13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 2280

APPLICATION OF NON-STATIONARY SEISMOLOGICAL MODELS TO THE DETERMINATION OF STOCHASTIC RESPONSE SPECTRA

Ignacio FERRER[1] and Carlos R. SÁNCHEZ-CARRATALÁ[2]

SUMMARY

This paper presents an analytical method for calculating stochastic response spectra from non-stationary seismological models. The method takes into account the non-stationary character of the seismic input and the transient-response phase; this makes it possible to apply the advances in the Seismology field to the most recent developments in the Earthquake Engineering field. These developments are related to the application of non-stationary stochastic processes to calculate the response of a single degree-of-freedom oscillator. The first part of the paper presents a brief review of stationary and non-stationary seismological models, emphasizing the application of the evolutionary process concept to the formulation of a non-stationary seismological model from an underlying stationary model in which a suitable duration definition has been used. Next sections deal with the characterization of the oscillator response when excited by a non-stationary seismic action and the subsequent calculation of the stochastic response spectrum. The final part of the paper involves a parametric study of the influence of some seismological and oscillator parameters on the response spectrum, using a European event as the reference earthquake; the resulting stochastic response spectra are then compared with the corresponding Eurocode 8 design response spectra.

INTRODUCTION

In recent decades, many researchers in the field of Engineering Seismology have been working on the stochastic characterization of seismic ground motion by the application of seismological models. At first, the proposed models were stationary, as the well-known Boore's model [1]. More complex non-stationary stochastic models have been developed based on seismological concepts that allow introduction of the time evolution of energy and/or frequency content by means of an evolutionary variance –power– spectral density function [2-6]. Seismological models are a very powerful tool when compared with conventional response spectra, because the seismic action is characterized by the physical parameters that control the process of generation and propagation of the earthquake signal to the prediction site, instead of using the structural response of a single degree-of-freedom system to characterize the phenomenon. Furthermore,

[1] Research Assistant. Department of Mechanics of Continuous Media and Structural Analysis, Technical University of Valencia, Valencia, Spain. E-mail: igferbal@doctor.upv.es
[2] Professor. Department of Mechanics of Continuous Media and Structural Analysis, Technical University of Valencia, Valencia, Spain. E-mail: csanchez@mes.upv.es

the non-stationary character of earthquakes can be incorporated into the model in a relatively straightforward manner.

At the same time, several techniques to estimate the maximum value of a non-stationary stochastic process have been developed in the context of Random Vibration Theory [4,7-10]. These techniques are the best way of calculating stochastic response spectra when the seismic action is defined with non-stationary seismological models, taking into account not only the permanent but also the transient-response phase of the structural response.

This paper presents an analytical method to calculate stochastic response spectra from non-stationary seismological models. The non-stationary character of the seismic ground motion is introduced by means of an amplitude modulation function and an evolutionary corner frequency; the latter allows modulation of the frequency content of the input signal. A parametric study is made of the influence on the response spectrum of the energy distribution in frequency and time, including the effect of variations in the magnitude, hypocentral distance, path properties and site characteristics. Response spectra obtained with this methodology show a reasonably good agreement with those proposed in present seismic codes and recommendations for different types of soil and damping ratios.

STATIONARY SEISMOLOGICAL MODELS

The earlier attempts to predict the amplitude spectrum of an earthquake –i.e., the module of the Fourier amplitude spectrum or complex amplitude spectrum– consisted in a Gaussian white noise representation. This hypothesis was improved by filtering the white noise with the response transfer function of the soil, defined by a characteristic soil damping and a characteristic soil natural frequency (e.g., Kanai-Tajimi model [11,12]; Clough-Penzien model [13]). At the same time, different seismological models of the source were developed (Haskell's model [14-15]; Savage's model [16]; Brune's model [17]; barrier model [18]; asperity model [19]; Hartzell's model [20]; Hisada's model [21]) to represent the earthquake rupture process. Some of them, like the Brune's, Haskell's, and barrier models, were developed to predict the amplitude spectrum of a particular event. However, these representations were not yet good enough, as they were focused on the source, and therefore, did not include the multiple transformations suffered by the earthquake signal in its travel to the ground surface.

Important advances in seismological models were made by Boore [1], who took into account the medium properties of the path covered by the seismic waves from the source to the prediction site. Basically, Boore's model considers three different parts in a site seismological model: the source model, a reduction function and an amplification function. Other authors have added additional factors that can be summarized in a regional correction function. In their most general expression, stationary seismological models can be formulated as:

$$A_{ag,s}(f) = C_g\, A_{as}(f)\, F_c(f)\, F_r(f)\, F_z(f) \tag{1}$$

where $A_{ag,s}(f)$ is the two-sided amplitude spectrum of the stationary acceleration time series in the prediction site, $a_{g,s}(t)$, –site amplitude spectrum–, $A_{as}(f)$ is the two-sided amplitude spectrum of the non-stationary acceleration time series in the prediction site but without considering the path propagation and surface effects, $a_s(t)$, i.e., the acceleration time series due only to the fault rupture as measured in the far field –source amplitude spectrum–, C_g is a scaling factor of the site, $F_c(f)$ is a regional correction function, $F_r(f)$ is a reduction function, $F_z(f)$ is an amplification function, and f is the cyclic frequency.

Source model
Brune's model is used in this paper to characterize the fault rupture with the following formulation:

$$A_{as}(f) = \frac{R_{\theta\phi}}{4\pi \rho_s \beta_s^3 r_h} \frac{M_0}{1+\left(\frac{f}{f_s}\right)^2} (2\pi f)^2 \tag{2}$$

where $R_{\theta\phi}$ is the radiation pattern, M_0 is the seismic moment, ρ_s is the density in the source region, β_s is the S-wave propagation velocity in the source region, r_h is the hypocentral distance, and f_s is the source corner frequency, which can be calculated with:

$$f_s = 4.9 \cdot 10^6 \, \beta_s \left(\frac{\Delta\sigma}{M_0}\right)^{1/3} \tag{3}$$

where $\Delta\sigma$ is the effective stress released in the rupture process in bar, f_s is in Hz, β_s is in km/s, and M_0 is in dyne-cm. The following formula is commonly used to estimate the seismic moment from the moment magnitude M_w [22]:

$$M_w = \frac{2}{3} \log M_0 - 10.7 \tag{4}$$

Scaling factor
The scaling factor incorporates the effect of free surface through the factor K_F, and the partition of energy into two orthogonal horizontal directions through the factor K_D, being calculated as:

$$C_g = K_F K_D \tag{5}$$

Common values for the free surface and directional partition factors are $K_F=2$ and $K_D = 1/\sqrt{2}$.

Regional correction function
This is an empirical low-pass correction filter that takes into account some particular aspects of a specific seismogenetic region. Since a European seismic event will later be used in the numerical application, the following expression proposed by Faravelli [23] is considered:

$$F_c(f) = \frac{k_c}{\sqrt{1+\left(\frac{f}{f_c}\right)^2}} \tag{6}$$

where k_c is a normalization factor, and f_c is another corner frequency. The values $k_c=1.15$ and $f_c=2$ Hz have been suggested in [24].

Reduction function
In the reduction function two factors can be distinguished: an attenuation function $F_{rq}(f)$ and a filter of high frequencies $F_{rm}(f)$, thus giving:

$$F_r(f) = F_{rq}(f)\,F_{rm}(f) \tag{7}$$

The first factor takes into account the attenuation caused by the inelastic behaviour of the path medium, by using the S-wave inelastic attenuation factor Q_β. A frequency-dependent Q_β is commonly admitted [25,26]. According to Rovelli [26], $Q_\beta = Q_0 f^{\,n}$ may be used for Italian data, with n=1 and Q_0=40 s. With this hypothesis, the attenuation function has the following form:

$$F_{rq}(f) = \exp\left(-\frac{\pi\, r_h}{Q_0 \beta_p}\right) \tag{8}$$

where β_p is the S-wave propagation velocity in the path medium.

Actual amplitude spectra from recorded events have a rapid decay beyond a certain maximum frequency f_m. This phenomenon is included in the seismological model by the second filter in Eq. (7). This paper uses the following formula proposed by Boore [1]:

$$F_{rm}(f) = \frac{1}{\sqrt{1+\left(\frac{f}{f_m}\right)^8}} \tag{9}$$

Amplification function

As site effects are very important to evaluate the amplitude spectrum, an amplification factor is included in the general formulation of seismological models. A common practice is to consider two effects through two different filters: the impedance effect caused by the change in the density and S-wave propagation velocity from the source region to the prediction site, and the resonance effect due to the local configuration of the site (geotechnical, geological and topographical conditions). The amplification function can be expressed as:

$$F_z(f) = F_{zi}(f)\,F_{zr}(f) \tag{10}$$

where $F_{zi}(f)$ is the impedance function, and $F_{zr}(f)$ is the resonance function. The following expression fitted by Faravelli [23] to a tabulated amplification function calculated by Gusev [27] is used for the first filter:

$$F_{zi}(f) = \frac{2}{1+\left(\frac{f_{zi}}{f}\right)^2} \qquad f > f_{zi} \tag{11}$$

where f_{zi} is an empirical frequency, whose value is 0.32 Hz according to [23].

It is not possible to obtain a general analytical formula for the resonance effect that includes all the local conditions listed above, so a detailed study should be made for every specific prediction site. Nevertheless, geotechnical conditions are usually the most important ones, and past research has proposed some analytical models that provide rough evaluation of the soil response. This paper uses the following analytical formula proposed by Şafak [28] for a one layer medium, worked out from a discrete-time wave propagation technique:

$$F_{zr}(f) = \frac{(1 + C_R)\exp\left(-\pi f \tau_g / Q_{\beta g}\right)}{\sqrt{1 + 2C_R \cos\left(4\pi f \tau_g\right)\exp\left(-2\pi f \tau_g / Q_{\beta g}\right) + C_R^2 \exp\left(-4\pi f \tau_g / Q_{\beta g}\right)}} \tag{12}$$

where $Q_{\beta g}$ is the S-wave inelastic attenuation factor in the soil medium, τ_g is the travel time of waves in the soil layer, and C_R is the reflection coefficient for upgoing waves, given by:

$$C_R = \frac{\rho_r \beta_r - \rho_g \beta_g}{\rho_r \beta_r + \rho_g \beta_g} \tag{13}$$

where ρ is the density and β is the S-wave propagation velocity, with subscripts r and g standing, respectively, for bedrock and soil in the vicinity of the prediction site.

NON-STATIONARY SEISMOLOGICAL MODELS

All the aforementioned models have a limitation: they are not able to represent the evolution of energy and frequency content that occurs during an earthquake; obviously, this is an important drawback for a short and sudden action like a seismic event. Some researchers have tried to generalize stationary models to the non-stationary case. The most commonly used non-stationary models assume the following analytical approach:

$$A_{ag}(f, t) = I_{ag}(t)\, A_{ag,s}(f) \tag{14}$$

where $A_{ag}(f,t)$ is the two-sided amplitude spectrum of the non-stationary acceleration time series in the prediction site, $I_{ag}(t)$ is a normalized intensity function –also referred to as normalized amplitude modulation function or normalized envelope function– which represents the time variation of the standard deviation of the acceleration in the prediction site, $\sigma_{ag}(t)$. Two general types of time-domain intensity functions can be distinguished according to the variation of energy content over time: ICD type [29,30] (Increasing-Constant-Decreasing) and IPD type [3,31,32] (Increasing/Peak/Decreasing), depending on the presence or not, respectively, of a flat segment where the function remains constant, coinciding with the strong-motion phase of the accelerogram.

Carli [5,6], Faravelli [23], and Carli and Faravelli [33] have improved Eq. (14) in the context of seismological models, developing a non-stationary model that also includes the modulation of frequency content. This seismological model consists of separating the non-stationary character of the seismic action into two parts: the variation of the total energy over time, which is modelled with a trapezoidal ICD intensity function, and the variation of frequency content, which is introduced through a time-dependent source corner frequency as follows:

$$f_s(t) = \begin{cases} f_{s1} & 0 \le t < t_1 \\ f_{s1} \exp\left[-\left(\dfrac{t - t_1}{t_2 - t_1}\right)^k \ln \dfrac{f_{s1}}{f_{s2}}\right] & t_1 \le t < t_2 \\ f_{s2} & t_2 \le t \le T_{gt} \end{cases} \tag{15}$$

where t_1 and t_2 are the initial and final instants, respectively, of the flat part in $I_{ag}(t)$, T_{gt} is the total duration of the seismic record, f_{s1} and f_{s2} are the constant values of corner frequency assumed, respectively, for the increasing and decreasing parts of the accelerogram, and k is a parameter to fit the observed decay of the corner frequency in time. Later on, Carli [6] has proposed a continuous time-dependent source corner frequency function very similar to Eq. (15). Obviously, the model can be applied with any other type of ICD intensity function, as will be done further on in the numerical application. By using $f_s(t)$, the non-stationary seismological model becomes:

$$A_{ag}(f,t) = I_{ag}(t)\,\Phi(f_s(t))\,A_{ag,s}(f \mid f_s(t)) \tag{16}$$

where $A_{ag,s}(f \mid f_s(t))$ is the seismological model defined in Eq. (1) in which the constant f_s has been replaced by the time-dependent source corner frequency function $f_s(t)$ of Eq. (15), and $\Phi(f_s(t))$ is a normalization function added to the model in order to guarantee that the evolution of $\sigma_{ag}(t)$ is represented only by the time-domain intensity function, $I_{ag}(t)$. The amplitude spectrum of the evolutionary process $\{a_g(t)\}$ can also be expressed from the amplitude spectrum of the underlying stationary process $\{a_{g,s}(t)\}$ as:

$$A_{ag}(f,t) = I_{ag}(f,t)\,A_{ag,s}(f) \tag{17}$$

where $I_{ag}(f,t)$ is a real frequency-dependent intensity function. This paper uses the non-stationary seismological model in Eq. (16). From Eqs. (16) and (17), the following expression can be obtained for the calculation of the frequency-dependent intensity function (TCF standing for Time-dependent Corner Frequency):

$$I_{ag}(f,t) = \frac{I_{ag}(t)\,\Phi(f_s(t))\,A_{ag,s}(f \mid f_s(t))}{A_{ag,s}(f)} \tag{18}$$

VARIANCE SPECTRAL DENSITY FUNCTION

Evolutionary spectral representation

To calculate a stochastic response spectrum from a theoretical amplitude spectrum it is necessary to regard the earthquake as a non-stationary random process to take into account the evolution over time of energy and frequency content. The corresponding non-stationary process can be represented by a time-dependent autocovariance function, although for engineering purposes a time-dependent variance spectral density function –non-stationary variance spectrum– is more commonly used. The variance spectrum is of capital importance in the study of earthquakes since ground motion in the prediction site is the result of multiple reflections and refractions of the initial waves produced during the rupture process of the fault and it can therefore be regarded as a Gaussian process.

The evolutionary variance spectrum is a suitable non-stationary representation of the seismic excitation. Any realization of the stochastic process of accelerations at the prediction site can thus be expressed as follows [34]:

$$a_g(t) = \int_{-\infty}^{\infty} \tilde{I}_{ag}(f,t)\,\exp(i\,2\pi f\,t)\,d\tilde{Z}(f) \tag{19}$$

where i is the imaginary unit, $\tilde{I}_{ag}(f,t)$ is a complex-valued intensity function, and $\{\tilde{Z}(f)\}$ is a complex-valued stationary random process with orthogonal increments. The process $\{\tilde{Z}(f)\}$ is such that an underlying stationary process $\{a_{g,s}(t)\}$ can be assumed to exist with the following spectral decomposition:

$$a_{g,s}(t) = \int_{-\infty}^{\infty} \exp(i\,2\pi f\,t)\,d\tilde{Z}(f) \tag{20}$$

The condition that $\tilde{I}_{ag}(f,t)$ is a function with slow variation over time must be imposed to preserve the physical meaning of Eq. (19), which represents an instantaneous frequency decomposition of the non-stationary process. According to Priestley [34], the two-sided evolutionary variance spectral density function, $G_{ag}(f,t)$, can be expressed as:

$$G_{ag}(f,t) = I_{ag}^2(f,t)\,G_{ag,s}(f) \tag{21}$$

where $I_{ag}(f,t)$ is the module function of $\tilde{I}_{ag}(f,t)$, and $G_{ag,s}(f)$ is the two-sided variance spectral density function of the underlying stationary process $\{a_{g,s}(t)\}$.

Underlying stationary process

The two-sided variance spectrum of a stationary stochastic process $\{x(t)\}$ can be calculated as follows:

$$G_x(f) = \lim_{T\to\infty} \frac{E\left[A_{x,T}^2(f)\right]}{T} \tag{22}$$

where $E[\cdot]$ is the mathematical expectation, and $A_{x,T}(f)$ is the two-sided amplitude spectrum of a sample of duration T extracted from a realization x(t) of the stochastic process, i.e., the module of the finite Fourier transform of duration T. An approximate expression of the variance spectrum can be obtained from an ensemble of samples of duration T by eliminating the limit in Eq. (22). In real phenomena like earthquakes, only one sample of duration T is usually available, so that a common simplification consists of eliminating the mathematical expectation operator, thus giving:

$$G_x(f) \approx \frac{A_{x,T}^2(f)}{T} \tag{23}$$

If the process $\{x(t)\}$ were ergodic as well as stationary, this last step would be exact without introducing any error.

In our case, the stochastic process of accelerations is undoubtedly non-stationary, and therefore Eq. (23) should not be applied either to $\{a_s(t)\}$ or to $\{a_g(t)\}$. However, this seems to be the approach commonly used in the literature without any further justification [1,4], giving rise to the following practical expression for the two-sided variance spectrum of the ground acceleration obtained from a seismological model:

$$G_{ag,s}(f) = \frac{A_{ag,s}^2(f)}{T} \tag{24}$$

where T can be interpreted as a certain duration parameter of the seismic event.

The apparent discrepancy between non-stationary stochastic process theory and the practical expression given in Eq. (24) can be solved with some heuristic arguments. First of all, Eq. (24) should be considered valid only for the stationary underlying process $\{a_{g,s}(t)\}$ and not for the actual process $\{a_g(t)\}$. In principle, the amplitude spectrum $A_{as}(f)$ given by a source seismological model only gives the amplitudes of an infinite number of trigonometric functions whose superposition, with certain phases, would reproduce the original time series $a_s(t)$. Nevertheless, when transformed into $A_{ag,s}(f)$ by means of Eq. (1), it can also be interpreted as the amplitude spectrum of the underlying stationary stochastic process, assuming that the different filters applied to obtain the site spectrum have been fitted with this aim. If this is the case, a characteristic duration T_g should be used in the denominator of Eq. (24) instead of the total T_{gt}, to calculate the corresponding variance spectrum:

$$G_{ag,s}(f) = \frac{A^2_{ag,s}(f)}{T_g} \tag{25}$$

The duration T_g can be interpreted as a scaling parameter of the variance spectrum, therefore influencing the total energy of the earthquake and the peak ground acceleration. Several duration parameters of common use in the literature, such as bracketed duration [35] or effective duration [36], do not seem to be the best choice for T_g, as they have no physical interpretation in terms of equivalent destructive potential of the earthquake ground shaking. A properly defined equivalent stationary duration T_{gs} [37,38] should be used instead, to preserve the total energy content of the earthquake.

Non-stationary variance spectrum

By substituting Eq. (25) into Eq. (21), and using a real frequency-dependent intensity function with slow variation over time like the one proposed in Eq. (18), the following expression is obtained for the two-sided variance spectrum corresponding to the non-stationary seismological model used in this paper:

$$G_{ag}(f,t) = I^2_{ag}(t)\frac{\Phi^2(f_s(t))\,A^2_{ag,s}(f\,|\,f_s(t))}{T_{gs}} \tag{26}$$

EVALUATION OF OSCILLATOR RESPONSE

The displacement of a linear single degree-of-freedom oscillator of natural cyclic frequency f_n, excited by a ground acceleration time series $a_g(t)$, is given by the Duhamel's integral:

$$d(t) = -\int_{-\infty}^{\infty} h_{ag;d}(\tau)\,a_g(t-\tau)d\tau \tag{27}$$

where, $h_{ag;d}(t)$ is the impulse response function of the oscillator, which has the following form:

$$h_{ag;d}(t) = \frac{1}{2\pi f_d}\exp(-\xi\,2\pi f_n t)\,\mathrm{sen}(2\pi f_d t) \tag{28}$$

ξ is the damping coefficient, and f_d is the damped cyclic frequency defined as:

$$f_d = f_n\sqrt{1-\xi^2} \tag{29}$$

As the seismic action is a non-stationary random process, the oscillator response will be non-stationary too. The two-sided evolutionary variance spectral density function of the oscillator response, $G_d(f,t)$, can be expressed as:

$$G_d(f,t) = H^2_{ag;d}(f,t)\, G_{ag,s}(f) \tag{30}$$

where $H_{ag;d}(f,t)$ is the module function of $\tilde{H}_{ag;d}(f,t)$, the complex-valued non-stationary frequency response function of the oscillator. The latter already incorporates the non-stationary character of the input signal and can be calculated by substituting Eq. (19) into Eq. (27), thus obtaining:

$$\tilde{H}_{ag;d}(f,t) = -\int_{-\infty}^{\infty} h_{ag;d}(\tau)\, \tilde{I}_{ag}(f, t-\tau) \exp(-i\, 2\pi f\, \tau)\, d\tau \tag{31}$$

DETERMINATION OF STOCHASTIC RESPONSE SPECTRA

It is a common assumption that the probability distribution of the first barrier crossing in a stochastic process follows a Poisson process law. Although at the beginning only stationary excitation processes were considered –with or without a transient-response phase–, more recently non-stationary processes have also been included in an attempt to model better the transient-response phase of the oscillator. A generalized probability distribution of the stationary case was proposed by Amin *et al.* [39] for non-stationary random processes:

$$P(\eta, t) = \exp\left(\int_0^t r(\tau) d\tau \right) \tag{32}$$

where, $P(\eta,t)$ is the probability that $d(t)$ remains below the threshold level η during the interval of duration t, and $r(t)$ is the time-dependent decay rate whose meaning is the average frequency of η-crossings in a differential time interval. The decay rate proposed in [40] is used here, given that it includes the bandwidth of the process in the formulation:

$$r(t) = 2 f_z(t) \frac{1 - \exp\left(-\sqrt{\frac{\pi}{2}}\, q(t) \frac{\eta}{\sigma_d(t)} \right)}{\exp\left(\frac{1}{2} \frac{\eta}{\sigma_d(t)} \right) - 1} \tag{33}$$

where $f_z(t)$ is the time-dependent average cyclic frequency of zero-up-crossings, $q(t)$ is the time-dependent spectral bandwidth parameter, and $\sigma_d(t)$ is the time-dependent standard deviation of the oscillator displacement. The inclusion of a bandwidth parameter in Eq. (33) is important for the study of a non-stationary process in which the frequency content can change and the functional shape of the variance spectrum is wide-band or narrow-band depending on the time instant. According to Michaelov *et al.* [10], these parameters can be calculated from the autocovariance and cross-covariance coefficients of the oscillator response. In the classical stationary approach, these coefficients are constants and can be related to the spectral moments in virtue of the simple relationships existing between the variance spectral density function of a process and those of its derivative processes. Di Paola [41] suggested that a pre-envelope process should be introduced, by means of which the autocovariance and cross-covariance coefficients of the oscillator response can be calculated in the following manner [10]:

$$c_{jk}(t) = \frac{(-1)^k i^{j+k}}{2} \frac{\partial^{j+k} C_{\psi\psi}(t_1, t_2)}{\partial t_1^j \partial t_2^k}\bigg|_{t_1=t_2=t} \tag{34}$$

where the subindexes j and k represent a response process, with 1 and 2 values corresponding to the oscillator displacement and velocity, respectively, $\{\psi(t)\}$ is a non-stationary random process defined by the pre-envelope function of the non-stationary random process of the oscillator displacement, $\{d(t)\}$, and $C_{\psi\psi}(t_1,t_2)$ is the autocovariance function of the pre-envelope process, which can be expressed as:

$$C_{\psi\psi}(t_1, t_2) = 4\int_0^\infty \tilde{H}^*_{ag;d}(f, t_1)\, \tilde{H}_{ag;d}(f, t_2)\, G_{ag,s}(f) \exp[i\, 2\pi f(t_2 - t_1)] df \tag{35}$$

where the superscript (*) means complex conjugate, and $G_{ag,s}(f)$ is the variance spectrum of the seismological model calculated with Eq. (25).

By substituting Eq. (33) into Eq. (32), the probability of non-exceedance $P(\eta,t)$ can be obtained for a given threshold level. However, in order to calculate a stochastic response spectrum the maximum of the oscillator response associated with a given probability of non-exceedance has to be determined, so that Eq. (32) must be solved by an iterative procedure.

NUMERICAL APPLICATION

This section involves a parametric study of the influence of several characteristics of a seismological model on the stochastic response spectrum, using the analytical method described above. First of all, consideration is given to the influence of magnitude, hypocentral distance, path attenuation and site effects, after which the influence of time variation of energy and frequency content is analyzed. Finally, the influence of damping coefficient and non-exceedance probability is discussed.

The properties of the source, path medium and site conditions for the definition of the reference earthquake have been partly taken from the Irpinia earthquake November 23, 1980 [23] to make it possible to compare the normalized stochastic response spectra (NSRS) of accelerations, to be obtained using the proposed method, with the Eurocode 8 Part 1 normalized design response spectra (NDRS) [42]. The parameters of the reference model are: $R_{\theta\phi}=0.63$; $M_w=6$; $\rho_s=2600$ kg/m^3; $\beta_s=3500$ m/s; $r_h=20$ km; $\Delta\sigma=300$ bar; C_g of Eq. (5) with $K_F=2$ and $K_D=1/\sqrt{2}$; $F_c(f)$ of Eq. (6) with $k_c=1.15$ and $f_c=2$ Hz; $F_{rq}(f)$ of Eq. (8) with $Q_0=40$ s and $\beta_p=3500$ m/s; $F_{rm}(f)$ of Eq. (9) with $f_m=20$ Hz; $F_{zi}(f)$ of Eq. (11) with $f_{zi}=0.32$ Hz; $F_{zr}(f)=1$ assuming rock site conditions. The intensity function is of the IPD type [31], with an equivalent stationary duration $T_{gs}=6$ s according to [38], and a rise time $t_r=2$ s. NSRS are calculated for an oscillator with a damping coefficient $\xi=5\%$, and for a probability of non-exceedance P=90%.

The effect of hypocentral distance r_h and path attenuation Q_0 on the response spectrum is limited to a scaling of its ordinates. On the contrary, both magnitude and site effects not only produce a proportional variation in the ordinates but also modify the shape of the response spectrum. The influence of magnitude in the response spectrum has been studied using $M_w=5$, 6 and 7. The results are shown in Fig. 1, where the response spectrum is seen to become narrower as the magnitude decreases. This effect is consistent with the two different types of NDRS proposed in Eurocode 8: Type 1 for regions where the largest expected earthquake is $M_s \geq 5.5$, and Type 2 for regions where the expected earthquake is $M_s<5.5$. Worthy of note here is the good fitting of the calculated NSRS to the decreasing part of the NDRS. This is particularly

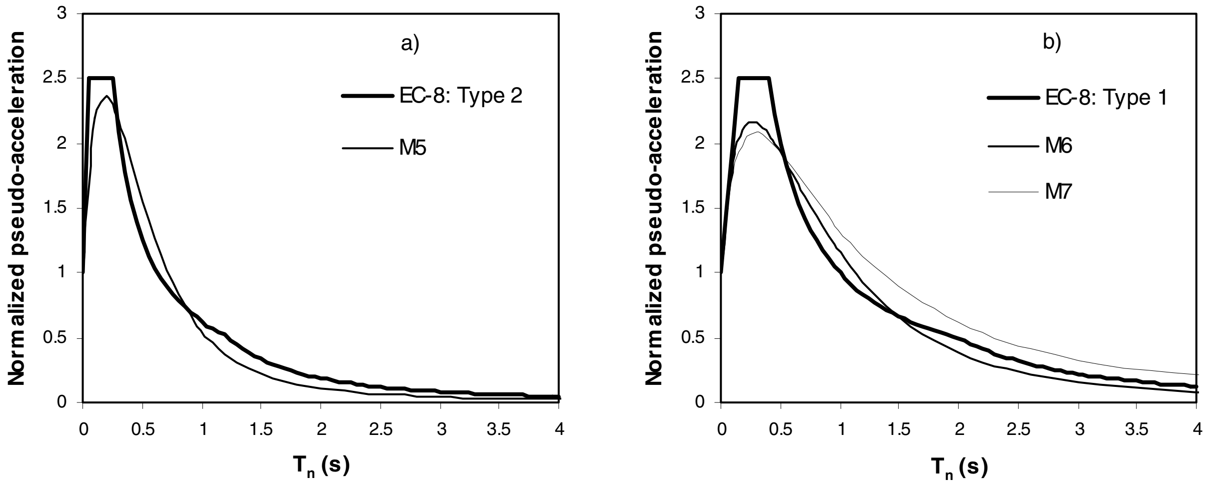


Figure 1. Comparison between normalized stochastic response spectrum and EC-8 normalized design response spectrum for different moment magnitudes: a) EC-8 Type 2 NDRS, and NSRS for M_w=5; b) EC-8 Type 1 NDRS, and NSRS for M_w=6 and M_w=7.

significant, since the estimation of response spectrum over long natural periods is greatly affected by the transient-response phase of the oscillator.

The inclusion of site effects has a great influence on the NDRS shape due to the well-known effect of enlarging the zone of constant acceleration in the response spectrum. In this paper, site effects have been included in the seismological model through Eq. (12) using ρ_r=2600 kg/m^3 and β_r=3500 m/s for bedrock characteristics. The average density of the soil layer is ρ_g=2000 kg/m^3, and the S-wave propagation velocity has been taken from the ranges specified in the Eurocode 8 for different soil conditions: specifically, soils C and D have been defined by β_g=250 m/s and 150 m/s, respectively. For the wave travel time, the values τ_g=0.40 s and 0.67 s, corresponding to a soil layer thickness of 100 m, are specified for both types of soil. The inelastic attenuation factor is set at $Q_{\beta g}$=50. The comparison of the resulting NSRS with Type 1 NDRS is shown in Fig. 2 for soils C and D. As can be seen, there is a reasonably good agreement between both spectra, taking into account the smoothed-out character of the design spectrum. In regards to the average level of NSRS in the constant spectral acceleration part of the NDRS, it is important to note that the ordinates of both spectra should be similar but not equal, since the process commonly followed to obtain a standard design spectrum is not completely consistent with the stochastic approach, due to the normalization of the different real response spectra prior to the averaging thereof. The ranges of natural periods contained in this part of the response spectrum in both spectra are also coincident for both types of soil. The NSRS of the reference earthquake, corresponding to rock site, is also represented to show the large amplification in the response spectrum attributable to site effects.

The next parametric study refers to the influence of the intensity function and source corner frequency modulation function. Fig. 3 shows the NSRS calculated for three different intensity functions: the first one, corresponding to the reference earthquake with an IPD intensity function; the second one for an ICD type intensity function [29], with the same equivalent stationary duration and rise time, and a duration of the constant part equal to $0.2T_{gt}$; the last response spectrum corresponds to an intensity function formulated according to Eq. (18), i.e., a TCF intensity function with the same embedded time-domain ICD intensity function as before, and a time-dependent source corner frequency as in Eq. (15), with f_{s1}=2 Hz, f_{s2}=f_s, where f_s is the corner frequency of the reference earthquake, and k=0.345. An increase of the maximum value in NSRS is clearly caused by the use of an ICD or TCF intensity function in comparison with the reference IPD function. The constant segment in an ICD function allows the oscillator to perform

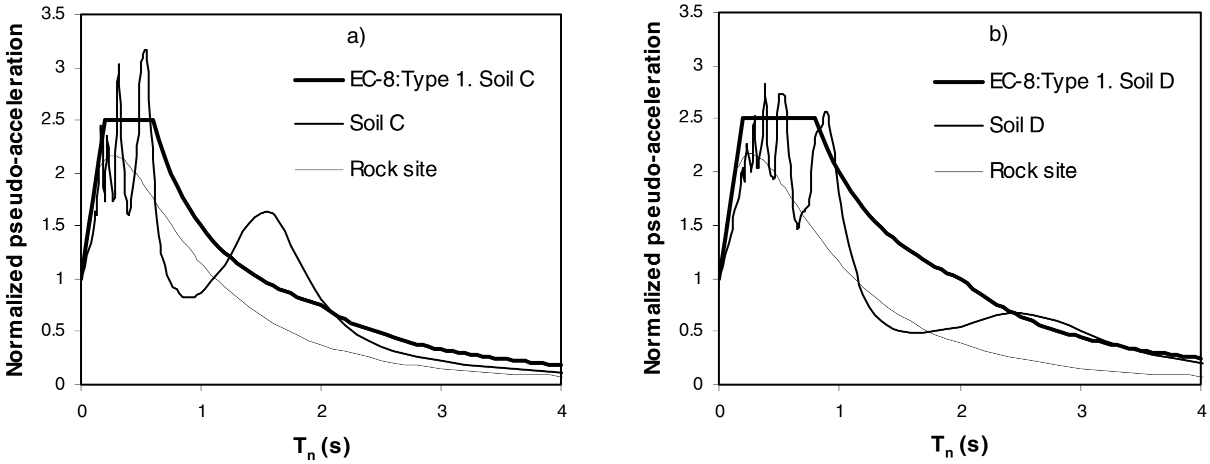


Figure 2. Comparison between normalized stochastic response spectrum and EC-8 normalized design response spectrum for different soils: a) EC-8 Type 1 NDRS for soil C, NSRS for a particular soil C, and NSRS for rock site; b) EC-8 Type 1 NDRS for soil D, NSRS for a particular soil D, and NSRS for rock site.

a greater number of cycles during the transient-response phase with a high level of excitation, so that as the natural period of the oscillator diminishes it is easier for the oscillator to reach the steady-response phase corresponding to the underlying stationary process for a given earthquake. The variation of frequency content obtained with a modulated source corner frequency produces an additional increase in the NSRS ordinates. This is so because the source corner frequency bears a very close relationship to the functional shape of the variance spectral density function, and hence also affects the shape of the response spectrum, specifically its maximum value.

Finally, the influence of the oscillator damping has been analysed. Seismic codes take into account the effect of damping through a correction factor η_ξ; in the case of Eurocode 8 this factor is given by:

$$\eta_\xi = \sqrt{\frac{10}{(5+\xi)}} \geq 0{,}55 \qquad (36)$$

Fig. 4 shows the NSRS obtained for oscillators with ξ=2.5%, 5% and 10%. For a NSRS, the damping

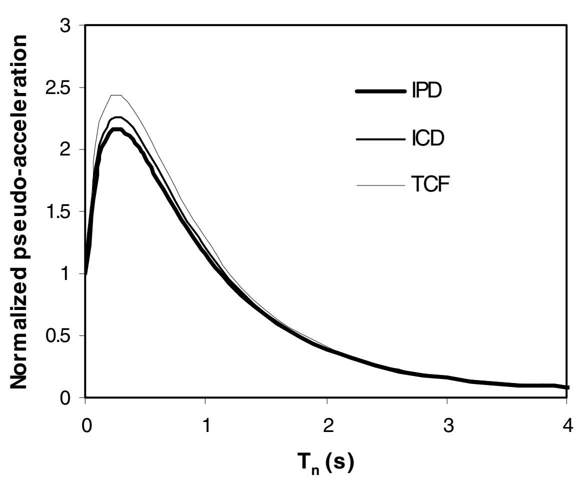
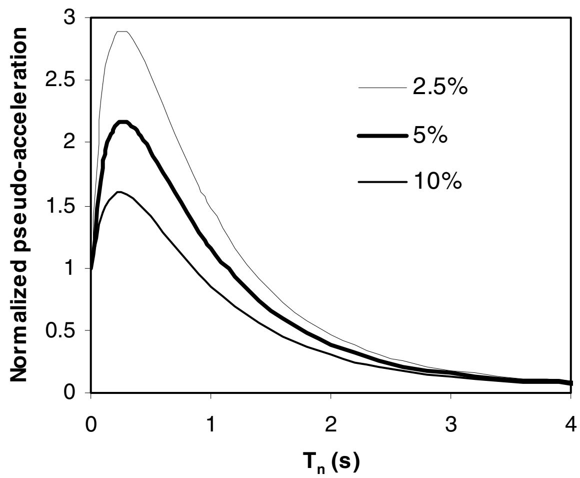


Figure 3. Influence of intensity function on normalized stochastic response spectrum.



ξ (%)	$\eta_{\xi,EC\text{-}8\ NDRS}$	$\eta_{\xi,NSRS}$
2.5	1.15	1.33
5	1.00	1.00
10	0.82	0.73

Figure 4. Influence of damping coefficient on normalized stochastic response spectrum.

correction factor is obtained as $\eta_\xi = R_{a;max}(\xi)/R_{a;max}(5\%)$, where $R_{a;max}(\xi)$ is the maximum value of the NSRS corresponding to a damping coefficient ξ. Eurocode 8 damping correction factors for NDRS and calculated NSRS damping correction factors are also given in Fig. 4. The values obtained for the NSRS are very close to the expression $\eta_\xi=(5/\xi)^{0.4}$, based on Random Vibration Theory, which is used in some codes. According to this, the empirical expression given in Eurocode 8 seems to be unsafe for small damping values.

CONCLUSIONS

An analytical method for the calculation of stochastic response spectra from non-stationary seismological models has been presented. The general formulation of the method allows the following factors to be taken into account: the variation of the energy content of the seismic action in both frequency and time, and the transient-response phase of the oscillator when subjected to a non-stationary input. This makes it particularly suitable for obtaining the shape of the response spectrum over long periods. Seismological models can be used to evaluate the response spectrum for specific conditions of the prediction site, as they are based on the mechanics of the fault rupture process and incorporate the transformation of the seismic waves in their propagation from the source to the earth surface. This advantage is particularly significant in low activity seismic regions, where the available data of relatively large earthquakes are limited or non-existent. An analysis is also made of the influence on stochastic response spectra of some parameters explicitly or implicitly considered in the formulation of design response spectra in modern seismic codes; the results are then compared with Eurocode 8 design response spectra. Consideration has also been given to the influence of the modulation of the energy content both in time and frequency by comparing stochastic response spectra obtained from different intensity functions. The agreement between the main characteristics of stochastic and design response spectra proves that seismological models can be considered at present as a reliable tool for determining site-specific stochastic response spectra for engineering purposes.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge the financial support provided by the Technical University of Valencia for carrying out this work, under Grant PPI-06-02/2928 of the Research Support Program.

REFERENCES

1. Boore DM. "Stochastic simulation of high-frequency ground motions based on seismological models of the radiated spectra." Bulletin of the Seismological Society of America, 1983; 73(6): 1865-1894.
2. Iyengar RN, Iyengar SR. "A nonstationary random process for earthquake accelerograms." Bulletin of the Seismological Society of America, 1969; 59(3): 1163-1188.
3. Shinozuka M, Sato Y. "Simulation of nonstationary random process." Journal of the Engineering Mechanics Division, 1967; 93(EM1): 11-40.
4. Şafak E. "Analytical approach to calculation of response spectra from seismological models of ground motion." Earthquake Engineering and Structural Dynamics, 1988; 16: 121-134.
5. Carli F. "Nonstationary models of earthquake accelerograms." Proceedings of the 10th World Conference on Earthquake Engineering, Madrid, Spain, 1992; 2: 829-834.
6. Carli F. "Smooth frequency modulating functions for strong ground motions." Proceedings of the 10th European Conference on Earthquake Engineering, Vienna, Austria, 1994; 155-160.
7. Corotis RB, Vanmarcke EH, Cornell CA. "First passage of nonstationary random processes." Journal of the Engineering Mechanics Division, 1972; 98(EM2): 401-414.
8. Langley RS. "Structural response to non-stationary non-white stochastic ground motions." Earthquake Engineering and Structural Dynamics, 1986; 14: 909-924.
9. Quek S-T, Teo Y-P, Balendra T. "Non-stationary structural response with evolutionary spectra using seismological input model." Earthquake Engineering and Structural Dynamics, 1990; 19: 275-288.
10. Michaelov G, Sarkani S, Lutes LD. "Spectral characteristics of nonstationary random processes - A critical review." Structural Safety, 1999; 21(3): 223-244.
11. Kanai K. "Semi-empirical formula for the seismic characteristics of the ground." Bulletin of the Earthquake Research Institute, University of Tokyo, Japan, 1957; 35: 309-325.
12. Tajimi H. "A statistical method of determining the maximum response of a building structure during an earthquake." Proceedings of the 2nd World Conference on Earthquake Engineering, Tokyo and Kyoto, Japan, 1960; 2: 781-798.
13. Clough RW, Penzien J. "Dynamics of structures." New York: McGraw-Hill Inc., 1993.
14. Haskell NA. "Total energy and energy spectral density of elastic wave radiation from propagating faults." Bulletin of the Seismological Society of America, 1964; 54(6): 1811-1841.
15. Haskell NA. "Total energy and energy spectral density of elastic wave radiation from propagating faults. Part II. A statistical source model." Bulletin of the Seismological Society of America, 1966; 56(1): 125-140.
16. Savage JC. "Radiation from a realistic model of faulting." Bulletin of the Seismological Society of America, 1966; 56(2): 577-592.
17. Brune JN. "Tectonic stress and the spectra of seismic shear waves from earthquakes." Journal of Geophysical Research, 1970; 75(26): 4997-5009.
18. Papageorgiou AP, Aki K. "A specific barrier model for the quantitative description of inhomogeneous faulting and the prediction of strong ground motion. Part I. Description of the model." Bulletin of the Seismological Society of America, 1983; 73(3): 693-722
19. Kanamori H. "A semi-empirical approach to prediction of long period ground motions from great earthquakes." Bulletin of the Seismological Society of America, 1979; 69(6): 1645-1670.
20. Hartzell S. "Comparison of seismic waveform inversion results for the rupture history of a finite fault: Application to the 1986 North Palm Springs, California, earthquake." Journal of Geophysical Research, 1989; 94(B6): 7515-7534.
21. Hisada Y. "A theoretical omega-square model considering the spatial variation in slip and rupture velocity." Bulletin of the Seismological Society of America, 2000; 90(2): 387-400.
22. Hanks TC, Kanamori H. "A moment magnitude scale". Journal of Geophysical Research, 1979; 84: 2348-2350.

23. Faravelli L. "Modeling the seismic input for a stochastic dynamic structural problem." Proceedings of the 5th International Conference on Application of Statistics and Probability in Soil and Structures, Vancouver, Canada, 1987; 230-237.
24. Faccioli E, Rovelli A, Fregonese R. "A study of the special characteristics of accelerograms from recent earthquakes in Italy and near regions." Proceedings of the 8th World Conference on Earthquake Engineering, San Francisco, California, 1984: 361-368.
25. Boore DM. "Short-period P- and S-wave radiation from large earthquakes: Implications for spectral scaling relations." Bulletin of the Seismological Society of America, 1986; 76(1): 43-64.
26. Rovelli A. "Frequency relationship for seismic Q_β of the central southern Italy from accelerograms for the Irpinia earthquake 1980." Physics of the Earth and Planet Interiors, 1983; 32: 209-217.
27. Gusev AA. "Descriptive statistical model of earthquake source radiation and its application to an estimation of short-period strong motion." Geophysical Journal of the Royal Astronomical Society, 1983; 74: 787-808.
28. Şafak E. "Discrete-time analysis of seismic site amplification." Journal of Engineering Mechanics, 1995; 121(7): 801-809.
29. Jennings PC, Housner GW, Tsai NC. "Simulated earthquake motions." Report of the Earthquake Engineering Research Laboratory, California Institute of Technology, Pasadena, California, 1968.
30. Amin M, Ang HS. "Non stationary stochastic model of earthquake motion." Journal of the Engineering Mechanics Division, 1968; 94(EM2): 559-583.
31. Saragoni GR, Hart C. "Simulation of artificial earthquakes." Earthquake Engineering and Structural Dynamics, 1974; 2(3): 249-267.
32. Tung ATY, Wang JN, Kiremidjian A, Kavazanjian E. "Statistical parameters of AM and PSD functions for the generation of site-specific strong ground motions." Proceedings of the 10th World Conference on Earthquake Engineering, Madrid, Spain, 1992; 2: 867-872.
33. Carli F, Faravelli L. "A nonstationary seismological model for strong ground motions." European Earthquake Engineering, 1990; IV(3): 29-42.
34. Priestley MB. "Evolutionary spectra and non-stationary processes." Journal of the Royal Statistical Society, Ser. B, 1965; 27: 204-237.
35. Bolt BA. "Duration of strong ground motion." Proceedings of the 5th World Conference on Earthquake Engineering, Rome, Italy, 1973; 1: 1304-1315.
36. Trifunac MD, Brady AG. "A study on the duration of strong earthquake ground motion." Bulletin of the Seismological Society of America, 1975; 65(3): 581-626.
37. Vanmarcke EH, Lai S-SP. "Strong motion duration and rms amplitude of earthquake ground motion." Bulletin of the Seismological Society of America, 1980; 70(4): 1293-1307.
38. Sánchez-Carratalá CR, Ferrer I. "Equivalent earthquake duration from amplitude modulation functions." Proceedings of the 13th World Conference on Earthquake Engineering, Vancouver, Canada, 2004. (in press)
39. Amin M, Tsao HS, Ang AH. "Significance of nonstationarity of earthquake motions." Proceedings of the 4th World Conference on Earthquake Engineering, Santiago, Chile, 1969; 97-113.
40. Vanmarcke EH. "First passage and other failure criteria in narrow band random vibration: A discrete state approach." Department of Civil Engineering, MIT, Research Report R69-68, 1969.
41. Di Paola M. "Transient spectral moments of linear systems." SM Archives, 1985; 10: 225-243.
42. ENV-1998-1:2001. "Eurocode 8: Design of structures for earthquake resistance. Part 1: General rules, seismic actions and rules for buildings." European Committee for Standardization, Draft No. 4, December 2001.