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NONLINEAR RESPONSE OF BASE ISOLATED STRUCTURE WITH HYSTERETIC HARDENING AND ENERGY ABSORPTION IN CHAOTIC DYNAMICAL SYSTEM

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SUMMARY

This paper presents a nonlinear hysteretic model of an isolator that reproduces real experimental curve with hardening and energy absorption and discusses its nonperiodic response behaviors and stability. First a mathematical nonlinear equation governing one mass system is given and two types of hysteretic curves adopting the polynomial and cubic functions of displacements are shown. The former has hysteretic energy absorption but the later doesn't. The equation can be solved by means of Runge-Kutta Method. Next the effect of hysteretic energy absorption is examined comparing time histories of the two models each other. Initial natural period of analytical models are set 3.0 seconds and damping coefficients are 0% and 5%. Here ground motions are sinusoidal waves, El Centro 1940, the 1995 Hyogo-Ken-Nanbu Earthquake and Miyagi-Ken-Oki earthquake of 1978 so that their maximum amplitudes may be modified 150 cm/sec and the time increment for computing is 0.001 second. Obviously hysteretic energy absorption reduces response after the range from 4.0 to 5.0 seconds in time histories. Attractors to sinusoidal motion with the same amplitude in phase plane show that it makes the chaotic behavior more stable.

1. INTRODUCTION

Practical studies aiming at development of base isolation system for building structures in 1980's by innovative pioneers opened the door to today's diffusion of response control devices in Japan. A large number of ideas to reduce response caused by earthquake were practiced in full-scale buildings afterwards. Advanced technology of material science in industry contributed to the development and made it possible to provide any hysteretic characteristics and dampings with them. Some of these structural devices show nonlinearity such as softening to middle range of deflection and hardening to large one. Therefore engineers have to face nonlinear equation in mathematical meaning essentially if they discuss the strict vibratory behaviors of the structure sustained by them. Authors [1] have already discussed nonlinear response of base isolated structure subjected to huge earthquake using Duffing's

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equation, enveloping the cubic function of displacement in order to idealize the hysteretic hardening of an isolator developed by Tada at al [2].

This paper presents an analytical model reproducing hysteretic hardening and energy absorption using the polynomial function and discusses complex nonlinear behaviors of a base isolated structure under earthquake motions with large amplitudes. Then nonperiodic behaviors of the structure and vibratory stability are considered related to damping of the system.

2. NONLINEAR EQUATION GOERNING DYNAMIC BEHAVIOR OF STRUCTURE

The mathematical nonlinear equation governing one-mass system shown in Figure 1 can be written as

$$m\ddot{x} + c\dot{x} + f(x) = -m\ddot{y}_0 \quad (1)$$

Here m, c, and y_0 denote mass, damping and ground motion respectively. Restoring force f(x) of isolators can be written as

$$f(x) = k_1 x + k_2 x^3 + \beta_1 (x \pm x_0)(x \mp x_0)^2 + \beta_2 (x \pm x_0)\left|(x \mp x_0)^{\alpha}\right| \quad (2)$$

Here k_1, k_2, α, β_1, β_2 and x_0 are constants set considering experimental results. Equation (1) is solved directly by Runge-Kutta Method, which is adopted generally when analyzing chaotic dynamical system.

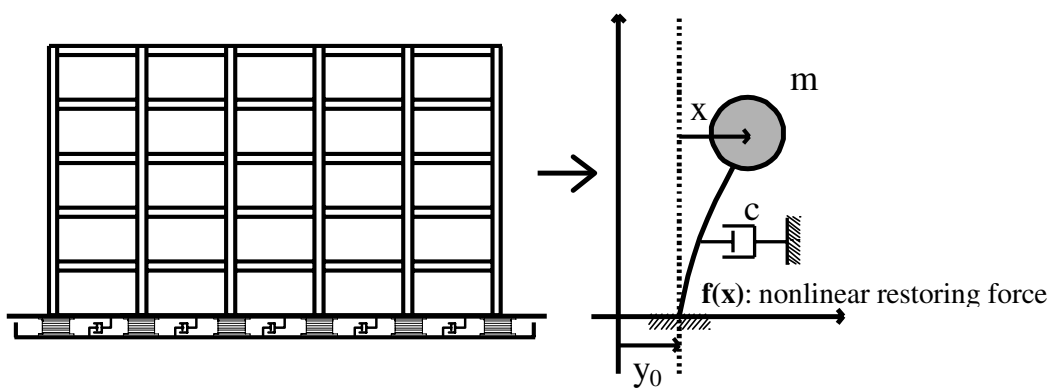


Figure 1: One-mass system sustained by isolators.

3. ANALYTICAL MODEL

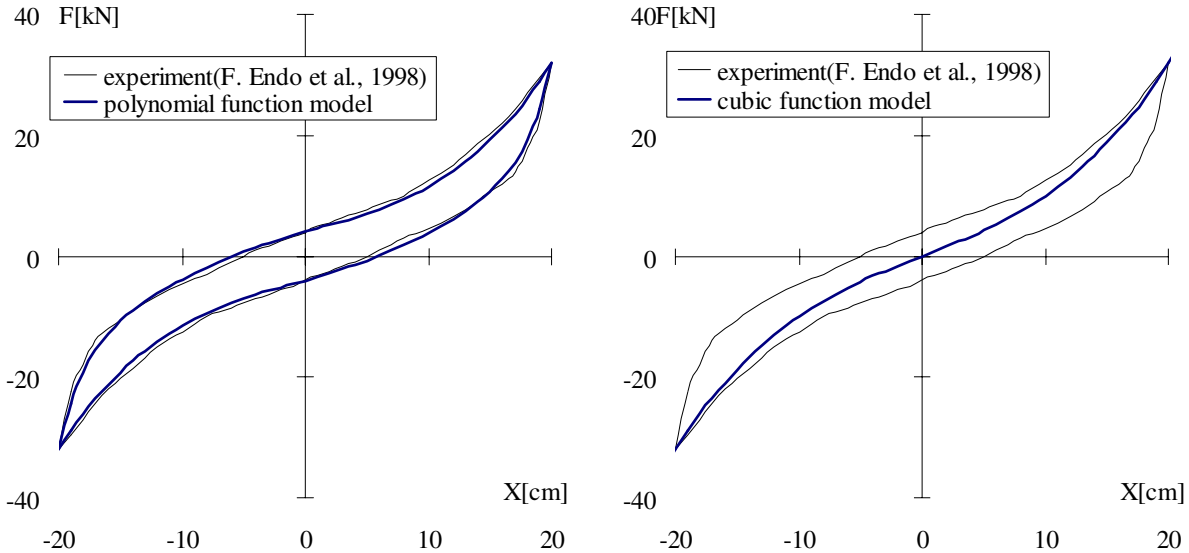
Figure 2 shows a hysteretic curve obtained through an experiment conducted by Endo, Sugiyama and Seki [3] and its analytical model idealized by the polynomial and cubic functions respectively. The restoring forces are expressed as

$$f(x) = 0.8x + 2.0\times10^{-3}x^3 + 0.5\times10^{-4}(x \pm x_0)(x \mp x_0)^2 + 3.0\times10^{-19}(x \pm x_0)(x \mp x_0)^{12} \quad (3)$$

and

$$f(x) = 0.8x + 2.0\times10^{-3}x^3 . \quad (4)$$

The last two terms of Equation (3) express hysteretic energy absorption and give good approximation to the original one.



(a) Polynomial function model **(b) Cubic function model**

Figure 2: Hysteretic models of nonlinear restoring force.

4. NONLINEAR RESPONSE AND ENERGY ABSORPTION IN TIME HISTORY

Figure 3 shows comparison of nonlinear responses of the model of hysteretic energy absorption with those idealized by the cubic function. Initial natural periods of the two models are set 3.0 seconds and damping coefficients are 5% respectively. Earthquake motions are El Centro 1940 NS, EW, the 1995 Hyogo-Ken-Nanbu Earthquake at JR Takatori Station NS, EW [4], Miyagi-Ken Oki Earthquake of 1978 NS and EW. Those maximum amplitudes are modified 150 cm/sec and the time increment for computing is 0.001 second. The time histories are shown in the above order, marked with (a), (b), (c), (d), (e) and (f). Obviously hysteretic energy absorption reduces amplitudes of responses after the range from 4.0 to 5.0 seconds in time history. Since the cubic function model cannot absorb the vibratory energy, the response displacements reach 100 centimeters. The natural period shortened by effect of hardening results in structural resonance with the high frequency components of earthquake motions as shown in Figure 3 (e) and (f).

Table 1 shows comparison of maximum response displacement of the polynomial model subjected to earthquake motions with one idealized by the cubic function. The hysteretic energy absorption doesn't seem to contribute to the reduction of the maximum displacements effectively in the case of the 1995 Hyogo-Ken-Nanbu Earthquake. Since the acceleration reaches the maximum value shortly after initial tremor, the system doesn't have enough time to act hysteretic mechanism fully. Therefore a sort of device that reduces response at the initial from 4.0 to 5.0 seconds of the ground shaking can be thought necessary if the superstructure has to be undamaged under the huge earthquakes.

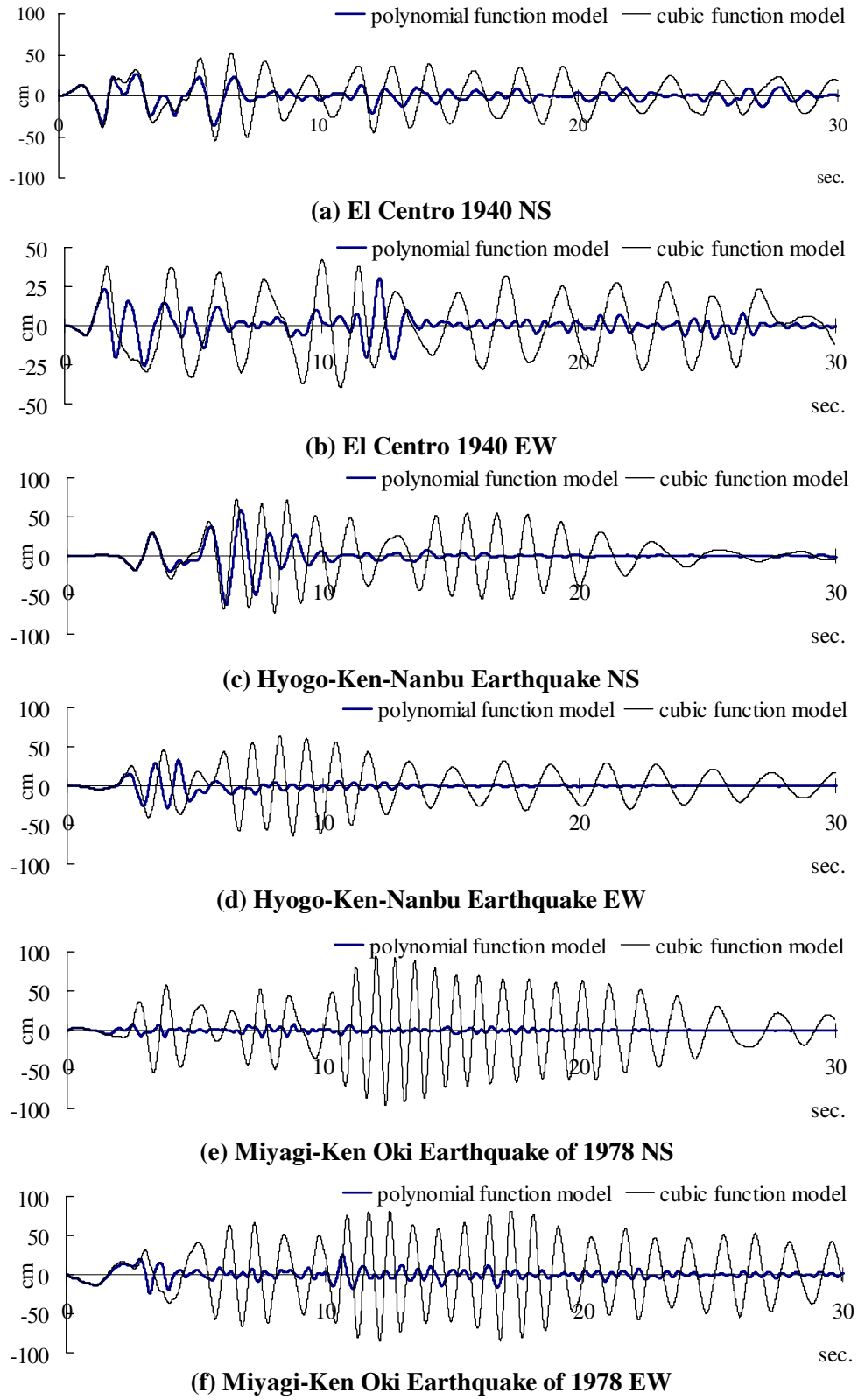


Figure 3: Comparison of the model of hysteretic energy absorption with one idealized by cubic function

Table1: Comparison of maximum displacement (unit: cm)

	cubic function model	polynomial function model
El Centro 1940 NS	54.75	36.75
El Centro 1940 EW	42.06	30.34
Hyogo-Ken-Nanbu Earthquake NS	72.61	62.31
Hyogo-Ken-Nanbu Earthquake EW	64.02	32.60
Miyagi-Ken Oki Earthquake of 1978 NS	94.91	8.80
Miyagi-Ken Oki Earthquake of 1978 EW	85.35	25.35

5. NONPERIODIC BEHAVIOR OF NONLINEAR RESPONSE

Figure 4 shows chaos attractors of responses of the one-mass system with the cubic hysteretic curve and initial natural period of 3.0 seconds subjected to nine sinusoidal ground motions of periods from 1.5 to 6.0 seconds. Damping is set 0%. They show nonperiodic behaviors of structural responses. Horizontal and vertical axes are displacement and velocity of them respectively. However scales are not unified on the figure because they are intended only to examine nonperiodic characteristics of the system. Obviously they have various patters, which suggest irregular vibrations governed by a certain rule, chaotic motions. Similarly Figure 5 shows chaos attractors concerning one-mass system with the polynomial hysteretic curve. Their orbits are smooth loops suggesting that the responses are more periodic and stable than the former case.

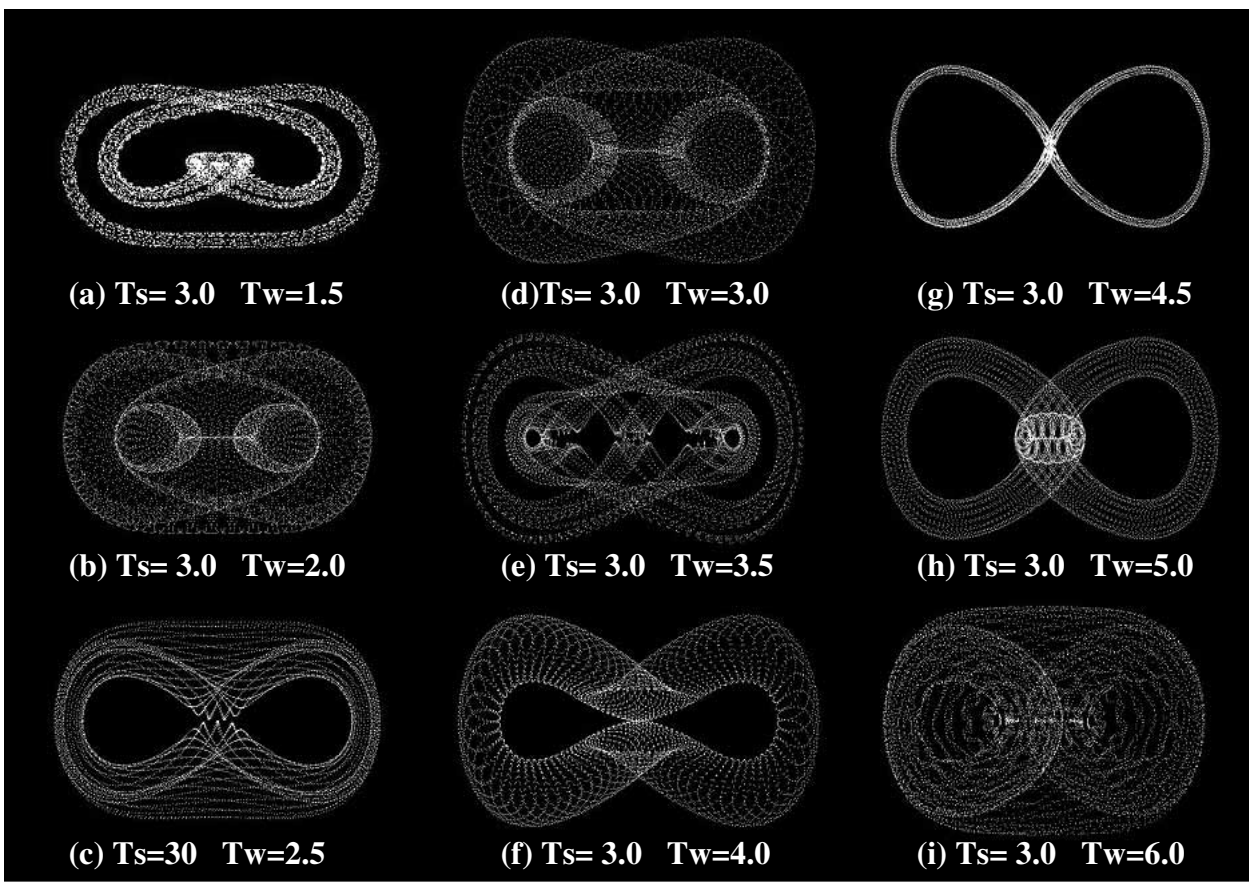


Figure 4: Chaos attractors of nonlinear responses of one-mass system with the cubic hysteretic curve subjected to sinusoidal ground motions.

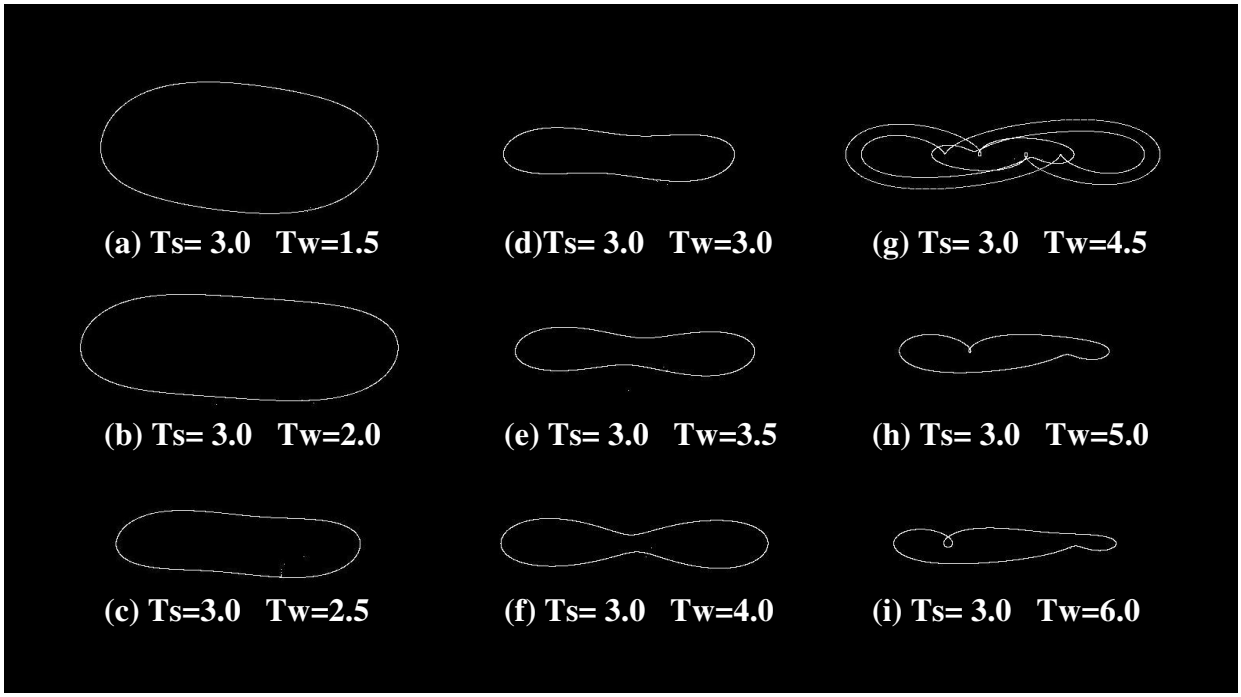


Figure 5: Chaos attractors of nonlinear responses of one-mass system with the polynomial hysteretic curve subjected to sinusoidal ground motions.

Generally chaos phenomenon arises from periodic input motion like a sinusoidal wave. Therefore nonlinear structural responses to earthquake motions don't seem to be chaos because they are assemblage of various frequency components and cannot be thought simple periodic motions. However authors examined them similarly using the 1995 Hyogo-Ken-Nanbu Earthquake. Figure 6 and 7 show their attractors. The analytical model with the cubic hysteretic function has overall patterns of attractors while the one with the polynomial function shows complex configuration. It suggests that the nonlinear earthquake responses are more irregular and nonperiodic than chaotic motions.

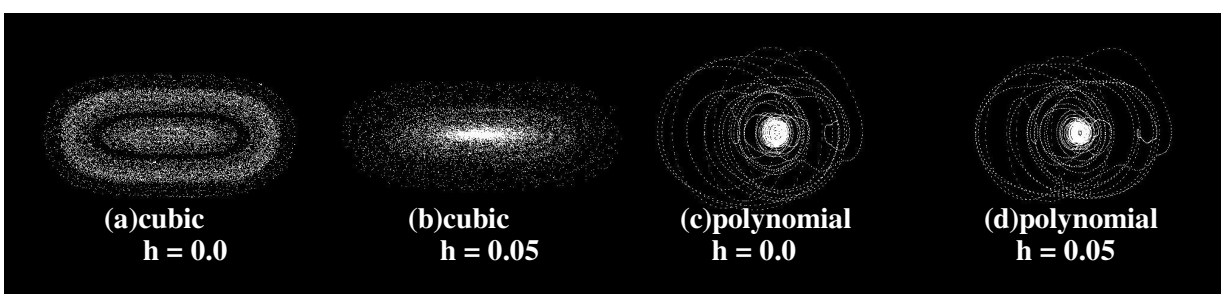


Figure 6: Attractors of nonlinear responses of one-mass system subjected to earthquake motions (The 1995 Hyogo-Ken-Nanbu Earthquake EW).

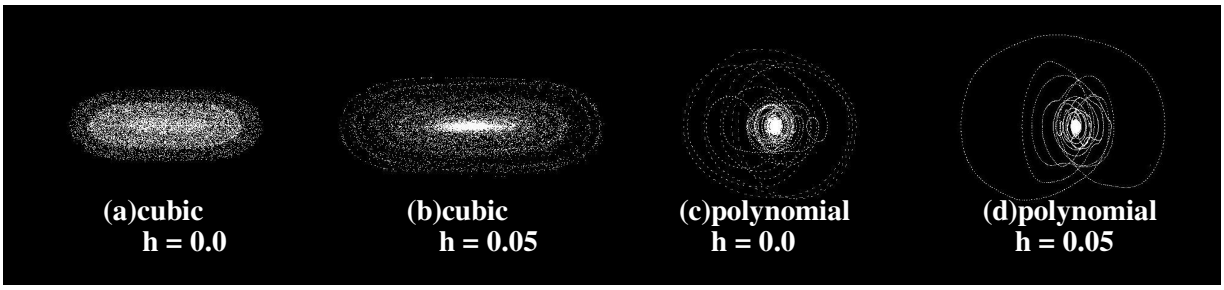


Figure 7: Attractors of nonlinear responses of one-mass system subjected to earthquake motions (The 1995 Hyogo-Ken-Nanbu Earthquake NS).

6. CONCLUSION

1. The model with hysteretic energy absorption using the polynomial function can reproduce an experimental hysteretic curve more accurately than one idealized by the cubic function.
2. Hysteretic energy absorption reduces response after the range from 4.0 to 5.0 seconds in time history. However it is not effective to the large amplitude that occurs at the beginning of the earthquake motion.
3. Chaos attractors of responses of the model with the polynomial hysteretic curve are more periodic and stable than one with the cubic hysteretic curve because of the hysteretic damping.

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