish and isolate particular modes from their closely spaced counterparts obtained from the identification of the different datasets.

Table A1 summarizes the values of all relevant environmental variables for each dataset and the modal frequencies retrieved by SSI for eight dominant vibration modes of the bridge. The statistical properties of the identified frequencies, damping ratios and mode shapes are given in Table 3.

**Table 3: Statistical properties of identified frequencies, damping ratios and mode shapes retrieved in the environmental effects case study**

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8
Frequency average (Hz)	0.474	0.665	1.634	1.820	2.769	3.387	4.641	5.143
std (Hz)	0.007	0.010	0.018	0.031	0.033	0.074	0.093	0.057
std (% of mean)	1.5	1.5	1.1	1.7	1.2	2.2	2.0	1.1
Damping average $\xi$ (%)	2.641	2.945	2.103	2.637	1.583	2.257	2.672	2.283
std ( $\xi$ %)	3.177	1.321	1.344	1.900	0.684	1.529	1.449	0.931
std ( $\xi$ % of mean)	120	45	64	72	43	68	54	41
MAC average	0.948	0.944	0.982	0.957	0.954	0.878	0.899	0.925

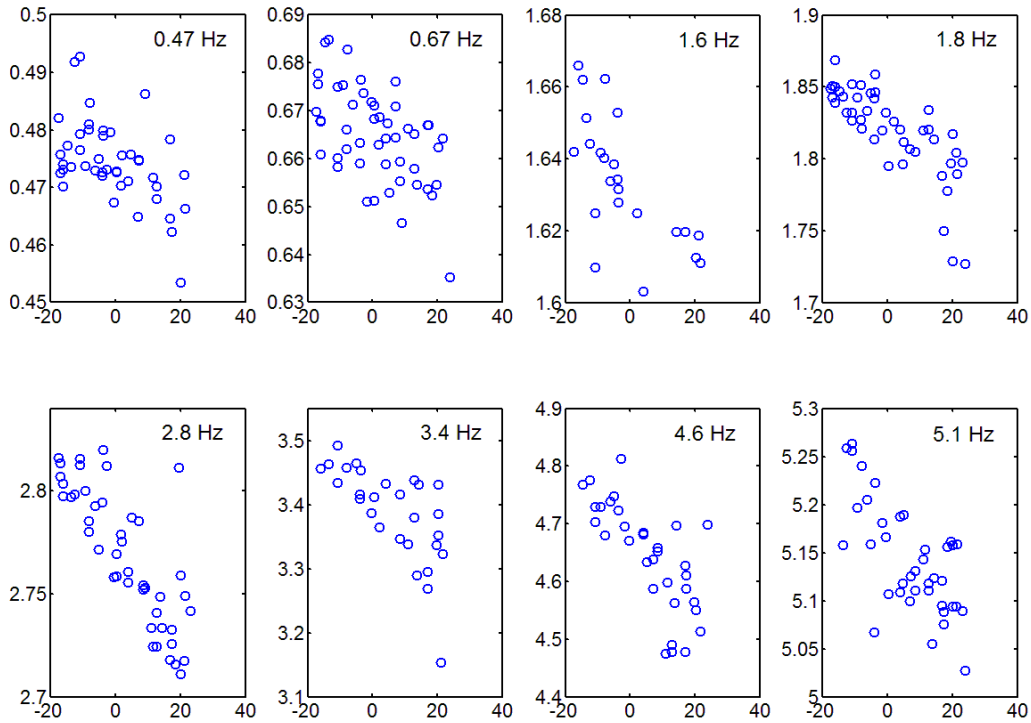
The frequencies of the eight different modes considered in this case study are plotted against temperature in Figure 5. As can be observed from the plots of Figure 5 there is a clear trend of reduction in the vibration frequencies with increase in temperature. This is likely a reflection of the changes in the material properties with temperature, in particular the elastic modulus of concrete.

A multiple linear regression analysis is carried out to investigate the correlation of the natural modal vibration frequencies with the environmental and amplitude variables. The linear regression model used in this case study is the following

$$Y_i = \beta_0 + \beta_1 \cdot x_{1i} + \dots + \beta_k \cdot x_{ki} + E_i \quad \text{for } i = 1, \dots, n \quad (16)$$

where  $n$  is the number of data points,  $x_j$  are the predictor variables,  $\beta_j$  are the regression coefficients,  $Y_i$  is the dependent variable and  $E_i$  is the error. In the present study the vibration modal frequency is the dependent variable, and the average concrete temperature, cross-section temperature differentials, average amplitude RMS, and wind speed are the predictor variables. The results of the regression analysis are presented in Table 4. The 95% confidence intervals for the regression coefficients are calculated, which are also presented in Table 4. These confidence intervals allow the identification of the variables most significantly correlated with the variations in the frequencies. Highlighted in Table 4 are the variables found to have a high correlation with frequency. For seven out of the eight modes, temperature shows the highest degree of correlation. As an example, for the 4.6 Hz mode, the frequency varies by 1.8 % for a 10°C change in temperature, which can amount to a 7.2 % yearly variation due to the annual temperature cycle at the bridge site of the Northumberland Strait. This observation demonstrates that the temperature

effects on the natural vibration frequencies are important aspects which should be considered in structural health monitoring of structures subjected to changing environments.



**Figure 5: Variation of the modal vibration frequencies (Hz) against temperature (°C) for the eight different modes retrieved in the environmental effects case study**

**Table 4: Results of multiple linear regression analysis**

		$\beta_0$	$\beta_{RMSv}$	$\beta_{RMSl}$	$\beta_T$	$\beta_{\Delta T_x}$	$\beta_{\Delta T_y}$	$\beta_{Wind\ Sp.}$
<b>0.47 Hz</b>	$\beta$	0.473	0.487	-0.544	<b>-0.000810</b>	0.00250	<b>0.00142</b>	0.000042
	95% c.i. for $\beta$	0.009	1.407	2.634	<b>0.000403</b>	0.00542	<b>0.00125</b>	0.000317
	95% c.i. for $\beta$ (%)	2.0	289	484	<b>50</b>	216	<b>88</b>	758
<b>0.67 Hz</b>	$\beta$	0.677	0.867	<b>-3.897</b>	0.000139	-0.00336	<b>-0.00224</b>	0.000096
	95% c.i. for $\beta$	0.012	1.752	<b>3.163</b>	0.000559	0.00545	<b>0.00165</b>	0.000425
	95% c.i. for $\beta$ (%)	1.7	202	<b>81</b>	402	162	<b>73</b>	445
<b>1.6 Hz</b>	$\beta$	1.633	0.642	0.277	<b>-0.001167</b>	-0.00991	0.00018	-0.00052
	95% c.i. for $\beta$	0.043	6.720	10.007	<b>0.001462</b>	0.02354	0.00425	0.00148
	95% c.i. for $\beta$ (%)	2.6	1047	3608	<b>125</b>	238	2409	284
<b>1.8 Hz</b>	$\beta$	1.809	2.409	1.088	<b>-0.001623</b>	-0.00358	-0.00161	<b>-0.00087</b>
	95% c.i. for $\beta$	0.030	4.680	8.100	<b>0.001541</b>	0.01334	0.00426	<b>0.00119</b>
	95% c.i. for $\beta$ (%)	1.6	194	744	<b>95</b>	373	265	<b>137</b>
<b>2.8 Hz</b>	$\beta$	2.776	<b>-3.629</b>	3.996	<b>-0.001522</b>	<b>0.01133</b>	-0.00068	0.00045
	95% c.i. for $\beta$	0.034	<b>4.937</b>	7.981	<b>0.001600</b>	<b>0.01254</b>	0.00443	0.00111
	95% c.i. for $\beta$ (%)	1.2	<b>136</b>	200	<b>105</b>	<b>111</b>	655	244
<b>3.4 Hz</b>	$\beta$	3.484	-4.798	-4.114	<b>-0.005784</b>	0.02017	0.00162	-0.00176
	95% c.i. for $\beta$	0.162	19.288	34.297	<b>0.005883</b>	0.07341	0.01705	0.00481
	95% c.i. for $\beta$ (%)	4.6	402	834	<b>102</b>	364	1053	274
<b>4.6 Hz</b>	$\beta$	4.716	<b>-21.123</b>	<b>28.288</b>	<b>-0.008383</b>	0.01449	0.00836	-0.00142
	95% c.i. for $\beta$	0.125	<b>19.150</b>	<b>37.231</b>	<b>0.005989</b>	0.05114	0.01856	0.00404
	95% c.i. for $\beta$ (%)	2.7	<b>91</b>	<b>132</b>	<b>71</b>	353	222	284
<b>5.1 Hz</b>	$\beta$	5.165	<b>7.829</b>	-11.649	<b>-0.004924</b>	0.01885	0.00253	-0.00057
	95% c.i. for $\beta$	0.073	<b>11.697</b>	21.009	<b>0.003836</b>	0.03287	0.01093	0.00265
	95% c.i. for $\beta$ (%)	1.4	<b>149</b>	180	<b>78</b>	174	433	469

## **PLATFORM APPLICATION FOR HEALTH MONITORING OF THE CONFEDERATION BRIDGE**

An application platform has been developed to facilitate the processing, analysis and interpretation of the large datasets associated with continuous monitoring. The development of such application tool is essential for rapid structural evaluations after the occurrence of severe events such as earthquakes. The case studies presented in this paper relied upon this tool to greatly reduce the amount of time and effort needed to obtain numerical results and observation findings from the monitoring data. The application platform consists of data visualization and animation, and data processing and analysis engines. The overall efficiency of the data analysis is improved by automation and centralizing the tasks of data processing, analysis, interpretation and visualization in one central computing platform.

The processing, visualization and analysis modules of the monitoring platform are all accessible through a single graphical user interface (GUI). The processing module receives as input the accelerometer measurements and automatically performs all pre-processing tasks discussed earlier. A copy of the data after each stage of processing is kept for further processing by the visualization and analysis modules. Once the processing of the acceleration time histories is complete they are numerically doubly integrated to obtain the corresponding displacement time histories.

The visualization modules include extensive plotting capabilities which provide a convenient environment to study large datasets within a reasonable time frame. Displacement and acceleration time histories and spectral plots can be easily manipulated to obtain the desired information on the behaviour of the structural system. Data may be viewed at any stage of processing to qualitatively evaluate the processing results. This module also includes a 3D bridge model for animation of bridge displacement responses and mode shapes. Bridge responses during an earthquake may be visualized, which provides valuable insight into the bridge behaviour. The animation module permits flexible user interaction. Parameters of the animations include scaling factor, view angle and playback speed. The animation and plotting capabilities are seamlessly integrated. There is also an option to record animation sequences for playback on common media players.

The analysis module incorporates the stochastic subspace identification algorithm presented earlier in this paper. User specified input parameters include: the number of time lags of the data time-correlations, the maximum state-space model order, reference channels, and the choice of background cross-spectra for the stabilization diagrams. The system identification GUI encompasses the process of identifying modes by pole-picking from the stabilization diagram, visualization of the identified mode shapes, and model validation by comparison of the non-parametric cross-spectra with plots of parametric cross-spectrum estimates constructed from the identified modes. Figure 6 shows the graphical interface of the system identification and visualization modules.

The processing and animation modules are designed and adapted to run in a real-time mode. A data management tool has been developed to automatically handle and redirect incoming data to the processing module and to animate the resulting bridge displacements in real-time. Responses of the bridge under normal and severe conditions may be viewed and assessed by operators of the bridge in near real-time with minimal time delay as limited by the network speed. The ability to visualize the bridge response in near real-time adds a significant amount of engineering value to the monitoring data especially during the occurrence of a major seismic event. The objective of the integrated real-time data processing and analysis platform is to facilitate timely condition assessment of the monitored facility based on evaluation of continuous dynamic monitoring data.

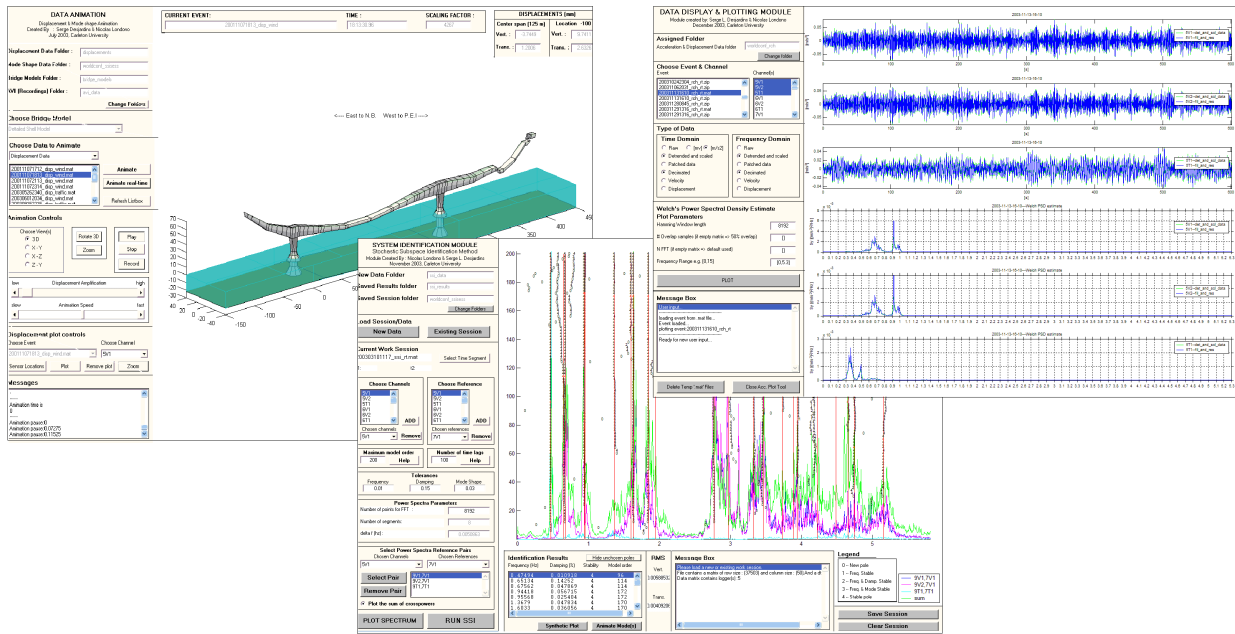


Figure 6: Graphical interfaces of some of the health monitoring platform application modules

## CONCLUSIONS

This paper presented the stochastic subspace method and case studies involving its application to the Confederation Bridge monitoring data. The case studies identified and addressed the normal variability of the dynamic properties of the bridge. The findings regarding the variability of the identified modal parameters are generally applicable to highway bridges subjected to varying environmental and loading conditions. Natural frequencies of the Confederation Bridge are found to have a significant correlation with the average temperature of the concrete. The understanding of this variability is essential and valuable for the ongoing development of vibration based condition assessment of highway bridges that can be applied reliably in the field.

## REFERENCES

1. Peeters B, De Roeck G. "Reference Based Stochastic Subspace Identification for Output-Only Modal Analysis." *Mechanical Systems and Signal Processing* 1999; 13(6): 855-878.
2. Peeters B. "System Identification and Damage Detection in Civil Engineering." PhD thesis, Department of Civil Engineering, Katholieke Universiteit Leuven, Belgium, 2000.
3. Montreuil MA, Lau DT, Brown TG. "A Distributed Data Acquisition System for Monitoring the Confederation Bridge." *Proceedings of the 44<sup>th</sup> International Instrumentation Symposium*, Reno, Nevada, U.S.A. May 3-7 1998; 318-333.
4. Cheung MS, Tadros GS, Brown T, Dilger WH, Ghali A, Lau DT. "Field Monitoring and Research on Performance of the Confederation Bridge." *Canadian Journal of Civil Engineering* 1997; 24: 951-962.
5. Bogunović Jakobsen J. "Fluctuating Wind Load and Response of a Line-like Engineering Structure with Emphasis on Motion-Induced Wind Forces." Ph.D. Thesis, Department of Structural Engineering, The Norwegian Institute of Technology, University of Trondheim, Norway, 1995.

6. Londoño NA, Lau, DT. "Verification of the Dynamic Properties of the Confederation Bridge." Proceedings of the annual conference of the Canadian Society for Civil Engineering, Moncton, Nouveau-Brunswick, Canada, June 4-7 2003.
7. Londoño NA, Lau DT, "Variability of dynamic properties from Confederation Bridge Monitoring Data." Proceedings of the first international conference on structural health monitoring and intelligent infrastructure, Tokyo, Japan, November 13-15, 2003.
8. Li D, Maes MA, Dilger WH. "Design Temperature Profiles for Bridge Decks: Experimental Data and Extreme Value Analysis." Proceedings of the annual conference of the Canadian Society for Civil Engineering, Moncton, Nouveau-Brunswick, Canada, June 4-7 2003.
9. Huang NE, Shen Z, Long SR, Wu MC, Shih HH, Zheng Q, Yen NC, Tung CC, Liu HH. "The Empirical Mode Decomposition and the Hilbert Spectrum for Nonlinear and Non-Stationary Time Series Analysis." Proceedings of the Royal Society of London. Series A. Mathematical, physical and engineering sciences, 1998; 454: 903-995.
10. Yang JN, Lei Y, Pan S, Huang N. "System identification of linear structures based on Hilbert-Huang spectral analysis. Part 2: complex modes." Earthquake Engineering and Structural Dynamics 2003; 32:1533-1554.

**Table A 1. Summary of parameters for environmental effects case study datasets**

Y	M	D	H	RMS <sub>t</sub> (milig)	RMS <sub>v</sub> (milig)	Temp (°C)	ΔT <sub>x</sub> (°C)	ΔT <sub>y</sub> (°C)	WS (m/s)	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6	Mode 7	Mode 8
2003	3	10	11	0.60	0.42	-5.1	0.91	-3.9	6.3	0.475	-	1.639	1.846	2.772	3.465	4.748	5.159
2003	5	26	23	0.63	0.31	11.0	-0.49	4.2	1.6	-	0.666	-	1.820	2.734	3.338	4.475	5.143
2003	5	30	8	1.10	0.56	12.9	0.10	4.0	5.1	0.468	0.658	-	1.820	2.741	3.438	4.477	5.118
2003	5	30	8	1.10	0.55	12.9	0.10	4.0	5.1	0.470	0.665	-	1.834	2.724	3.381	4.490	5.110
2003	6	17	13	0.76	0.54	14.3	-0.27	6.0	0.7	-	-	1.620	1.814	2.733	3.432	4.697	5.124
2003	6	23	9	1.14	0.57	21.2	0.47	9.2	0.4	0.472	-	1.619	1.805	2.717	3.153	-	5.094
2003	7	18	9	0.87	0.37	21.6	0.39	5.8	2.3	0.466	0.664	1.611	1.789	2.749	3.322	4.513	5.159
2003	8	13	22	0.39	0.25	24.0	0.22	10.1	0.7	-	0.635	-	1.727	-	-	4.699	5.027
2003	8	21	13	0.84	0.51	23.0	0.06	8.3	6.7	-	-	-	1.798	2.742	-	-	5.090
2003	8	25	9	0.75	0.53	17.0	0.07	2.3	12.8	0.465	0.667	-	-	2.718	3.269	4.627	5.121
2003	8	25	9	0.75	0.53	17.0	0.07	2.3	12.8	0.478	0.654	1.620	1.788	-	3.296	4.478	5.095
2003	9	11	19	0.80	0.33	17.3	-0.41	6.9	0.2	0.462	0.667	-	-	2.733	-	4.610	5.088
2003	9	11	19	0.80	0.33	17.3	-0.41	6.9	0.2	-	-	-	1.750	2.725	-	4.588	5.076
2003	9	12	18	0.79	0.26	18.4	-0.21	7.5	2.6	-	0.652	-	1.778	2.716	-	-	5.156
2003	9	25	22	0.63	0.39	20.3	0.82	6.4	2.6	0.453	-	1.612	1.817	2.711	3.431	-	-
2003	9	25	22	0.63	0.39	20.3	0.82	6.4	2.6	-	-	-	-	2.759	3.351	4.551	5.094
2003	9	25	22	0.63	0.39	20.3	0.82	6.4	2.6	-	0.662	-	1.728	-	3.385	4.551	5.158
2003	9	26	11	0.83	0.42	19.7	1.02	4.7	1.5	-	0.655	-	1.797	2.811	3.337	4.565	5.161
2003	10	16	7	0.59	0.33	13.8	0.71	3.8	5.0	-	0.655	-	-	2.748	3.289	4.563	5.055
2003	10	17	13	0.95	0.51	11.5	0.64	3.0	6.4	0.472	-	-	-	2.724	-	4.598	5.153
2003	10	19	16	0.56	0.26	9.1	-0.02	3.1	6.2	0.486	0.647	-	-	2.753	-	-	-
2003	10	24	23	0.49	0.28	7.0	0.46	1.0	15.4	0.465	0.676	-	1.807	-	-	4.587	5.100
2003	11	6	20	0.88	0.57	8.6	0.25	3.0	0.8	-	0.655	-	1.805	2.754	3.416	4.652	5.110
2003	11	6	20	0.88	0.57	8.6	0.25	3.0	0.8	-	0.659	-	-	2.752	3.346	4.659	5.131
2003	11	9	5	1.04	0.66	1.8	-0.12	-1.8	18.9	0.470	0.663	-	-	2.779	-	-	-
2003	11	13	16	1.27	0.71	4.7	0.05	1.4	16.9	0.476	0.667	-	1.796	-	-	-	5.118
2003	11	26	13	0.97	0.60	5.2	2.04	-0.2	5.6	-	0.653	-	1.812	2.787	-	4.633	5.190
2003	11	28	8	0.72	0.39	4.0	0.52	0.0	8.8	0.471	0.659	-	-	2.756	-	4.682	5.109
2003	11	28	8	0.72	0.39	4.0	0.52	0.0	8.8	-	0.664	1.603	1.821	2.761	3.432	4.684	5.187
2003	11	29	13	0.53	0.47	7.2	0.02	2.9	18.7	0.475	0.664	-	-	2.785	-	-	-
2003	11	29	13	0.53	0.47	7.2	0.02	2.9	18.7	0.475	0.671	-	-	-	-	4.639	5.126
2003	12	4	2	0.81	0.51	-2.7	0.14	-2.5	18.4	0.473	0.674	-	-	2.812	-	4.813	-
2003	12	9	10	0.87	0.47	-0.4	-0.05	-0.1	2.6	0.467	0.672	-	1.832	2.758	3.387	4.671	5.166
2003	12	12	2	0.60	0.39	0.5	0.56	0.3	19.9	0.473	0.671	-	1.795	2.769	3.412	-	5.107
2003	12	12	2	0.60	0.39	0.5	0.56	0.3	19.9	0.473	0.668	-	-	2.759	-	-	-
2003	12	12	2	0.60	0.39	0.5	0.56	0.3	19.9	-	0.651	-	-	-	-	-	-
2003	12	12	19	0.64	0.53	2.1	-0.05	1.3	12.2	0.476	0.669	1.625	1.826	2.775	3.365	-	-
2003	12	15	17	0.42	0.39	-3.7	0.00	-1.3	5.4	0.480	-	1.628	1.847	2.820	3.454	4.723	5.223
2003	12	15	17	0.42	0.39	-3.7	0.00	-1.3	5.4	0.479	0.677	1.632	1.859	-	-	-	-
2003	12	16	8	1.03	0.60	-1.6	-0.24	0.2	20.5	0.480	0.651	-	1.820	-	-	4.696	5.181
2004	1	7	16	0.78	0.49	-6.1	0.83	-3.6	9.0	0.473	0.671	1.634	1.833	2.793	-	4.739	5.205
2004	1	8	7	0.78	0.55	-9.1	0.12	-4.9	16.7	0.474	0.675	1.642	1.842	2.800	-	4.729	5.196
2004	1	9	0	0.68	0.42	-13.6	-0.06	-7.0	19.4	0.474	0.685	1.651	1.843	2.797	3.464	-	5.158
2004	1	10	22	0.47	0.42	-14.5	0.03	-5.2	8.6	0.477	0.684	1.662	1.847	-	-	4.768	-
2004	1	13	10	0.85	0.55	-12.4	0.81	-4.0	2.7	0.492	-	1.644	1.832	2.798	-	4.776	5.259
2004	1	14	5	0.75	0.50	-10.9	-0.07	-2.7	19.9	0.477	0.675	1.625	1.827	2.816	3.492	4.730	5.264
2004	1	14	5	0.75	0.50	-10.9	-0.07	-2.7	19.9	0.479	0.660	1.610	1.832	2.813	3.434	4.704	5.256
2004	1	14	20	0.70	0.70	-15.9	-0.57	-6.1	14.7	0.474	0.668	-	1.869	2.803	-	-	-
2004	1	14	20	0.70	0.70	-15.9	-0.57	-6.1	14.7	0.470	0.668	1.666	1.839	-	-	-	-
2004	1	14	20	0.70	0.70	-15.9	-0.57	-6.1	14.7	0.473	0.661	-	1.850	2.798	3.456	-	-
2004	1	14	22	0.40	0.46	-16.8	-0.52	-6.6	13.7	0.473	0.676	-	1.843	2.807	-	-	-
2004	1	14	22	0.40	0.46	-16.8	-0.52	-6.6	13.7	0.476	0.678	-	1.851	2.813	-	-	-
2004	1	16	23	0.67	0.54	-17.5	0.50	-4.5	18.5	0.482	0.670	1.642	1.849	2.816	-	-	-
2004	1	17	11	1.02	0.60	-10.9	0.09	-0.2	16.7	0.493	0.658	-	1.852	-	-	-	-
2004	1	17	18	0.88	0.46	-8.2	-0.01	1.0	21.5	0.481	0.662	-	1.852	2.780	-	-	-
2004	1	17	18	0.88	0.46	-8.2	-0.01	1.0	21.5	0.480	0.666	1.640	1.827	2.785	3.458	-	-
2004	1	17	20	0.66	0.32	-7.7	-0.04	1.0	20.4	0.485	0.683	1.662	1.821	-	-	4.681	5.241
2004	1	20	13	0.74	0.62	-4.0	-0.14	-1.1	14.2	0.472	0.659	1.653	1.814	-	3.410	-	-
2004	1	20	13	0.74	0.62	-4.0	-0.14	-1.1	14.2	0.473	0.663	1.634	1.842	2.795	3.417	-	5.068