13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 2600

A PROBABILISTIC APPROACH FOR SEISMIC ASSESSMENT OF R.C. STRUCTURES. I THEORY

Renato GIANNINI[1] **Gianmarco de FELICE**[1]

SUMMARY

A methodology for seismic safety assessment of r.c. structures is proposed, which is based on the estimate of a polynomial relation (response surface) giving the probability of failure of the structure as a function of the random variables that affect the response. The seismic fragility or risk is then evaluated through numerical integration.

INTRODUCTION

Evaluation of seismic risk to structures (whether in planning or existing) is a difficult but at the same time highly important task. As regards structural behaviour, although fairly reliable models are today available in the non-linear field, they tend not to be included in the framework of probabilistic analysis, also because of the long computation times involved. As regards the model of action, the use of samples of a stochastic process underwent severe criticism, since it was considered incapable of reproducing the whole complex reality and variability of seismic motion.

The only approach capable of utilising models, of both structure and action, which are adequately accurate and realistic, is simulation. However, as was underlined earlier, the Monte Carlo method, proves to be exceedingly onerous from the computational point of view, especially in the case of complex structures simulated by refined programmes, whose randomness depends on many variables.

In order to calibrate the partial safety coefficients used in the FEMA-350 guidelines, an efficient method [1] has been developed for evaluating the seismic risk of a structure. This method is based on a simple analytical formulation for the probability of failure, whose parameters are obtained on the basis of a limited number of analyses with different earthquakes, scaled to the intensity of the expected value for the ordinate of the site response spectrum, corresponding to the fundamental period of the structure. However so much efficiency is achieved only at the cost of restrictive hypotheses, that is by not taking account of the randomness of the structure's mechanical parameters and considering one failure mechanism only.

An alternative way of reducing the number of structural analyses required by simulation, already proposed several years ago by Veneziano [2] and recently taken up again [3], consists of using the statistic technique of the response surface. This method, which will be described hereafter, consists of determining, with a limited number of targeted experiments, a simple relation, generally of polynomial type, between the random variables and a structural response parameter measuring the performance of the structure. Such measurement, if expressed in terms of ground motion intensity (e.g. peak ground

[1] University Roma Tre, Department of Structures, via Vito Volterra, 62 – 00146 Rome, Italy

acceleration or spectral acceleration at the fundamental period of the structure), may be directly compared with the seismic hazard. The probability that the structural response will exceed a certain limit can be estimated by means of convolution with the probability density function of the hazard at the site and of the structural random variables, using as a model of structural behaviour the simple polynomial relation obtained earlier. The limit of the Response Surface Method (RSM) lies in the fact that it loses efficiency if the number of random factors considered is not sufficiently small (6-8). In fact the number of simulations necessary in order to construct the response surface grows exponentially with the number n of variables, so that if n is large, the number of experiments necessary becomes comparable with that required by the Monte Carlo method.

Recently an alternative procedure was proposed, known by the acronym EFA (Effective Fragility Analysis) by Giannini et al. [4], by means of which it is possible to achieve a significant reduction in the number of variables on which the response surface depends. In the versions presented earlier, the method did not have recourse to the response surface technique; the dependence of the probability of failure on the variables considered was expressed by means of a Taylor series expansion truncated after the second order. In the present work the EFA is integrated with the RSM in order to permit a less local estimate of probabilities and to further reduce the number of simulations necessary, taking into account seismic variability too.

Hereafter EFA will be briefly presented in its original version, then after a rapid illustration of the RSM, we will clarify how the two methods are joined together.

THE EFA METHOD

Assumptions

The ultimate resistance of a reinforced concrete structure depends on a large number of independent random variables: properties of the materials, effective capacity of the structural elements, lack of correlation between the effective capacities and the empirical formulae of assessment. This makes it very onerous to use the simulation methods mentioned heretofore. In fact every simulation requires repetition of the dynamic analysis of a non-linear model of the structure which, despite the important advances in computation mechanics, takes quite a considerable time which, multiplied by thousands of simulations, makes this approach almost impracticable.

However it is easy to see that many of these uncertainties have a negligible or even nil influence on the dynamic response of the structure. In fact the computation models, even the most refined, generally fail to directly foresee the limit state of the structure, but entrust such control to a subsequent comparison between certain response parameters (local or global deformation, tension, etc.) with corresponding capacities or resistances. The latter are determined by means of empirical formulae, the uncertainty of which depends on the degree of correlation with experimental results. Obviously a variation of these capacities has no influence on the structural response.

The mechanical properties of the materials, concrete and steel, certainly alter, not only the resistance, but also the dynamic response. However, if we separate randomness of the mechanical properties into two parts, one, average for the whole structure, and the other which expresses its local fluctuations, we may venture the hypothesis that the latter, while affecting the resistance, has only a limited influence on the response.

Therefore the random variables are divided into two groups, the first comprising the variables (defined as *internal* and indicated hereafter by X), which it is assumed do not influence the dynamic response of the structure, and the second consisting of what are called the *external* variables (indicated by Y), responsible among other things for the dynamic behaviour. The expedient of separating the random variables makes it possible to greatly reduce the number of analyses implied by the simulation process; in fact, after fixing the variables of the second group (external) and the action, the structural response proves to be deterministic and thus calculation of the probability of failure (conditioned by the values of the external variables) may be carried out relatively rapidly.

Calculation of conditioned probability

With reference to a limit state defined by the equation $g(\mathbf{X},t)=0$, the condition of survival of the structure may be expressed in the form $\min_{t\in T}\{g(\mathbf{X},t)\}>0$ and accordingly

$$P_S = \Pr\left[\min_{t\in T}\{g(\mathbf{X},t)\}>0\right] \tag{1}$$

Since the function g has an explicit form and the parameters depending on time are known, the value of P_S may be obtained in an inexpensive way using the Monte Carlo (MC) method improved, if necessary, with importance sampling. In fact each simulation merely requires computation of function g, which often has a fairly simple form.

Such computation becomes more rapid when the limit state equation can be expressed in the form

$$g(\mathbf{X},t) = c(\mathbf{X}) - d(t) \tag{2}$$

where $c(\mathbf{X})$ represents the capacity of the structure, not dependent on time t, while $d(t)$ expresses the deterministic demand, obtained from the results of the dynamic analysis. In such case obviously:

$$P_S = \Pr\left[\min_{t\in T}\{g(\mathbf{X},t)\}>0\right] = \Pr\left[c(\mathbf{X}) - \max_{t\in T}\{d(t)\}>0\right] = \Pr[c(\mathbf{X}) - d_{\max} > 0] \tag{3}$$

This probability, or its complementary $P_F = \Pr[c(\mathbf{X}) - d_{\max} \le 0]$, may be computed using the usual methods of time-invariant reliability, for example by means of the First Order Reliability Method (FORM).

Obviously in a real structure, account must be taken of many alternative mechanisms, which concern different elements or failure modes. If the structure is treated as a system in series, it fails as a result of the failure of the weakest component and therefore the condition for survival is that no limit state has been exceeded, that is:

$$P_S = \Pr\left[\bigcap_{k=1}^{n}\left(\min_{t\in T}\{g_k(\mathbf{X},t)\}>0\right)\right] \tag{4}$$

and accordingly the probability of failure

$$P_F = \Pr\left[\bigcup_{k=1}^{n}\left(\min_{t\in T}\{g_k(\mathbf{X})\}\le 0\right)\right] \tag{5}$$

By applying the MC method, the calculation of P_F could be carried out contemporaneously for all the mechanisms. However if, as often happens, many of the mechanisms are not very well correlated, convergence of the procedure could be considerably slowed down. In this case it is preferable to compute separately the probabilities of failure for each mechanism, and for the intersections two by two, that is:

$$P_{F_k} = \Pr\left[\left(\min_{t\in T}\{g_k(\mathbf{X}_k)\}\le 0\right)\right] \tag{6}$$

$$P_{F_{kj}} = \Pr\left[\left(\min_{t\in T}\{g_k(\mathbf{X}_k)\}\le 0\right)\cap\left(\min_{t\in T}\{g_j(\mathbf{X}_j)\}\le 0\right)\right] \tag{7}$$

and thus estimate the probability of the union using, for example, the bounds of Ditlevsen [5]. In (6) and (7) $\mathbf{X}_k$ indicates the appropriate sub-set of random variables (r.v.) $\mathbf{X}$ involved in the k-th mechanism. When the limit state equations can be expressed in form (2), eqs. (6) and (7) can be evaluated by FORM using formulation (3).

The probability of failure computed in the abovementioned way is obviously conditioned to the value $\mathbf{y}$ of external r.v. $\mathbf{Y}$ and to the particular accelerogram $a(t)$ utilised in the analysis. It is therefore necessary to make this probability no longer conditional on the external random variables. This could be achieved by using the MC method. The drastic reduction in the number of external r.v. gathered in $\mathbf{Y}$, speeds up

convergence of the method and accordingly reduces the number of simulations necessary to estimate P_F, notwithstanding which this number remains very large, and the cost of their computation high.
In the first version of the method, the dependence of P_F on $\mathbf{Y}$ was approximated by using a quadratic expansion of P_F around a design point $\mathbf{y}_0$:

$$P_F(\mathbf{y}) = P_F(\mathbf{y}_0) + \frac{\partial P_F}{\partial \mathbf{y}} \cdot (\mathbf{y} - \mathbf{y}_0) + \frac{1}{2}\left[\frac{\partial^2 P_F}{\partial \mathbf{y}^2}(\mathbf{y} - \mathbf{y}_0)\right] \cdot (\mathbf{y} - \mathbf{y}_0) + o\left(|\mathbf{y} - \mathbf{y}_0|^2\right) \tag{8}$$

The coefficients of the expansion $\frac{\partial P_F}{\partial \mathbf{y}}$ and $\frac{\partial^2 P_F}{\partial \mathbf{y}^2}$, were computed using a finite differences scheme around the design point. However this approach presents some drawbacks: it is not coherent, since P_F may assume values that are either negative or greater than one, and it is reliable only if the Probability Density Function (PDF) of $\mathbf{Y}$ diminishes rapidly with $|\mathbf{y} - \mathbf{y}_0|$.
In the present work the dependence of P_F on $\mathbf{y}$ is estimated using the RSM. This method not only provides a rational criterion for obtaining a non-local approximation of the variable sought, but makes it possible to estimate the lack-of-fit of the model and to take account of the random effects due to earthquake samples with a small number of experiments.
In the next paragraph a brief description will be given of the RSM, while in the following one an attempt will be made to clarify its application to EFA.

THE RESPONSE SURFACE METHOD

The response surface method is a statistic technique for determining the parameters of a function that approximate an unknown function $Y(\mathbf{x})$, in the neighbourhood of a point $\mathbf{x}_0$, on the basis of the values of Y obtained by means of appropriate experiments.
The approximating function is generally expressed in quadratic form with respect to the variables $\mathbf{x}$:

$$Y = \theta_0 + \sum_{i=1}^{m} \theta_i x_i + \sum_{i=1}^{m}\sum_{k=1}^{m} \theta_{ik} x_i x_k + \varepsilon \tag{9}$$

ε is a zero-mean random variable which takes account of the lack of fit of the model. The (9) may be more concisely written in the form:

$$Y = \mathbf{v}(\mathbf{x})\boldsymbol{\theta} + \varepsilon \tag{10}$$

where

$$\mathbf{v}(\mathbf{x}) = \begin{bmatrix} 1 & x_1 & \cdots & x_m & x_1^2 & x_1 x_2 & \cdots & x_m^2 \end{bmatrix} \tag{11}$$

and $\boldsymbol{\theta}$ is a vector constructed with the parameters θ_i and θ_{ij}.
The model expressed by (10) is linear in the parameters $\boldsymbol{\theta}$: as a result it gives a r.v. whose average is $\mathbf{v}(\mathbf{x})\boldsymbol{\theta}$ while the dispersion is expressed by means of the term ε. By carrying out n experiments in connection with the values $\mathbf{x}_i$ of the variables, we obtain n values of response y_i, which may be gathered in the vector $\mathbf{y}$. Accordingly we have:

$$\mathbf{y} = \mathbf{V}\boldsymbol{\theta} + \boldsymbol{\varepsilon} \tag{12}$$

in which $\mathbf{V}$ is the matrix constructed with $\mathbf{v}(\mathbf{x}_i)$:

$$\mathbf{V} = \begin{bmatrix} \mathbf{v}(\mathbf{x}_1) \\ \vdots \\ \mathbf{v}(\mathbf{x}_n) \end{bmatrix} \tag{13}$$

and $\boldsymbol{\varepsilon}$ is a vector of n independent realisations of the r.v. ε, which measures the discrepancy between observation and the model.

The coefficients $\boldsymbol{\theta}$ may be determined on the basis of (12) by applying the criterion of maximum likelihood or, more simply, with the method of least square. Assuming that $\boldsymbol{\varepsilon}$ is a vector of Gaussian variables, both methods give the same estimate for $\boldsymbol{\theta}$:

$$\hat{\boldsymbol{\theta}} = \left(\mathbf{V}^T\mathbf{V}\right)^{-1}\mathbf{V}^T\mathbf{y} \tag{14}$$

The variance of ε may be estimated on the basis of the discrepancies between the observed values and the forecasts of the model $\hat{\mathbf{y}} = \mathbf{V}\hat{\boldsymbol{\theta}}$. Given $\mathbf{r} = \mathbf{y} - \hat{\mathbf{y}}$, an estimate of the variance of ε without bias is the quadratic difference :

$$s_\varepsilon^2 = \frac{\mathbf{r}\cdot\mathbf{r}}{n-m} \tag{15}$$

where n indicates the number of experiments and m the number of parameters gathered in the vector $\boldsymbol{\theta}$. The equation (15) shows that, in order to reduce the dispersion of error, it is necessary to carry out a considerably larger number of experiments than that of the parameters.

If in (14) we substitute for the vector of results of the experiments $\mathbf{y}$, a vector of $\mathbf{Y} = \mathbf{V}\boldsymbol{\theta} + \boldsymbol{\varepsilon}$, the result $\hat{\boldsymbol{\theta}}$ is a random vector with average $E\left[\hat{\boldsymbol{\theta}}\right] = \boldsymbol{\theta}$ and covariance

$$\mathrm{Cov}\left[\hat{\boldsymbol{\theta}}\right] = \left(\mathbf{V}^T\mathbf{V}\right)^{-1}\mathbf{V}^T\,\mathrm{Cov}[\mathbf{Y}]\mathbf{V}\left(\mathbf{V}^T\mathbf{V}\right)^{-1} = \sigma_\varepsilon^2\left(\mathbf{V}^T\mathbf{V}\right)^{-1} \tag{16}$$

since $\mathrm{Cov}[\mathbf{Y}] = \sigma_\varepsilon^2\mathbf{I}$. It follows that the estimate of response $\hat{y} = \mathbf{v}(\mathbf{x})\hat{\boldsymbol{\theta}}$ is in its turn a r.v. with variance

$$\mathrm{Var}[\hat{y}] = \mathbf{v}(\mathbf{x})\mathrm{Cov}\left[\hat{\boldsymbol{\theta}}\right]\mathbf{v}^T(\mathbf{x}) = \sigma_\varepsilon^2\mathbf{v}(\mathbf{x})\left(\mathbf{V}^T\mathbf{V}\right)^{-1}\mathbf{v}^T(\mathbf{x}) \tag{17}$$

Equation (17) shows that the variance of $\hat{y}$ depends, not only on the variance of ε and of the matrix $\mathbf{V}$, but also on point $\mathbf{x}$ where the estimate is made. The design of experiments which makes the variance of $\hat{y}$ dependent exclusively on the distance from the central point, is defined as rotatable. The conditions for rotatability may be found in the specialised texts (Box and Draper [6]).

The minimum number of experiments necessary for determining $\hat{\boldsymbol{\theta}}$ is equal to the number of the parameters but, as has been seen, this number must be increased if the estimate of $\boldsymbol{\theta}$ is to be sufficiently reliable; accordingly a compromise must be found between the two contrasting requirements of accuracy and economy. The choice of a rational project for the experiments is thus an essential point in the procedure.

A complete factorial project consists of fixing two levels for each of the variables m and then carrying out 2^m experiments corresponding to all of their possible combinations. This project does not explore the effects of the variations of a single variable at a time and accordingly does not permit an accurate estimate of pure quadratic terms of (9). Therefore it is customary to combine the factorial design with a "star" design, forming what is called a *central composite design* (Fig.1). Denoting by x_i^- e x_i^+ the two levels for each variable in the factorial design, the central point has coordinates $x_{0i} = \frac{x_i^- + x_i^+}{2}$ and thus, utilising the coordinate transformation $x_i = x_{0i} + \Delta_i\xi_i$, in which $\Delta_i = \frac{x_i^+ - x_i^-}{2}$, a standardised space is obtained in which the points of the factorial experiments are the apices of a hypercube centred on the origin located in the points of coordinates $\xi_i = \pm 1$. The star part of the experiment is represented by points of coordinates $\left\{\xi_i = \pm\alpha,\ i = 1,\ldots,m;\ \xi_j = 0, \forall j \neq i\right\}$. The value of α is established by the conditions of rotatability of the design: in the absence of repetitions $\alpha = 2^{m/4}$.

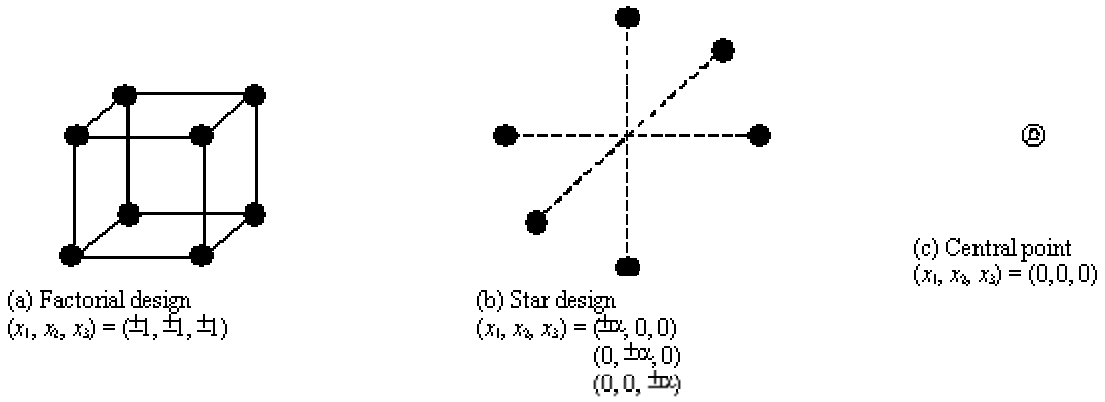

Figure 1. Diagram of a central composite design of experiments

When the experiment consists of a simulation with a mathematical model, if the variables in $\mathbf{x}$ completely define the model, the response is clearly deterministic and accordingly repetition of the experiment is useless. When on the contrary the experiment is carried out in the field, the result always depends on a large number of parameters, many of which cannot be controlled, so that, if the experiment is repeated for the same values of $\mathbf{x}$, in general different results are obtained. This fact may also be considered in the case of numerical simulations, by introducing other parameters, not included in vector $\mathbf{x}$, which vary randomly between one simulation and another. Clearly, in order for this procedure to be efficient, random effects should not mask the fixed effects depending on $\mathbf{x}$.

In order to take random effects into account, an additive random term δ should be added into equation (9). One possible way of estimating this term consists of repeating the whole block of experiments, varying the random factors. However this procedure is unattractive because of the large number of experiments required. A more performing method consists of subdividing (the factorial part of) the experimental design into as many blocks as there are random factors.

The division of the experiments into blocks implies a certain loss of information; it could be shown that, assuming random effects to be of additive type, block design will mask the interactions between some of the variables governing the fixed effects. The order of effects that are masked depends on the dimensions of the blocks and on the way in which the experiments are divided into the blocks. The division of experiments into blocks must ensure that only interactions of the of the higher order are masked, as illustrated in the specialised literature [6]. In any case, the blocks cannot be too small; therefore, if the number of blocks necessary for estimating random effects is large compared to the number of factorial experiments, it would be advisable to repeat twice or more the whole programme of experiments, using different random factors in the various repetitions.

If $\mathbf{y}$ is the vector of the results of experiments, $\mathbf{V}(\mathbf{x})$ is the matrix of the polynomial terms already seen in (12) and $\boldsymbol{\theta}$ is the vector of the unknown parameters, the model with random effects could be formulated as follows:

$$\mathbf{y} = \mathbf{V}(\mathbf{x})\boldsymbol{\theta} + \mathbf{B}\boldsymbol{\delta} + \boldsymbol{\varepsilon} \tag{18}$$

in which $\mathbf{B}$ is a matrix $n \times b$ which combines the b random effects contained in $\boldsymbol{\delta}$ with the n experiments, according to the diagram of division into blocks. Thus, if experiment i is combined with block k, this gives B_{ik} =1, otherwise B_{ik}=0. Matrix $\mathbf{B}$ introduces a correlation between the terms of vector $\mathbf{y}$ that does not allow determination of the coefficients $\hat{\boldsymbol{\theta}}$ independently from the variances of δ and of ε. The procedure for estimating $\hat{\boldsymbol{\theta}}$, σ_δ and σ_ε, based on the criterion of maximum

likelihood, becomes more complex, requiring an iterative method, since the problem is non-linear. More details may be found in the specialised literature (Searle et al. [7]).
In conclusion, for the given the vector of r.v. $\mathbf{X}$, once the dependence of the response parameter Y on $\mathbf{X}$ has been approximately established on the basis of the response surface, it should be possible to express Y in the form:

$$Y = \mathbf{v}(\mathbf{x})\hat{\boldsymbol{\theta}} + \delta + \varepsilon \tag{19}$$

which clearly shows that Y is a r.v. depending not only on $\mathbf{x}$, but also on δ and ε which represent the random factors introduced by the random effects and by the lack-of-fit of the model, respectively.

APPLICATION OF RSM TO EFA

As we have seen in a previous section, the probability of failure P_F conditional on given values of the external variables $\mathbf{Y}$ can be computed for a particular accelerogram according to the EFA method. In order to obtain the unconditional probability of failure, a functional relationship between of P_F and $\mathbf{Y}$ should be established using the RSM.
As a first step utilising the RSM in this context, we should ensure that no values of P_F less than zero or greater than one are attained. This will be done by applying to P_F a transformation which projects the interval [0.1] over the whole real axis. The most obvious of these transformations is the inverse of the standard normal Gauss distribution Φ^{-1}, so that P_F is transformed into the reliability index β:

$$\beta = -\Phi^{-1}(P_F) \tag{20}$$

The subsequent considerations are accordingly applied to β instead of P_F directly.
The result of each simulation obviously depends not only on the values assigned to the external variables, but also to the ground motion utilised in the analysis. In order to account for such a dependence, an intensity measure of the earthquake, such as a peak ground or an elastic spectral value, should be defined and included among the random variables responsible for the fixed effect. However this does not cover completely the effect of ground motion variability on randomness of the response, since different profiles of ground motion with the same earthquake intensity may produce different outputs giving rise to different values of P_F. The effect of ground motion profile, with parity of ground motion intensity, may carry out the role of the random effect in the RSM described in the previous paragraph. As illustrated heretofore, account may be taken of this effect without increasing the number of experiments, that is, of dynamic analyses of the structure. The design of numerical simulations to be carried out is divided into as many blocks as the number of earthquake samples used.

EARTHQUAKE SAMPLES AND SEISMIC INTENSITY MEASURE

The applicability of the RSM to EFA is conditioned by the fact that the random effect introduced by the choice of a ground motion profile should not be too great, so as not to cover the fixed effects due to the parameters controlled in the simulation. The dispersion introduced by the variation of ground motion profile from sample to sample, at parity of intensity, depends to a large extent on the chosen intensity measure. This problem is not very important if artificially generated accelerograms as samples of a random process are employed, since the response scatter is quite low; however this way of proceeding is today generally refused, since it is considered that these samples do not satisfactorily describe the actual variability of seismic action. Alternatively, recourse may be had to recorded natural accelerograms, but there are some difficulties in ensuring the standardisation of natural accelerograms by adopting a single measure of ground motion intensity. Various criteria have been proposed (Kurama and Farrow [8]), but, besides its efficiency in reducing the variability in structural response, the chosen intensity, should also

ensure hazard computability. Nowadays the hazard is normally provided either in terms of peak ground acceleration, velocity or displacement, or in terms of the ordinates of the elastic response spectrum. At present, the spectral value for the fundamental elastic mode of the structure (Shome et al. [1]), appears to provide the lowest scatter. For elastic structures dominated by the first mode, the criterion clearly provides almost nil dispersion; but when the structure becomes non-linear, it modifies its characteristics, and efficacy is less guaranteed, especially for structures with a small period ($0.2 \div 0.4$ sec). In this range the spectra vary very rapidly with period, and even a slight change in the mechanical properties (either as an effect of non-linear behaviour, or as a result of the variation in mechanical properties) may produce a high scatter in the response.

In order to divide the experiments into blocks, in particular as regards its factorial part consisting of 2^m analyses, the number of random effects must be a power of 2. It would seem reasonable that the number of natural accelerograms samples to be considered, should be either 8 or 16.

The design of experiments depends to a large extent on the number of variables contained in the vector $\mathbf{y}$. If this is high, the total number of experiments required is high, and accordingly also a division into 8 or 16 blocks may be accepted; if not, it would be advisable to divide the experiment into a smaller number of blocks and to repeat it integrally, combining this with a further series of accelerograms. For example, if the terms in $\mathbf{y}$ are 6, the factorial part consists of $2^6 = 64$ experiments, which can be subdivided into 8 blocks, each of which consists of 8 experiments. The star part of the design could combine 6 of the 8 samples, one for each variable, while at the central point of the design, the analyses for all the samples could be repeated. In this way it would be necessary to carry out $2^6 + 2 \times 6 + 8 = 84$ analyses for the whole experimental plan. In the case of 4 variables only, the number of factorial experiments is $2^4 = 16$, which clearly cannot be divided into 8 blocks without confusing the fixed effects. It is advisable in that case to repeat the whole experimental plan, dividing each one into 4 blocks, so that the total number of analyses becomes $2 \times \left(2^4 + 2 \times 4 + 4\right) = 56$, which is not very different from that of the preceding case.

COMPUTATION OF UNCODITIONAL PROBABILITY

Once the relation between β and $\mathbf{y}$ has been established in explicit form, the probability of failure P_F is obtained by inverting (20):

$$P_F(\mathbf{Y}, \delta, \varepsilon) = \Phi[-(\mathbf{v}(\mathbf{Y})\boldsymbol{\theta} + \delta + \varepsilon)] \tag{21}$$

where δ and ε are Gaussian r.v. with zero-mean and known standard deviations σ_δ e σ_ε. Vector $\mathbf{Y}$ collects the variables controlling the fixed effects. In order to compute unconditional probability, it is therefore necessary to carry out a multidimensional convolution between the function (21) and the probability density functions of the variables to be saturated (δ , ε and the components of $\mathbf{Y}$). If the variables in $\mathbf{Y}$ include seismic intensity, we are faced with two possible alternatives. Either the probability of failure is not saturated with respect to seismic intensity, giving rise to the fragility curve of the structure or, a convolution with the hazard curve at the site is performed, giving rise to the seismic risk.

Denoting by $\overline{\mathbf{Y}}$ the vector, sub-set of $\mathbf{Y}$, of the variables to be saturated, the probability of failure, conditional on the remaining $\tilde{\mathbf{Y}}$, is :

$$P_F\left(\tilde{\mathbf{Y}}\right) = \int_{R^m} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_F\left(\mathbf{Y}, \delta, \varepsilon\right) f_{\overline{\mathbf{Y}}}\left(\overline{\mathbf{y}}\right) f_\delta\left(\delta\right) f_\varepsilon\left(\varepsilon\right) d\overline{\mathbf{y}} d\delta d\varepsilon \tag{22}$$

If $\tilde{\mathbf{Y}}$ is reduced to the parameter of seismic intensity alone, (22) gives the function of fragility of the structure.

The integral (22) should be computed numerically. Given the nature of the integrated functions, one appropriate method could be the Monte Carlo method itself.

The fact that, at the end of this complex procedure, we have reverted to the MC method may appear circular. But we should bear in mind that now the integrand function in (22) has a simple explicit form and its evaluation requires only few operations at a limited computational cost.

CONCLUSIONS

In this work a procedure for evaluating the seismic risk of structures is presented, which makes it possible to obtain accurate results, using refined models for the structural analysis and natural accelerograms at low computational cost. This method is based on a combination of the EFA (Effective Fragility Analysis) method and the RSM (Response Surface Method). Random variables both for the structural model and for the action are taken into account. The random variables referring to the structure are divided into two groups, one of which contains all of those variables (internal) that, while influencing the resistance, have little effect on the dynamic response; whereas the other collects the remaining structural variables (external) that strongly affect the dynamic response. Once the external variables have been fixed, and a ground motion at the base of the structure selected, the conditioned probability of failure (P_f) is computed using standard reliability methods (FORM). The response surface method is then used to establish a function approximating the dependence of P_f on external variables and on seismic intensity. Random effects, due to the variability between the seismic samples used in the simulations, are taken into account by dividing the experimental plan into blocks, without increasing the number of analyses. The fragility curve of the structure is finally obtained by convolution with the probability density functions of external random variables.
The procedure is then applied in a companion paper [9] to assess the seismic fragility of three existing r.c. bridges in the Italian highway network.

REFERENCES

1. Shome N., Cornell C., Bazzurro P., Carballo J. "Earthquakes, records, and nonlinear responses". *Earthquake Spectra*, 1998, **14**: 469–500.
2. Veneziano D., Casciati F., Faravelli L. "Method of seismic fragility for complicated systems", *Proc. 2nd Committee of Safety of Nuclear Installation (CNSI) Specialist Meeting on Probabilistic Methods in Seismic Risk Assessment for NPP,* Lawrence Livermore Laboratory, CA, 1983.
3. Franchin P., Lupoi A., Pinto P.E. "Seismic fragility of reinforced concrete structures using a response surface approach". *Journal of Earthquake Engineering,* 2003, **7**: 45-77.
4. Giannini R., Pinto P.E., de Felice G. "Seismic reliability assessment of RC structures". Submitted to *Asce Jorunal of Structural Engineering*, 2003.
5. Ditlevsen O. "Narrow reliability bounds for structural systems". *Journal Struct. Mech.*, 1979, **7**(4): 453–472.
6. Box G.E.P., Draper N.R. *Empirical Model-Building and Response Surfaces*, John Wiley & Sons, 1987.
7. Searle S.R., Casella G., McCulloch C.E. *Variance Components,* Prentice-Hall, 1992.
8. Kurama Y.C., Farrow K.T. "Ground motion scaling methods for different site conditions and structure characteristics". *Earthquake Engineering and Structural Dynamics*, 2003, **32**: 2425–2450.
9. de Felice G., Giannini R., Rasulo A. "A probabilistic approach for seismic assessment of r.c. structures: application to highway bridges", *13th WCEE*, Vancouver, Canada. Paper no. 2601, 2004.