13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 1578

RELIABILITY OF DECENTRALIZED ENERGY MARKET-BASED STRUCTURAL CONTROL

Jerome Peter LYNCH[1]

SUMMARY

Widespread adoption of structural control systems has been limited by the perception that systems might prove to be unreliable over their operational life spans. To address this concern, a novel decentralized control implementation that is responsive to actuation failures is proposed. Termed energy market-based control (EMBC), a control system is modeled as a free market economy where actuators act as market buyers and system energy supplies are modeled as market sellers. At every time step, each market participant simultaneously optimizes their respective utility functions resulting in a Pareto optimal control solution. As a piece-wise approximation of a dynamic optimization control solution, such as centralized linear quadratic regulation (LQR), energy market-based control is responsive to changes in the underlying system including possible failures in the initial actuation configuration. To illustrate the robustness of the EMBC control solution to actuation failures, a 20-story benchmark structure controlled by semi-active variable dampers is selected. Using cost functions as measures of optimality, changes in cost function final values are tracked during scenarios of failed actuation configurations for both an LQR and EMBC controller. Based on cost function changes, the EMBC control solution is shown to be responsive to failed actuators resulting in greater solution reliability.

INTRODUCTION

Since the inception of the concept in 1972 by Yao [1], structural control technologies have been employed by engineers to limit the response of civil structures to large external forces such as those associated with strong winds and earthquakes. The design of all structural control systems entail the use of sensors to measure structural responses, controllers to calculate control forces and actuators to apply actuation forces. Structural control systems can be classified as either active or semi-active depending upon the type of actuator used to apply control forces in the system. Early control system designs were defined as active because of the use of electro-mechanical or electro-hydraulic actuators that applied forces directly to the structure. To deliver forces sufficient to control large civil structures during small or moderate earthquakes, active actuators are large devices that consume considerable amounts of power, often on the order of tens of kilowatts [2]. To overcome the inability of active control systems to control structures during large earthquakes, semi-active structural control was proposed. Actuators used in semi-active

[1] Assistant Professor, Department of Civil and Environmental Engineering, University of Michigan. Email: jerlynch@umich.edu

structural control are essentially passive energy dissipation devices, modified with mechanisms that change their dissipation properties in real-time. The variable damper is a popular example of a semi-active control actuator [3, 4]. In contrast to active actuators whose control forces are applied directly to the structure, semi-active actuators modify global structural properties, such as damping or stiffness, to drain energy out of the system. Since power is only used to change the energy dissipation properties of the actuation device, semi-active actuators consume less power compared to active counterparts, often on the order of tens of watts [2]. Another attractive feature of semi-active control devices is they are bounded-input bounded-output (BIBO) stable; in other words, semi-active actuators cannot destabilize the system during improper operation [5].

Even though concepts of controlling civil structures were first introduced over three decades ago, the technology has yet to experience wide-spread adoption. The large gap that exists between structural control research and full-scale industry implementation can be attributed to a belief that structural control systems can prove to be unreliable over their expected operational lives [6]. In response to this belief, numerous investigators have addressed the reliability of structural control systems from many different perspectives. In the first, the reliability of the control solution is evaluated with respect to uncertainties surrounding the structural properties of the system. Often termed robust control analysis, bounds are placed on the uncertainty of structural parameters that are then propagated through the design of the controller to arrive at a worst-case assessment of the control system's performance [7]. Through this analysis, the robustness of the control solution with respect to design uncertainties can be ascertained. In other approaches, first- and second-order reliability methods (FORM/SORM) have been applied to the problem to make an assessment of the probability of instability that could result from the control solution [8]. Another approach to considering the robustness of the control solution is from the perspective of the reliability of the individual actuation devices. For example, some have taken a fault-tree approach in measuring the probability of actuation availability when the control system is activated [6].

As semi-active actuators continue to diminish in cost and size, semi-active control systems will be designed with an increasing number of actuators within a single structure. With respect to actuation failures, semi-active control systems defined by high actuation densities are inherently more reliable than systems employing a small number of actuators. While reliability improves with larger systems, the capacity of the centralized controller reduces faster than at a linear rate [9]. From observing similar trends in other fields, control engineers have explored decentralizing control system architectures to provide robust control solutions in complex large-scale problems. Lynch [10] has recently investigated optimal and sub-optimal decentralized control system architectures in civil structures and found decentralized control system performance was comparable to traditional centralized control solutions such as the linear quadratic regulator (LQR). One particular decentralized control solution that has shown promise when used in structural control systems is energy market-based control (EMBC) [11]. EMBC is a control solution that models the control system as a free market economy where autonomous market buyers compete with one another to purchase scarce system resources from market buyers. When applied to structural control, semi-active actuators are modeled as market buyers and portable power supplies (e.g. batteries) are modeled as market sellers with the scarce market resource being battery energy. Modeling the control system as a free market economy is a decentralized control solution because it replaces the decision authority of a centralized controller with a community of autonomous decision makers (market buyers and sellers). An attractive feature of EMBC is its highly adaptive nature; adaptation renders the EMBC solution sensitive to changes in the underlying problem and decision context, such as when actuators might fail in the control system.

In this study, the robustness of EMBC solutions to actuation failures will be quantitatively studied. A 20-story benchmark structure designed for the seismic Los Angeles area is utilized in the study. A total of 36 semi-active variable damping devices designed by Kajima Corporation, Japan [4] are installed throughout the structure to control structural deflections resulting from large seismic events. A centralized LQR control solution is formulated and is first applied in the benchmark structure. Next, a decentralized EMBC control solution is implemented. The performance of the two control solutions are compared to one another by calculating the final value of a cost function that decreases with improved control performance and increases with the energy consumed by system actuators. After the baseline performance of both control solutions has been determined for the fully functional system, various configurations of failed actuators are assumed. For each failed actuation configuration, the performance of the centralized LQR and decentralized EMBC solutions are compared using the final value of the cost function. As will be shown, EMBC exhibits excellent robustness during the occurrence of actuation failures as a result of its adaptive nature. This will be in contrast to the centralized LQR solution which assumes all the actuators are functional during its derivation; hence, any failure in the actuators renders the solution sub-optimal.

STATIC OPTIMIZATION OF FREE MARKETS

Economics is essentially the study of allocating scarce resources between competing parties. Many of the resource allocation problems encountered by economists are considered static and do not require inclusion of time effects in the behavioral models used to describe the economic system. To determine the optimal allocation of resources in economies, static optimization techniques are widely used. Static optimization entails the maximization of a scalar objective function, *F(x)*, confined by solutions found within a prescribed opportunity set, *X*:

$$\max F(\boldsymbol{x}) \text{ subject to } \boldsymbol{x} \in X \tag{1}$$

If an economy is composed of consumers seeking to purchase quantities of a single commodity, *x*, from market sellers, the optimal pricing of the commodity, *p*, can be determined by posing the system in the form of Equation (1). The objective function, $F(\boldsymbol{x})$, governing the behavior of the market buyers is the utility function, *U(x)*. The utility function is a mathematical convenience for measuring how much utility a buyer obtains in purchasing a particular set of goods or services. To determine the optimal amount of goods a buyer should buy, the utility function of the buyer is statically optimized. The constraining opportunity set, *X*, for the market buyer is the size of its available budget. The budget prevents the purchase of more goods than what the buyer's income can support. If the amount of goods, *x*, can be purchased at a price *p* and the household's available income is *r*, the budget of the household would constrain the maximization of the utility function by $px \le r$. Therefore, the static optimization of the household's utility reduces to a linear programming problem:

$$\max U(x) \text{ subject to } px \le r, \quad x \ge 0 \tag{2}$$

Provided that the optimization is dependent upon the price, *p*, and the solution is unique, the maximization of the utility described by Equation (2) results in the demand function of each buyer (as a function of price) [12].

Market sellers represent the manufacturing engine of the economy with goods and services produced for buyer consumption. Similar to the market buyer, the behavior of the seller is determined by its objective function. The objective function for the seller is the profit function, *Π*, which is simply revenue, *R*, minus production costs, *C*:

$$\Pi = R - C = pq - C \tag{3}$$

Revenue is the money obtained from market buyers for the quantity of commodity they purchase, *q*. The output of each seller is determined by maximizing the profit function of Equation (3). In the long-term, the maximization of the cost function is unconstrained except by the firm's ability to manufacture a sufficient quantity of goods without exceeding its manufacturing capacity, *b*. The result is a classical programming problem.

$$\max \Pi(q) \text{ subject to } q \le b \tag{4}$$

The solution of the optimization problem posed by Equation (4) yields the firm's supply function for each of its outputs.

Economies are often comprised of many market buyers and sellers that compete with one another by maximizing their respect utility and profit functions. Mathematically, the optimal allocation of the scarce market resource is provided by the simultaneous solution of Equations (2) and (4) for all sellers and buyers. In addition, market-clearing constraints exist within the market with the sum of all demands equaling the sum of supplies of a particular good [12]. The final pricing solution determined by the static optimization procedure is termed the Pareto optimal solution. Pareto optimal is defined by a market in competitive equilibrium where no market participant can reap the benefits of higher utility or greater profit from a change in the resource allocation without causing harm to the other participants [13].

Dynamic optimization differs significantly from static optimization by considering optimality within the context of time. Dynamic optimization is widely used in multiple disciplines including economics and control theory. Analogous to the objective function used in static optimization, dynamic optimization centers on the maximization of a prescribed scalar cost function, *J*, which is an integral of an instantaneous objective function, *I*, defined over a desired time interval. The cost function is generally a function of the state of the model, $\boldsymbol{x}$, control variables, $\boldsymbol{u}$, and time, *t*.

$$J = \int_{t_0}^{t_1} I(\mathbf{x}(t), \mathbf{u}(t), t)\, dt \tag{5}$$

In optimizing the cost function, *J*, the system is often constrained by the first-order differential equation that specifies the dynamic equilibrium of the system:

$$\max J = \max \int_{t_0}^{t_1} I(\mathbf{x}(t), \mathbf{u}(t), t)\, dt \text{ subject to } \dot{\boldsymbol{x}} = f(\boldsymbol{x}, \boldsymbol{u}, t) \tag{6}$$

The final solution, is then an optimal trajectory of the system behavior, $\boldsymbol{x}$, in time-space. Linear quadratic regulation (LQR), the widely used centralized control law, is formulated in the same manner as Equation (6). For LQR, the cost function is written as an integral sum of the time-history response of the structure and the applied control force history. The matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$ and used to adjust the relative emphasis between the response of the system, $\boldsymbol{x}(t)$, compared to the control effort, $\boldsymbol{u}(t)$. Optimization of the cost function, *J*, is constrained by the equation of motion of the system when stated in state-space form with $\boldsymbol{A}$ as the system matrix and $\boldsymbol{B}$ as the control location matrix:

$$\max J = \max \int_{t_0}^{t_1} \left(\mathbf{x}^T Q \mathbf{x} + \mathbf{u}^T R \mathbf{u}\right) dt \text{ subject to } \dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \tag{7}$$

The motivation behind considering the use of market allocation mechanisms for controlling civil structures is partially attributed to the general Weierstrass theorem which states that dynamic optimization problems can be posed as piece-wise static optimization problems solved at sequential points in time [12]. In lieu of considering the optimal control of a civil structure as a dynamic optimization problem, as done in designing an LQR controller, an approximate dynamic optimization solution can be achieved by exploiting a piece-wise static optimization approach. Piece-wise static optimization is attractive because an optimization is re-performed at each step in time rendering the method very responsive to any changes that might occur in the underlying system. This is in contrast to dynamic optimization which determines the optimal control law *a priori*. After formulation, the dynamic optimization solution is optimal only if the system remains unchanged; any change to the system, such as the existence of failed actuators, renders the solution sub-optimal. To reap the benefits of piece-wise static optimization, the structural control system will be posed as a free market economy which will serve solely as a theoretical construct. The ecological community formed by the market will sacrifice closed-form *a priori* control laws for a more phenomenological approach with greater resilience to system changes.

ENERGY MARKET-BASED CONTROL

The marketplace construct used to deliver a piece-wise static optimization control solution is centered upon the scarce market commodity of control energy. The actuators in the control system are modeled as market buyers who seek to acquire control energy from market sellers. The control energy they purchase is then used to apply control forces to the system. System battery supplies are given the role of market sellers offering control energy in exchange for money. Before implementation, the demand and supply functions that dictate the behavior of the market buyers and sellers is stated. To remain consistent with the scarce market commodity, the demand and supply functions are written in terms of measurable quantities of system response energies. To simplify the implementation, the energy market-based control solution models the structure as a lumped mass shear structure with each actuator assigned to a translational degree-of-freedom.

The actuator's demand function is written to encapsulate two behavioral features expected of a market buyer. First, if control energy, CE, is free (price, p, is zero) the i^{th} buyer will seek to purchase enough control energy, CE_i, to counteract the input energy to the actuator's translational degree-of-freedom resulting from the seismic ground excitation, x_g. Second, as the price of control energy grows to infinity, the demand of market buyers will asymptotically converge to zero. These defining behavioral characteristics suggest the demand function be written in a decaying exponential form:

$$CE_i = W_i \left| \ddot{y}_i(t) m_i dx_g \right| e^{\frac{-2p\alpha}{\dot{y}_i^2 m_i + x_i^2 k_i}} \tag{8}$$

The displacement of the actuators translation degree-of-freedom relative to the ground motion is denoted as x_i while the absolute response of the degree-of-freedom is $y_i = x_g + x_i$. The lump mass associated with each translational degree-of-freedom is m_i while the elastic restoring force of the degree-of-freedom is denoted as k_i. The demand function's y-axis intercept is equal to the instantaneous input energy of the ground motion at a particular degree-of-freedom. Each market buyer is provided with an amount of money, termed instantaneous wealth, W_i, from which it can purchase control energy. The exponential decay of the demand function is dependent upon the kinetic and strain energy of the system as depicted by the exponential term's denominator. As the response of the system increases due to greater kinetic and strain energy, the rate of decay decreases. A tuning constant, α, is provided to control the sensitivity of the demand function before its implementation in the control system. As α increases, the demand function's decay becomes more rapid.

The system energy source is assumed to be a single battery servicing all of the control system actuators. The behavior of this energy source is as a market seller whose actions will be described by a supply function. The market seller has in its possession a certain amount of control energy, CE. Again, two observations of the market seller's behavior are required before specifying a suitable supply function. First, if the price of power is set to zero, the market seller will be unwilling to sell control energy. Second, as the price grows to infinity, the market seller will desire to sell all of its remaining control energy, L. As a result, the following supply function is proposed:

$$CE = L\left(1 - e^{-\beta p}\right) \tag{9}$$

Equation (9) provides an origin intercept in addition to an asymptotic convergence to the remaining control energy, L, at very large market prices. Similar to the demand function, the constant β is used to provide a means of adjusting the supply function's sensitivity before implementation in a structure.

Assuming that the control system to be implemented in a discrete-time environment, the individual demand functions of the market buyers (actuators) are aggregated at each time step to form a global demand function. The point at which the demand function equals the supply function establishes the competitive equilibrium pricing of the scarce control energy commodity. After the Pareto optimal commodity price is established, each buyer buys the amount of control energy specified by the price and their own individual demand functions. The control force then applied by each actuator at a time step k is simply:

$$u_i(k) = CE_i(k) / \Delta x_i(k) \tag{10}$$

Once control energy has been purchased, the amount of money used to make that purchase is removed from the buyer's instantaneous wealth:

$$W_i(k+1) = W_i(k) - CE_i(k) * p(k) \tag{11}$$

Figure 1 provides a summary of the EMBC control implementation for controlling a structure during seismic events.

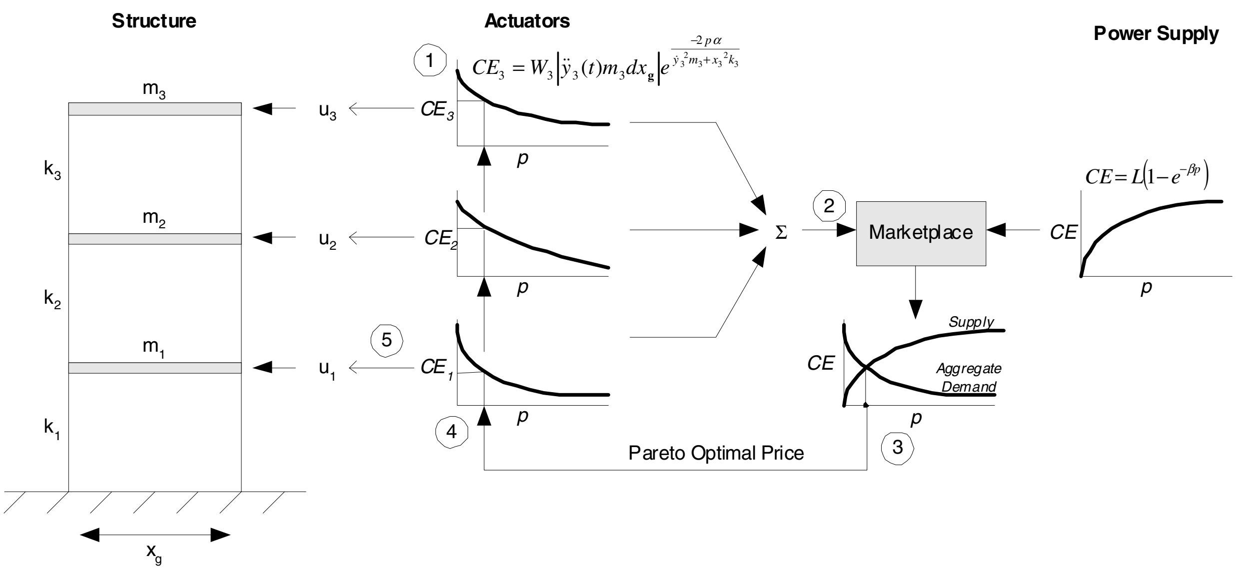
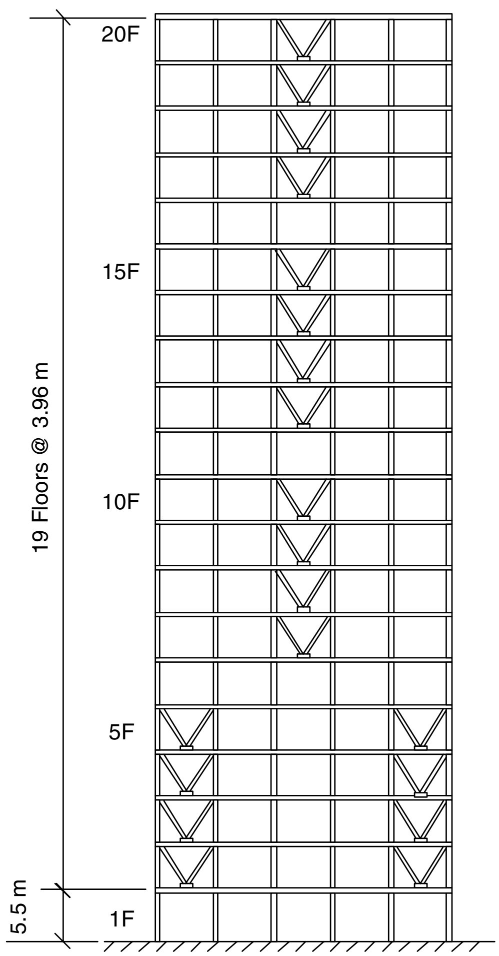


Figure 1. Energy market-based control of a civil structure during a seismic disturbance: 1) each actuator defines a demand function, 2) all buyers' demand functions are aggregated to derive a global demand function, 3) the competitive price of control energy is found where demand equals supply, 4) based on the Pareto optimal price, each actuator purchases control energy, and 5) the control energy purchased is used to apply a control force

BENCHMARK 20-STORY STEEL STRUCTURE

In this study, the 20-story steel structure designed for the Structural Engineers Association of California (SAC) project is selected to observe the performance of the EMBC solution [14]. Having been designed using current seismic building design codes, the building depicted in Figure 2 represents a realistic design for the southern California region. To simplify the implementation of the control solution, the structure is modeled as a lumped mass shear model sustaining elastic deformations. While the stiffness and mass properties are tabulated in Figure 2, the structure's natural damping is chosen to be 5% of critical damping in the first mode.



Floor	Seismic Mass	Story Stiffness	# of Actuators
1	1.126×10^6 kg	862.07×10^3 kN/m	-
2	1.100×10^6 kg	862.07×10^3 kN/m	4
3	1.100×10^6 kg	862.07×10^3 kN/m	4
4	1.100×10^6 kg	862.07×10^3 kN/m	4
5	1.100×10^6 kg	862.07×10^3 kN/m	4
6	1.100×10^6 kg	554.17×10^3 kN/m	-
7	1.100×10^6 kg	554.17×10^3 kN/m	2
8	1.100×10^6 kg	554.17×10^3 kN/m	2
9	1.100×10^6 kg	554.17×10^3 kN/m	2
10	1.100×10^6 kg	554.17×10^3 kN/m	2
11	1.100×10^6 kg	554.17×10^3 kN/m	-
12	1.100×10^6 kg	453.51×10^3 kN/m	2
13	1.100×10^6 kg	453.51×10^3 kN/m	2
14	1.100×10^6 kg	453.51×10^3 kN/m	2
15	1.100×10^6 kg	291.23×10^3 kN/m	2
16	1.100×10^6 kg	291.23×10^3 kN/m	-
17	1.100×10^6 kg	291.23×10^3 kN/m	1
18	1.100×10^6 kg	256.46×10^3 kN/m	1
19	1.100×10^6 kg	256.46×10^3 kN/m	1
20	1.170×10^6 kg	171.70×10^3 kN/m	1

Figure 2. The 20-story SAC benchmark structural model

To control the structure during seismic disturbances, a total of 36 semi-active hydraulic damping (SHD) devices are strategically placed throughout the structure. In particular, SHD devices designed and manufactured by the Kajima Corporation, Japan are considered [4]. The Kajima SHD devices are capable of applying a maximum control force of 1,000 kN to the structure with the SHD damper typically installed between the low point of a stiff K-brace and the floor. Given a command control force, the SHD calculates the damping coefficient by dividing the command force by the relative velocity between the two floors to which the SHD is connected. If the relative velocity between the two floors is in the opposite direction of the desired control force, then the control force is applied. If the response is in the same direction, no control force is applied and the damper is set to its default minimum damping setting. In the idealized lumped mass model, each SHD device is modeled as a Maxwell damping element whose damping coefficient is adjusted in real-time [15]. To evaluate the effectiveness of the control strategies implemented, the El Centro (1940 NS) earthquake record is utilized. The peak absolute velocity of the record is scaled to 50 cm/s resulting in a peak ground acceleration of 3.07 m/s^2.

ROBUSTNESS OF PIECE-WISE STATIC OPTIMIZATIONS

The centralized LQR control solution is based on the assumption of complete knowledge of a fixed parameter system. On the other hand, EMBC has the distinct advantage of performing an approximate dynamic optimization solution by solving a static optimization problem at each point in time. Therefore, the piecewise static optimization solution has an opportunity to account for changes in the system as they may arise, resulting in a robust control approach. The adaptive nature of the EMBC approach can potentially reap great benefits when the approach is applied to the robust control problem. When a component of the control system fails, the EMBC approach has indirect knowledge of the failure and will find an optimal solution for the new actuation configuration.

Measure of Optimality

Optimality measures of the control solution are required for robustness assessment of the EMBC method. In reference to the derivation of the LQR controller, optimality is measured with respect to a total control cost index, J, represented by the cost function of Equation (7). We will use the same optimality measure, J, with the weighting matrices, $\boldsymbol{Q}$ and $\boldsymbol{R}$, selected to represent total energy in the system. First, the cost function is divided into two components, J_X and J_U, respectively, representing the energy of response and the energy associated with the applied set of control forces. As presented by Equation (12), the first optimality measure, J_X, represents the total kinetic and strain energy of the system where $\boldsymbol{X}$ is a state space representation of the system response, $\boldsymbol{X} = \{\boldsymbol{x} \quad \dot{\boldsymbol{x}}\}^T$:

$$J_X = \sum_{i=1}^{N} \boldsymbol{X}^T \begin{bmatrix} \boldsymbol{K} & 0 \\ 0 & \boldsymbol{M} \end{bmatrix} \boldsymbol{X} \tag{12}$$

The second optimality measure, J_U, is the cost associated with the application of controls to the system. As shown in Equation (13), the cost of control is a function of the control energy applied by the system actuators [16]:

$$J_U = \sum_{i=1}^{N} \boldsymbol{U}^T \boldsymbol{K}^{-1} \boldsymbol{U} \tag{13}$$

The total measure of optimality is the sum of Equations (12) and (13) resulting in Equation (14):

$$J = J_X + J_U \tag{14}$$

The LQR solution is optimal because the solution derived represents a minimization of the cost function, J. It should be noted that even though the optimal cost index is originated from the LQR derivation, it can be readily applied to other control approaches so as to measure their optimality relative to the LQR solution. This point will be instrumental in assessing the robustness of the EMBC control solution relative to the centralized LQR control solution.

Assessment of the Robustness of the Control Solution

In assessing the robustness of the EMBC solution, the 20-story benchmark structure will be used to simulate control systems that have partial actuation failures. Failure of a portion of the actuation system is applied to the model at the problem outset. The behavior of the LQR control solution is well understood by a strong closed-form mathematical solution. Conversely, the derivation of the EMBC approach is more phenomenological. To assess the robustness of the EMBC controller, the LQR controller is used to observe its robust behavior during failures to the actuation system. By drawing a parallel to the LQR behavior, the robustness of the EMBC controller is proven. As such, the following analysis will be performed in order to illustrate the robustness qualities of the EMBC solution. First, the robustness of the LQR solution is quantified by performing the following tasks:

1. An optimal LQR controller, reflected by the minimization of the cost index, J, is designed. Once the LQR solution is implemented, the cost index is computed and recorded for a given seismic disturbance.

2. To represent a scenario of actuation failure, a portion of the control system's actuators is intentionally disabled before the seismic disturbance is applied. During the excitation, the control forces are determined by the controller's original feedback gain matrix but applied only by those actuators that are working. Using the original LQR gain matrix that was derived under the assumption that all actuators are working, the failed actuators result in a sub-optimal controller performance as observed by a change in the cost index value. Whether an increase or decrease of the cost index occurs cannot be conclusively stated in this case. If all actuators are properly functioning but the solution is sub-optimal, an increase in the cost index would be observed. However, as a result of some actuators not using control energy, the cost index might decrease even though it is a suboptimal solution.

3. The control solution can be re-optimized if a new LQR solution is calculated based on knowledge of the failed actuators. The performance of the new LQR controller is optimal for the current configuration of working and failed actuators. The seismic excitation is applied to the structure. A decrease in the cost index is noted for this new LQR solution compared to that from the case of the original LQR solution applied to the failed actuation system in the previous step.

This analysis is purely academic since no mechanism exists for the LQR controller to recalculate an optimal feedback gain once the failure of actuators is identified. The intention is to only show how the LQR solution behaves given failures in the actuation configuration. This behavior will serve as a key component in assessing the robustness qualities of the EMBC approach.

Next, attention is turned to assessing the robustness of the EMBC controller:

1. An EMBC solution is devised under the assumption that all actuators are reliable and will work when required. The solution devised by the EMBC controller is optimal in a Pareto optimal sense. The same seismic disturbance used in the LQR analysis is imposed to the structural system and the cost index, J, is measured using Equations (12), (13) and (14).

2. Next, actuators are purposely disabled prior to application of the seismic disturbance. The EMBC controller is artificially constrained to perform market competitions without awareness of the actuator failures; control energies purchased by defective actuators are not used for control. This is equivalent to step 2 of the LQR analysis where the original LQR controller is used to apply control forces only by actuators functioning properly. The result is a sub-optimal performance of the EMBC solution as manifested by a change in the cost index. Again, a decrease or increase in the cost index can occur as a result of the failed actuators not using control energy. Forcing the EMBC controller to behave in this manner during actuation failures is spurious and is done only for illustration purposes. Naturally, the marketplace can observe the loss of an actuator by seeing that control energy purchased is not being applied. In a situation where the actuator is not functioning, the actuator's market buyer would not spend its wealth nor receive income. The

isolation of the market buyer associated with the failed actuator is the natural internal mechanism of the EMBC controller for indirect identification of failed actuators.

3. In the next set of analyses, the EMBC solution is allowed to behave as designed; market buyers associated with broken actuators are permitted to add their demand to the global market demand function, but no wealth is transferred from or to the failed market participant. The result is an EMBC solution that is Pareto optimal with respect to the loss of the actuators. This fact will be observed by a decrease in the optimal cost index, J, when compared to the cost index calculated for the spurious implementation of EMBC in the previous step.

In summary, the analysis will illustrate two important issues. First, when actuators fail in the control system, the original optimal LQR control solution has become sub-optimal as reflected by a change in the cost index. To return to an optimal solution as reflected by a reduction in the suboptimal cost index value, a new LQR solution needs to be calculated with *a priori* knowledge of the failed system actuators. Second, if the energy market-based control approach is forced to ignore the existence of failures in the control system's actuators, it too will experience a change in the cost index. By allowing the EMBC controller to behave as it is designed, a decrease in the cost index from the artificially constrained EMBC implementation can be achieved.

If the LQR controller could identify the existence of failed actuators in the system and recalculate its control solution, it would represent a robust controller with respect to actuation failures. Unfortunately, no implementation exists that allows the LQR controller to behave in this manner. On the other hand, EMBC has the unique ability to sense the existence of actuation failure by identifying market-participants that have become isolated in the market (no wealth is transferred between the buyer and the market). As a result, the EMBC solution optimizes around the isolated participant. Proof of the EMBC approach's optimization and subsequent robustness is supported solely by the behavior of the EMBC total cost index value, but the cost index's behavior is compared to that of the LQR controller in order to provide stronger support since the LQR controller is well understood. The unique behavior of the EMBC approach observed makes the approach robust in the face of system failures since it can optimize the control solution in real-time to account for failures. This is additional evidence of the advantages of performing an approximate dynamic optimization through recalculation of the static optimization problem at each time step.

Robustness of Control Solutions in the 20-Story Benchmark Structure

To observe the robust behavior of the EMBC solution in a large-scale structure, the 20-story benchmark structure, as shown in Figure 2, will be used. Actuation failures will be grouped into larger regions that span multiple floors. For this set of analyses, the actuators located in the building are grouped into four zones. In the first zone, actuators on floors 2 through 5 are grouped. Likewise, the second, third and fourth zones group the actuators of floor 7 through 10, floors 12 through 15 and floors 16 through 20, respectively. A total of six failure scenarios are considered; the first four failure scenarios are of each of the zones experiencing total actuation failure, the fifth failure scenario is of the actuators in zones 2 and 3 all not working while the last scenario is of actuators in zones 1 and 4 all not working.

For the EMBC controller, the market parameters $\alpha = 1$, $\beta = 1$, and $L = 8 \times 10^{11}$ J are employed. For the initial distribution of instantaneous wealth, each actuator in floors 2 through 5 are provided with $W = 250$ while those on floors 7 through 15 are provided with $W = 500$. The actuators on floors 17 through 20 are provided with $W = 1000$. As a result, each floor has attributed to it a total instantaneous wealth of $W_{floor} = 1000$. The weighting matrices, $\boldsymbol{Q}$ and $\boldsymbol{R}$, of the LQR controller are selected as those presented in Equations (12) and (13).

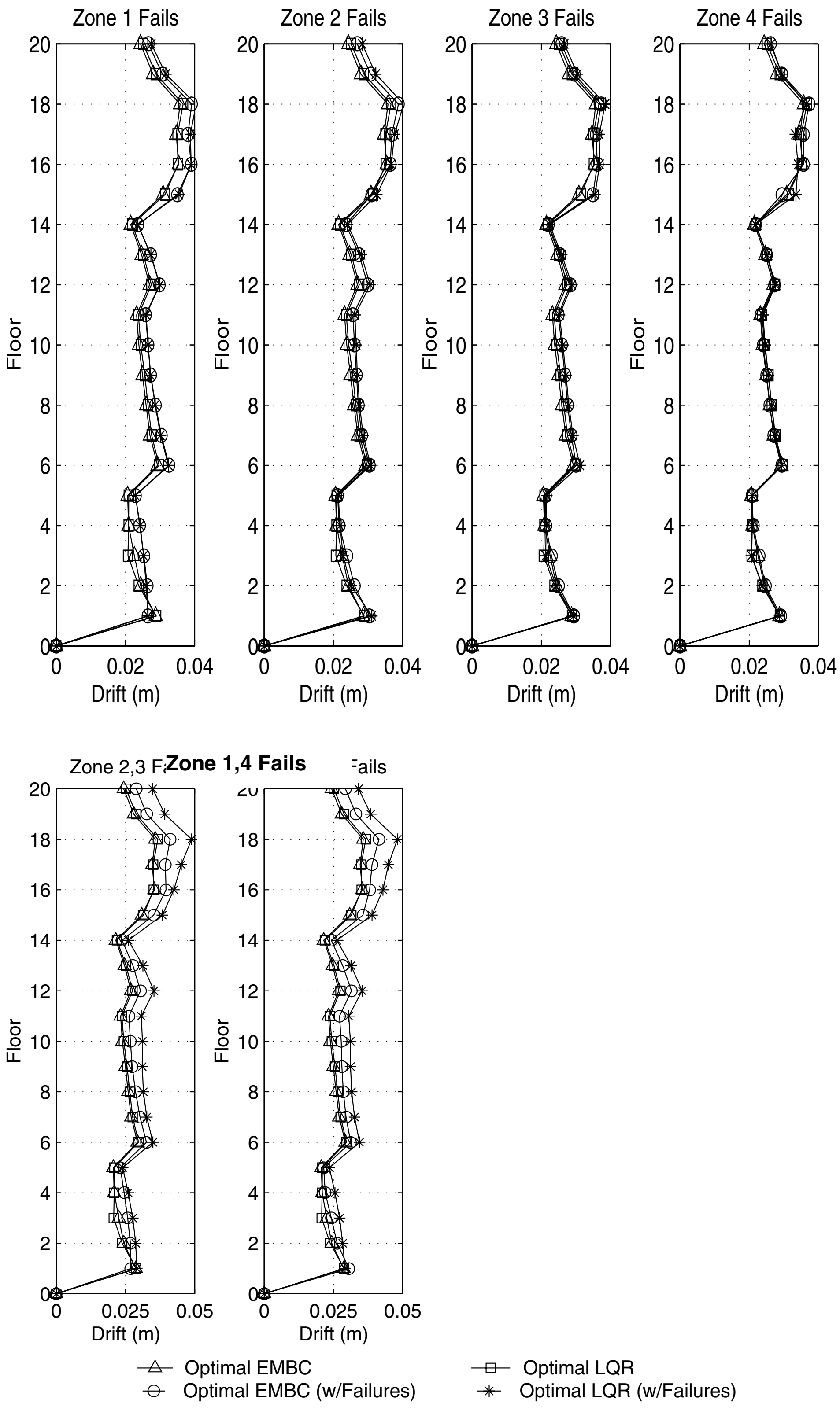


Figure 3. Controlled 20-story benchmark structure interstory drift response to actuator failures

The maximum absolute interstory drift response of the benchmark structure to the scaled El Centro seismic record is presented in Figure 3. The actuation failure of the last two cases (multiple zone failures) is most dramatically felt by the structure with the EMBC controller outperforming the LQR controller for these cases.

Assuming no actuation failure, the cost indices for the LQR controller are:

$$
\begin{aligned}
J_X &= 3.7209 \ x \ 10^9 \ \text{J} \\
J_U &= 3.7154 \ x \ 10^9 \ \text{J} \\
J &= 7.4363 \ x \ 10^9 \ \text{J}
\end{aligned}
\qquad (15)
$$

After determination of the initial cost index values, the actuation failure scenarios are applied to the benchmark structure. The analyses results for the normalized El Centro record are summarized in Table 1. Similar to the Kajima-Shizuoka Building's results, the cost index on structural response, J_X, increases and the cost index on control effort, J_U, decreases as a result of actuation failure. The trend of importance is the reduction of the total cost index, J, from application of the original controller to application of the re-optimized controller for each failure scenario.

Table 1. Benchmark structure LQR performance (actuation failure) – El Centro

Floor	J_X	J_U	$J = J_X + J_U$
Fail Z1, Centralized LQR	4.5975 x 10^9 J	2.8078 x 10^9 J	7.4054 x 10^9 J
Re-optimized LQR (w/o Z1)	4.5778 x 10^9 J	2.8010 x 10^9 J	7.3789 x 10^9 J
Fail Z2, Centralized LQR	4.3681 x 10^9 J	2.1539 x 10^9 J	6.5219 x 10^9 J
Re-optimized LQR (w/o Z2)	4.3504 x 10^9 J	2.1270 x 10^9 J	6.4774 x 10^9 J
Fail Z3 Centralized LQR	4.3135 x 10^9 J	1.8665 x 10^9 J	6.1800 x 10^9 J
Re-optimized LQR (w/o Z3)	4.3211 x 10^9 J	1.8706 x 10^9 J	6.1917 x 10^9 J
Fail Z4 Centralized LQR	3.7829 x 10^9 J	3.5603 x 10^9 J	7.3432 x 10^9 J
Re-optimized LQR (w/o Z4)	3.7255 x 10^9 J	3.5931 x 10^9 J	7.3186 x 10^9 J
Fail Z1,Z4 Centralized LQR	5.0869 x 10^9 J	0.7080 x 10^9 J	5.7949 x 10^9 J
Re-optimized LQR (w/o Z1, Z4)	5.3092 x 10^9 J	0.0070 X 10^9 J	5.3172 x 10^9 J
Fail Z2,Z3 Centralized LQR	4.6583 x 10^9 J	2.7424 x 10^9 J	7.4007 x 10^9 J
Re-optimized LQR (w/o Z2, Z3)	6.2995 x 10^9 J	1.3479 x 10^9 J	6.3130 x 10^9 J

In the next set of analyses, the EMBC solution is applied to the benchmark structure. With the actuators all working properly, the EMBC cost indices are determined from the response of the structure to the El Centro seismic record.

$$
\begin{aligned}
J_X &= 3.7412 \ x \ 10^9 \ \text{J} \\
J_U &= 3.6391 \ x \ 10^9 \ \text{J} \\
J &= 7.3803 \ x \ 10^9 \ \text{J}
\end{aligned}
\qquad (16)
$$

Note the total cost index is lower for EMBC than that of the LQR controller; the ability of the EMBC controller to have better optimality measures than LQR can be attributed to the nonlinear solution presented by EMBC. LQR only provide the optimal control solution for a linear controller.

To assess the robustness of the EMBC solution, the actuation failure scenarios are applied to the structure at the outset of the analysis. The artificially constrained and natural EMBC solutions are implemented and the cost indices calculated as presented in Table 2. Evident is a reduction in the total cost index, J, from the constrained to the natural EMBC implementations. With the reductions identical to those in the set of LQR analyses, it can be concluded that the EMBC solution is optimized to the current failed system configuration. Therefore, with real-time optimization capabilities, the energy market-based control solution exhibits an element of robustness that the centralized LQR solution is lacking. Again, it is also observed that the reductions of the LQR cost index appear to be more significant than to those of the EMBC controller.

Table 2. Benchmark structure EMBC performance (actuation failure) – El Centro

Floor	J_X	J_U	$J = J_X + J_U$
Fail Z1, EMBC Ignore Failure	4.6209×10^9 J	2.6854×10^9 J	7.3063×10^9 J
EMBC (w/o Z1)	4.6305×10^9 J	2.6739×10^9 J	7.3044×10^9 J
Fail Z2, EMBC Ignore Failure	4.3978×10^9 J	2.0015×10^9 J	6.3993×10^9 J
EMBC (w/o Z2)	4.3879×10^9 J	2.0002×10^9 J	6.3881×10^9 J
Fail Z3, EMBC Ignore Failure	4.2931×10^9 J	1.8446×10^9 J	6.1377×10^9 J
EMBC (w/o Z3)	4.2981×10^9 J	1.8537×10^9 J	6.1518×10^9 J
Fail Z4, EMBC Ignore Failure	3.8094×10^9 J	3.5111×10^9 J	7.3205×10^9 J
EMBC (w/o Z4)	3.7763×10^9 J	3.5046×10^9 J	7.2809×10^9 J
Fail Z1,Z4 EMBC Ignore Failure	5.0468×10^9 J	0.6752×10^9 J	5.7219×10^9 J
EMBC (w/o Z1, Z4)	5.0197×10^9 J	0.6947×10^9 J	5.7144×10^9 J
Fail Z2, Z3 EMBC Ignore Failure	4.6889×10^9 J	2.6213×10^9 J	7.3102×10^9 J
EMBC (w/o Z2, Z3)	4.6836×10^9 J	2.5860×10^9 J	7.2696×10^9 J

CONCLUSIONS

In this study, the reliability of a structural control system was enhanced by employing a piece-wise static optimization control solution that is responsive to changes in the underlying structural system. Termed energy market-based control (EMBC), natural measures of energy in the dynamic structural system were used to derive a powerful approach to the control problem. While deriving the market-based control approach using measures of energy is intuitive, other market models could be adopted that might provide superior control performances. When implemented in a 20-story benchmark structure, the EMBC control solution proves to be equivalent to the centralized LQR control solution. However, the robustness of the EMBC control approach was illustrated to be superior to that of LQR. Robustness with respect to actuation failures was illustrated for the EMBC approach by observing the behavior of the cost index, J, during the LQR and EMBC implementations and drawing parallels between the two controllers. Conceptually, the robustness of the EMBC approach is directly derived from the fact that static optimization is being performed at each step in time. Changes in the system can be observed and incorporated in the static optimization solutions immediately. The failure of actuators in the system represents only one form of change in the system's properties. Therefore, the ability of the approximate dynamic optimization of the EMBC control problem could have potential in the realm of nonlinear controls, which also represents property changes in the system.

REFERENCES

1. Yao, J. T. P. (1972). "Concept of structural control." *Journal of Structural Division*, ASCE, 98(7): 1567-1574.
2. Symans, M. D., and Constantinou, M. C. (1999). "Semi-active control systems for seismic protection of structures: a state-of-the-art review." *Engineering Structures*, Elsevier, 21(6): 469-487.

3. Spencer, B. F., Yang, G., Carlson, J. D., and Sain, M. K. (1998). "Smart dampers for seismic protection of structures: a full-scale study." *Second World Conference on Structural Control*, International Association for Structural Control, Kyoto, Japan, pp. 417-426.
4. Kurata, N., Kobori, T., Takahashi, M., Niwa, N., and Midorikawa, H. (1999). "Actual seismic response controlled building with semi-active damper system." *Earthquake Engineering and Structural Dynamics,* John Wiley & Sons, 28(11): 1427-1447.
5. Housner, G. W., Bergman, L. A., Caughey, T. K., Chassiakos, A. G., Claus, R. O., Masri, S. F., Skelton, R. E., Soong, T. T., Spencer, B. F., and Yao, J. T. P. (1997). "Structural control: past, present, and future." *Journal of Engineering Mechanics*, ASCE, 123(9): 897-971.
6. Battaini, M., Casciati, F., and Faravelli, L. (2000). "Some reliability aspects in structural control." *Probabilistic Engineering Mechanics*, Elsevier, 15(1): 101-107.
7. Chandrasekharan, P. C. (1996). *Robust control of linear dynamical systems*. Academic Press, London, UK.
8. Spencer, B. F., Sain, M. K., Won, C. H., Kaspari, D. C., and Sain, P. M. (1994). "Reliability based measures of structural control robustness." *Structural Safety*, Elsevier, 15(1): 111-129.
9. Lunze, J. (1992). *Feedback control of large-scale systems*. Prentice Hall, New York, NY.
10. Lynch, J. P. and Law, K. H. (2002). "Decentralized Control Techniques for Large-Scale Civil Structural Systems," *Proceedings of the 20th International Modal Analysis Conference (IMAC XX)*, Los Angeles, CA.
11. Lynch, J. P., and Law, K. H. (2004). "Decentralized Energy Market-Based Structural Control", *Structural Engineering and Mechanics,* Techno Press, 17(3), in press.
12. Intriligator, M. D. (1971). *Mathematical optimization and economic theory*. Prentice-Hall, Englewood Cliffs, NJ.
13. Mas-Colell, A., Whinston, M. D., and Green, J. R. (1995). *Microeconomic theory*. Oxford University Press, New York, NY.
14. Spencer, B. F., Christenson, R. E., and Dyke, S. J. (1998). "Next generation of benchmark structural control problems for seismically excited buildings." *Second World Conference on Structural Control*, International Association for Structural Control, Kyoto, Japan, pp. 1351-1360.
15. Hatada, T., Kobori, T., Ishida, M., and Niwa, N. (2000). "Dynamic analysis of structures with Maxwell model." *Earthquake Engineering and Structural Dynamics*, John Wiley & Sons, 29(2): 159-176.
16. Wong, K. F., and Yang, R. (2001). "Effectiveness of structural control based on control energy perspectives." *Earthquake Engineering and Structural Dynamics,* John Wiley & Sons, 30(12): 1747-1768.