



SOFTWARE FOR GENERATION OF SPECTRUM COMPATIBLE TIME HISTORY

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SUMMARY

One of the important and the final step of getting a site-specific synthetic ground motion is generation of spectrum compatible time history. This paper presents an algorithm for generation of acceleration spectrum compatible time history, which requires target response spectra for the site and duration of time history to be generated as input. The method is based on picking random phase in Fourier domain and for the first iteration, Fourier magnitude of ground motion is assumed to be equal to pseudo spectral velocity derived from target spectra. The signal is made non-stationary by multiplying output of inverse Fourier transform with an envelop function as per user's choice. The present version of the software has two choices, however, more envelop functions are proposed to be included in later versions of the computer program. The signal is band passed in the frequency range of 0.1 Hz to 25 Hz and also performs notch filter at the corner frequency of the target spectra. The method is iterative and the convergence is assumed to be over when the spectral velocities between 0.1 Hz and 25 Hz calculated at bin frequencies are within +/- 10% of corresponding target spectral velocities. Few spectrum compatible time histories corresponding to response spectra of an Indian Standard Code as well as comparison of computed and target spectra is also presented. Executable version of the computer program and its help file can be downloaded from <http://www.iitr.ernet.in/acads/depts/earthquake/faculty/facthtml/akmeqfef.shtml>.

INTRODUCTION

For the purpose of earthquake resistant analysis and design of structures, realistic ground motion is required. In most of the cases it may not be possible to have strong motion records at a given site. Even if such recordings are available, there is no basis to expect that a future earthquake might generate same or similar ground motion. It is, therefore, essential that for earthquake resistant time history analysis of a structure, synthetic time histories must be generated for specific sites. However, there are several uncertainties in arriving at such time histories and to overcome these uncertainties decisions are required to be taken in a scientific and purposeful manner. Some of the issues for generation of site specific synthetic accelerogram are shape of design response spectra, zero period acceleration (ZPA), duration of record, rise, strong motion and decay time (envelop function of time history), phase characteristic of record, number of zero crossings, realistic derived velocity and displacement history etc. Several authors

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have worked in this field and large amount of literature is available. In this paper, details of software to generate synthetic time history is presented where it has been assumed that target response spectra for the site and duration of time history are known. Author has also presented earlier version of this work during 12th Symposium on Earthquake Engineering at Roorkee (Kumar [1]). Based on response of users, some changes in the computer program are done and one additional choice of envelop function added.

BRIEF DESCRIPTION

The first iteration of the computer program begins with the approximation that the target velocity response spectrum is equal to Fourier magnitude spectrum of the history. Frequency domain description (real and imaginary parts) of signal corresponding to Fourier magnitude spectrum is obtained using random phase at each bin frequency. Signal is band passed in the frequency range of 0.1 Hz to 25 Hz and notch filtering is also done at frequency where there is abrupt change in the target spectra. The notch filter frequency is automatically picked up by computer program. The operation of band pass and notch filter is performed in the frequency domain. Inverse DFT of this frequency domain description of band passed and notch filtered signal is then performed to get the stationary time domain signal. This stationary signal is made to look like an earthquake excitation by multiplying it by an envelope function. In the present version of the computer program there is a choice of two types of envelop function. One that is given by Boore [2] and second using rise, strong motion and decay time. Details of these envelop functions are given in a later section. Response spectrum of this non-stationary signal is then computed at each bin frequency interval, which is then compared with the target spectra at each bin frequency. Correction factors in the frequency range 0.1 Hz to 35 Hz thus obtained are multiplied to the real and imaginary parts of previously used frequency domain description of signal for the next iteration. This ensures that the random phase at different bin frequencies as was picked up during the first iteration, remains same for all subsequent iterations. Convergence of iteration is assumed to be over when the pseudo spectral velocities between 0.1 Hz and 25 Hz calculated at bin frequencies are within +/- 10% of corresponding target spectral velocities.

BAND PASS AND NOTCH FILTER

It is essential to remove very low frequencies from the synthetic accelerogram so that the velocity and displacement histories of the synthetic acceleration history are realistic. Also frequencies more than 25 Hz have been found to have negligible contribution in earthquake accelerograms. Notch filtering is essential as it has been found that spectra of the computed time history will always have undesired spikes at the frequency where the target spectra has sharp edge. In this work, band pass and notch filtering operations are done in the frequency domain.

Band pass filter is performed using Butterworth function (Lam [3]). For low pass operation the function is

$$f(\omega) = \frac{1}{1 + \left(\frac{\omega}{\omega_l}\right)^{2n}} \quad \text{where } \omega_l \text{ is the cut off frequency of low pass filter and } n \text{ is the order of}$$

filter.

For high pass operation the function is

$$f(\omega) = \frac{1}{1 + \left(\frac{\omega_h}{\omega}\right)^{2n}} \quad \text{where } \omega_h \text{ is the cut off frequency of high pass filter}$$

As mentioned above, the cut off frequency of the low pass filter is taken as 25 Hz and for high pass filter it is taken as 0.1 Hz. In this work, the order of filter is taken as 2.

Notch filtering operation is performed in frequency domain. The frequency of the sharp edge of the target spectrum is picked up first. The filter function for notch filter comprises of a weight of 0.1 for the notch frequency and weights of 0.4, 0.7 and 0.9 for three corresponding bin frequencies before and after the notch frequency.

Overall filter function, for band pass as well as notch filtering operation for IS1893: 2002 spectra [4] as target spectra, is given in Fig. 1.

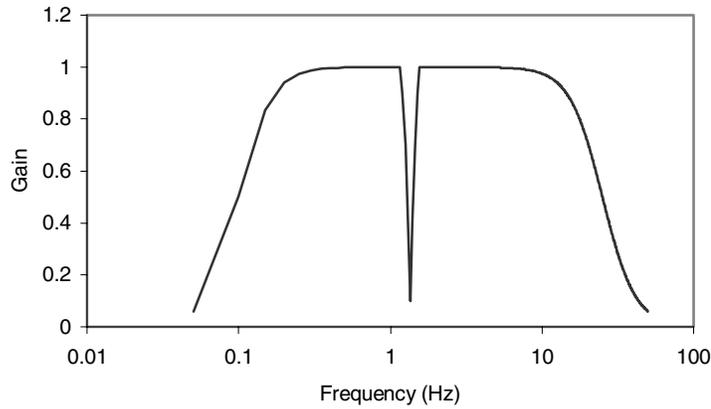


Fig. 1 Filter Function for Band Pass and Notch Filter

ENVELOP FUNCTION

Inverse discrete Fourier transformation (IDFT) of any frequency domain description will provide a stationary time domain signal whereas earthquake time histories are essentially non-stationary. Also such a signal will not have rise and decay as in any earthquake signal. For last few decades several authors have worked to model the ground motion. Initially some of the authors modeled strong motion by a non-stationary random process represented by sum of decaying sinusoid. However, the most popular method is to get the non-stationary process by multiplying the stationary process by an envelop function i.e.

$$\ddot{Z}(t) = E(t) \ddot{x}(t) \quad (1)$$

Where $\ddot{x}(t)$ is stationary accelerogram obtained through IDFT, $E(t)$ is the envelop function and $\ddot{Z}(t)$ is the required non-stationary accelerogram. The choice of the envelop function $E(t)$ is made to agree with different types of recorded accelerograms. In this work, users of the software have choice between two envelop functions details of which are given below. More envelop functions will be added in the subsequent version of the software.

Closed Form Function

One of the envelop function used is a closed form function as suggested by Boore [2], which is

$$E(t) = a t^b e^{-ct} H(t) \quad (2)$$

Where $H(t)$ is heavy side function and other parameters are given as

$$b = \frac{-\varepsilon \ln \eta}{1 + \varepsilon(\ln \varepsilon - 1)} \quad (3)$$

$$c = \left(\frac{b}{\varepsilon T_w} \right) \quad (4)$$

(where T_w is total duration of generated time history)

$$a = \left(\frac{e^1}{\varepsilon T_w} \right)^b \quad (5)$$

In the above formulation, peak of envelop occurs at some fraction ε of specified duration T_w and the amplitude at time T_w is reduced to a fraction η of the maximum amplitude. In this work ε is taken as 0.2 and η is taken as 0.05.

Papageorgiu et. al [5] in their strong ground motion simulation code for Eastern North America have also used this envelop function. Some recent work done by Cramer and Kumar [6] have suggested similarities of attenuation characteristics of Bhuj and Eastern North America and it has biased the decision of using this envelop function here also. For duration of ground motion of 20 seconds the above envelop function is given in Figure 2.

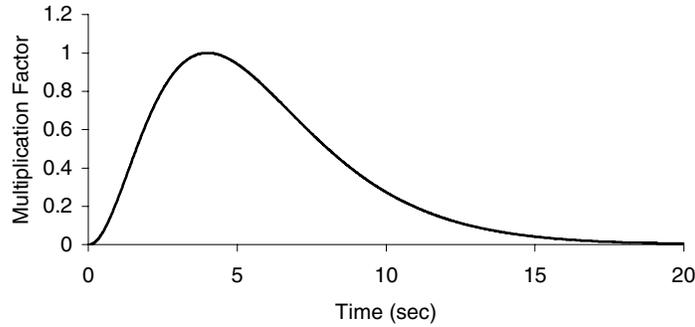


Fig. 2 Closed Form Envelop Function for a Duration of 20 Seconds

Function with Rise, Strong motion and Decay Time

The software has choice of another envelop function in which user can define rise time, strong motion time and decay time of the accelerogram. This function has parabolic rise and exponential decay as is being used by Jennings et.al [7]. The parabolic rise has a factor of 0 at $t=0$ and a factor of 1 at the end of rise time (or beginning of strong motion time). The exponential decay reduces the amplitude by a factor of 1 at the end of strong motion time (or beginning of decay time) to a factor of 0.05 at the end of accelerogram. A typical envelop function for a 20 second duration time history with rise time of 5 seconds, strong motion time of 8 seconds and decay time of 7 seconds is shown in Figure 3.

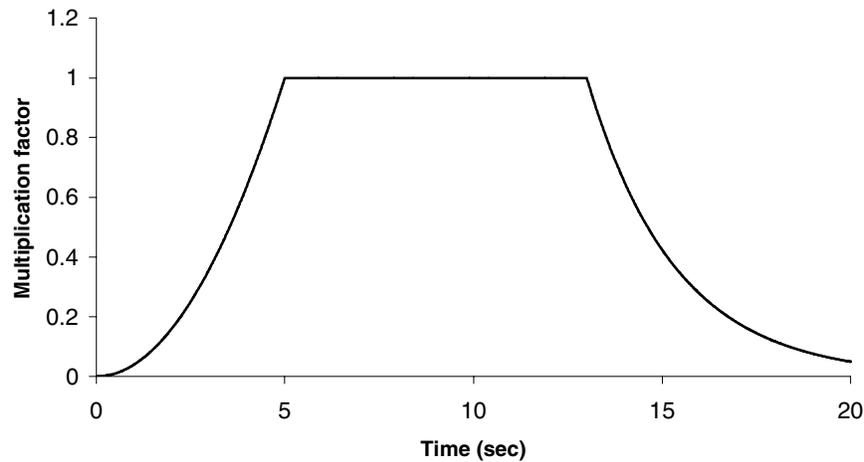


Figure 3 Envelop Function with Rise, Strong Motion and Decay Time

DETAILS OF COMPUTER PROGRAM

1. The developed software takes natural frequency vs. spectral acceleration of the target response spectrum as one of the input file (natural frequencies must be in ascending order). The last (i.e. the highest) natural frequency in the file decides the sampling rate of output time history, which is twice of this value. For example if the last natural frequency in the target response spectrum is 50 Hz then output of time history will be at 100 SPS. That means, for getting time history at 200 SPS the user should provide target response spectra upto 100 Hz. While specifying the target spectra it is highly recommended that all the frequencies above 35 Hz be assumed to have rigid spring and should have almost same spectral acceleration (the ZPA). The damping value of target response spectra is taken through terminal.
2. The user of the software is required to take decision on the duration of generated time history, which can be based on work done by different authors in this direction. The software demands duration of record in seconds, which is to be given through terminal.
3. In this computer program the user is asked to give choice between closed form envelop function and envelop function in which user can prescribe rise and decay time (with duration known from step 2, strong motion time is automatically calculate). The non-stationary signal is obtained by using the envelope function as per choice of the user. Details of both functions are given in the previous section.
4. First, number of data points in time history is determined as $n_{data} = \text{sampling rate} * \text{duration}$. Then bin frequency interval for frequency domain calculations are determined using $\text{binf} = \text{sampling rate} / n_{data}$. At this bin frequency, the target spectrum is linearly interpolated. Then this target acceleration response spectrum is converted into target velocity response spectrum.
5. For simplicity, this velocity spectrum is normalized, by dividing each of its ordinate by its peak value. This normalized velocity spectrum is considered as Fourier magnitude spectra. To develop the frequency domain description of the Fourier magnitude, random phase method is used. This means that for each frequency, a random number between -1 and 1 is picked up as real part. Its corresponding imaginary part is determined using the Fourier magnitude value of each frequency. Thus, real and imaginary pair for each bin frequency upto Nyquist frequency is determined which is then multiplied by peak value of velocity spectra, which was used for normalizations.
6. These pairs of real and imaginary parts are then multiplied by filter function. The filter function comprises of multiplication of high pass Butterworth function for cutoff frequency of 0.1 Hz, low

pass Butterworth function for cutoff frequency of 25 Hz and notch filter at the desired frequency. The frequency for the notch filter is automatically picked up by program from target spectra.

7. Real and imaginary pairs of Fourier spectra from Nyquist frequency to twice of Nyquist frequency (which are really negative frequencies of the frequency domain) are then determined by usual method for real uni-directional time history signal i.e. taking mirror image of real part and negative mirror image of imaginary part of values upto Nyquist frequency.
8. Inverse Fourier Transform of this set of complex number is then determined to get the accelerogram, which will be a stationary sequence.
9. This sequence is then multiplied by the envelop function given as selected in step 3 above, to get the desired non-stationary accelerogram.
10. Acceleration and velocity spectrum of this sequence are the determined at each bin frequency. In this work, spectral accelerations for different frequencies at bin frequency interval are determined through frequency domain approach.
11. Correction factor at each bin frequency between 0.1 Hz to 35 Hz is then determined by dividing the target velocity spectrum by obtained velocity spectrum.
12. This correction factor is then multiplied with the real and imaginary part of Fourier magnitude as determined at the end of step-5.
13. There can be several methods by which we can conclude that matching between computed and target spectrum is satisfactory. This program terminates, if all the ordinates of velocity spectra in the frequency range 0.1 Hz to 25 Hz have deviation of less than $\pm 10\%$. User has a choice to fix the number of iterations after the program will terminate.
14. The program uses DFT subroutine which takes much less computation time if number of data points in time history is equal to product of small prime numbers. Thus, the program will run faster if number of data points in the history will be 2000 or 2048 or 2187 or 2400 or 2401 or 3000 or 3600 or 4000 or 4096 or 4500 or 5000 or 6000 or 6561 or 8192.....
15. The program has a graphic interface, which displays the matching between target and computed spectra after each iteration. It also gives graphic display of the computed spectrum compatible time history.
16. One of the time history of duration of 20 seconds simulated for 5% damping response spectra (Type III Soil) of IS1893: 2002 [4] as target spectra is given in Figure 4. This time history has been simulated at 100 SPS with closed form envelop function. The target and achieved response spectra curves are given in Figure 5.
17. Another time history of duration of 20 seconds simulated for the same target spectra as of step 16 above is given in Figure 6. This time history has been simulated at 100 SPS with envelop function having attack time of 5 seconds, strong motion time of 8 seconds and decay time of 7 seconds. The target and achieved response spectra curves are given in Figure 7.
18. Executable version of the computer program and its help file can be downloaded from <http://www.iitr.ernet.in/acads/depts/earthquake/faculty/facthtml/akmeqfeq.shtml> (file names: spec.exe and spec.txt).

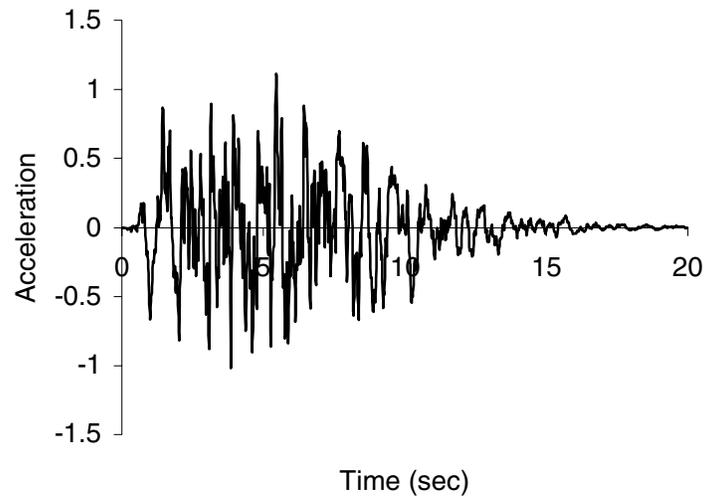


Fig. 4 Spectrum Compatible Time History for IS1893: 2002 Spectrum with Closed Form Envelop Function

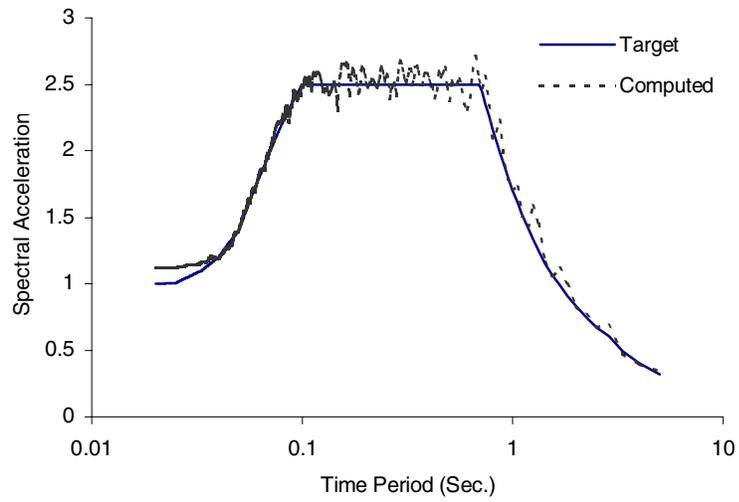


Fig. 5 Matching of IS1893: 2002 Spectra and Computed Spectra of Time History of Fig.3

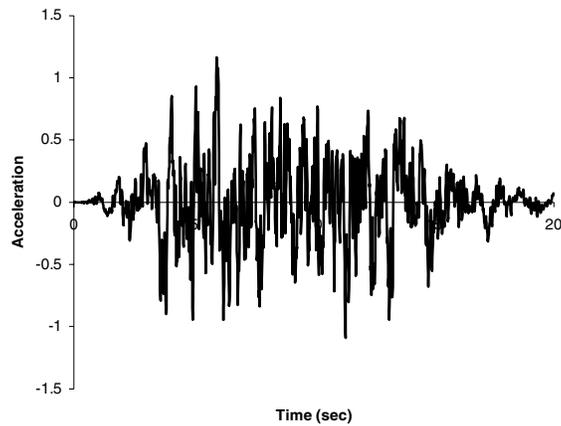


Figure 6 Spectrum Compatible Time History for IS1893: 2002 Spectrum with Prescribed Rise and Decay Time

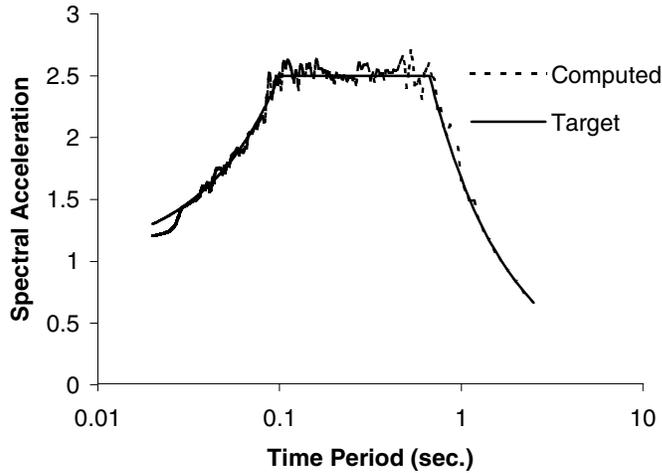


Fig. 7 Matching of IS1893: 2002 Spectra and Computed Spectra of Time history of Fig.5

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