



## IMPROVED CAPACITY-DEMAND-DIGRAM METHODS FOR ANALYSIS OF STRUCTURAL NONLINEAR SEISMIC RESPONSE

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### SUMMARY

Several disadvantages of the current capacity-demand-diagram method are presented in the paper. The first one is that the pushover curves cannot reflect the patterns of lateral loads that represent the real earthquake actions; the second is that the capacity-demand-diagram corresponding to the single nonlinear parameter cannot lead to the similar results to the nonlinear dynamic time-history analysis. To solve the problems, the method is being improved in the following aspects, to find factors that lead to discrepancies between the different capacity curves: the shape of lateral load distribution, assumption of member model, irregularity of the structures, and P- $\Delta$  effects. The most important of them is found by some analysis of structures. The order of the capacity curves' distribution and the data of the maximum base shear and the top displacement of the structures are found. The effect of nonlinear factors to the demand curves during the course of making equivalent linear systems is analyzed. Method of ATC-40 (Procedure-B) is improved; the improved method is more accurate in calculating the top displacements of the structures. The method of conversion relationship between the results of the capacity-demand curves method and the results of structural dynamic time-history analysis is established.

### INTRODUCTION

The capacity-demand-diagram method is an important improvement of the traditional capacity spectra method and is considered as one of the most effective methods for analysis of structural displacement response. The traditional capacity spectra method needs additional iteration to achieve an exact result which leads the points in the demand spectra with the right ductility factor and viscous damping ratio, while the capacity-demand-diagram method can avoid the iteration and reduce the errors caused by the approximately parallel to the horizontal ordinate section of demand spectrum. But the present capacity-demand-diagram method still should be improved in two aspects.

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Firstly, the excessively simplified bi-linear capacity spectrum cannot exactly describe the nonlinear characteristics of structures under the rarely occurred earthquake. The superiority of the real capacity spectrum cannot be embodied.

Secondly, there are some indeterminate factors in the traditional method. For example, the patterns of lateral load distribution and the model of structural analysis will affect the capacity spectrum; the nonlinear parameters and their correlation will affect the demand curves. The errors caused by these factors cannot be ignored.

Some scholars failed to find the best pattern of lateral load distribution and the relations of nonlinear parameters, in order to obtain the most exact capacity spectrum or demand curves, the results show that a certain kind of capacity spectra or demand curve may be only applicable in a certain range of parameters. A general method, which can be used for arbitrary case to establish the models of capacity spectrum and demand curves cannot yet be found. To solve those problems, the significance of uncertain factors influencing the results should be analyzed first.

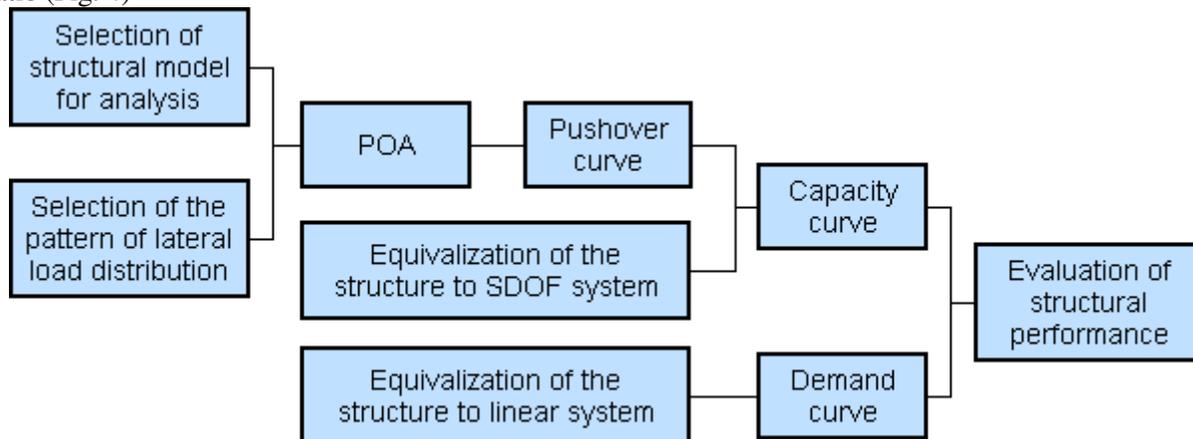
### THE TRADITIONAL METHODS OF CAPACITY-DEMAND-DIAGRAM AND THE IMPROVED METHODS

#### ATC-40 analysis procedure (Procedure B)

Different demand spectrum can be obtained according to different ductility factors if other factors are invariable. But on a demand curve, only one demand point has the same ductility factor or viscous damping ratio with the curve. These special points will be found by the capacity-demand-diagram methods; then the demand curve by connecting these special points can be plotted. At the same time, we have to consider the different discounting factors of the structural elastic stiffness or strength. The intersection between the demand curve and the capacity curve will show the largest top displacement of the analyzed structure. The main procedures of the analysis are shown in Fig.1.

Several vibration modes can be considered when the pushover curve is converted to a capacity diagram, because the fundamental mode of vibration is the most important, it is better to ignore other vibration modes for the simplification and convenience (Fig. 2). In the figure,  $V_b$  is the base shear,  $U_N$  is the top displacement,  $D$  is the factor of the fundamental mode,  $M_1^*$  is the effective modal mass for the fundamental vibration mode, and  $\Phi_{N1}$  is the peak top displacement of the fundamental vibration mode.

In ATC-40 analysis procedure, the nonlinear systems must be converted to equivalent linear system before the demand curves are plotted. The natural vibration period and the viscous damping ratio of the equivalent linear system are defined from the methods of secant stiffness (Fig. 3) and equivalent damping ratio (Fig. 4)



**Fig. 1** The procedure chart of the analysis by the method of capacity-demand curve

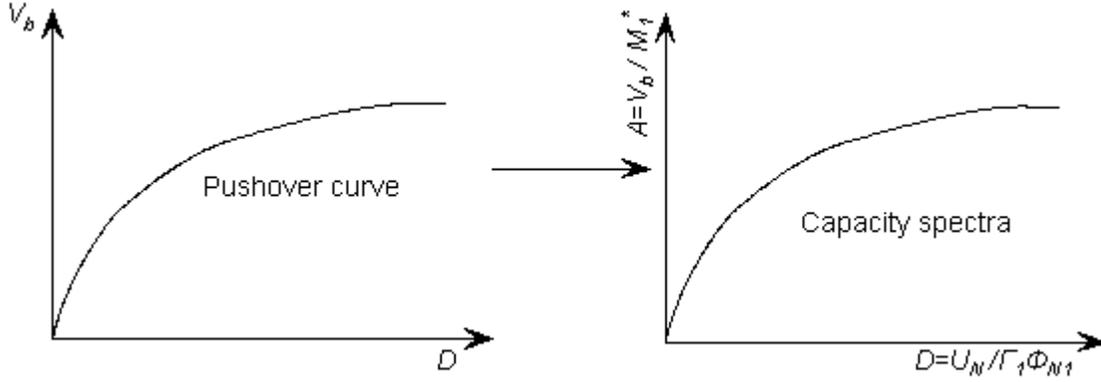


Fig. 2 Conversion of the pushover curve to capacity spectra

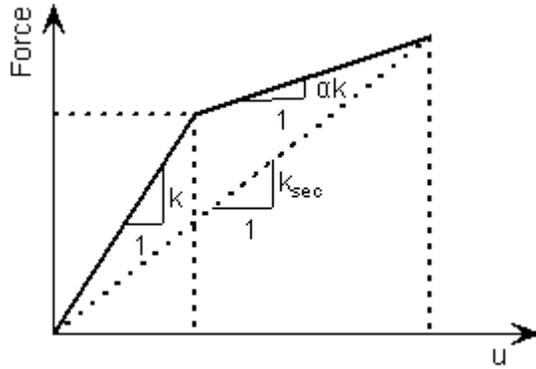


Fig. 3 The methods of secant stiffness

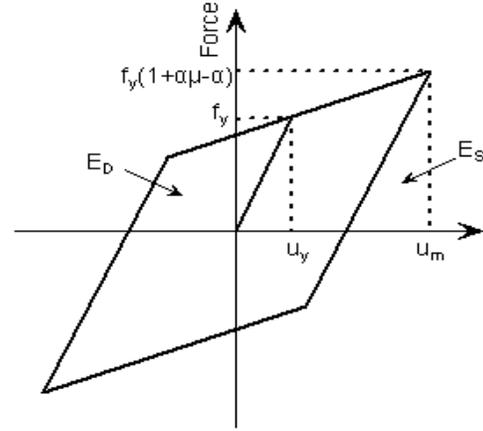


Fig. 4 The methods of equivalent damping ratio

$$T_{eq} = T_n \sqrt{\frac{\mu}{1 + \alpha\mu - \alpha}} \quad (1)$$

$$\hat{\zeta}_{eq} = \zeta + \kappa \zeta_{eq} \quad (2)$$

$$\zeta_{eq} = \frac{2(\mu - 1)(1 - \alpha)}{\pi(1 + \alpha\mu - \alpha)} \quad (3)$$

Where  $T_{eq}$  is the natural vibration period of the equivalent linear system,  $T_n$  is the natural vibration period of the system vibrating within its linearly elastic range,  $\mu$  is the ductility factor,  $\alpha$  is the strain-hardening ratio,  $\hat{\zeta}_{eq}$  is the equivalent viscous damping ratio of the nonlinear system,  $\zeta$  is the viscous damping ratio of the bilinear system vibrating within its linearly elastic range, and  $\zeta_{eq}$  is the additional equivalent viscous damping ratio of the nonlinear structure.  $\kappa$  is the damping modification factor[2].

### Main analysis procedure of the method

The main analysis procedures of the method of capacity-demand-diagram are,

- (1) To determine the strain-hardening ratio  $\alpha$ ;
- (2) To add the same increments on  $\mu$  iteratively;
- (3) To calculate the corresponding  $T_{eq}(\mu, \alpha)$  and  $\hat{\zeta}_{eq}(\mu, \alpha)$ ;
- (4) To calculate the corresponding  $A(T_{eq}, \hat{\zeta}_{eq})$  and  $D(T_{eq}, \hat{\zeta}_{eq})$ ; plot the demand points in the A-D format;
- (5) To connect these points, and then the demand curve is gotten;

(6) The earthquake-induced deformation demand of the analyzed structure is given by the D-value at the intersection between the demand curve and the capacity curve;

(7) To check and appraise the structural seismic performance of the buildings.

It is concluded that the precision of the capacity-demand-diagram method is affected by several factors, such as the original capacity curve, the simplification of the capacity curve and the methods of making equivalent linear structure system.

### **Several problems about the procedures**

The precision of capacity spectrum and demand curves affects the results of the capacity-demand-diagram method; the capacity spectrum is determined by the analysis of pushover and the SDOF modal of the structure system, the demand curves is determined by the equivalent linear system of the nonlinear structure system. Then, there are still some problems in practice discussed above.

(1) The modal for structural analysis

Different simplified skeleton curves of the element give different conclusion of the structure system analyses. The trilinear force-deformation element modal with the grade of strength degeneration can help us build more exact structural system modal of analysis than using other element modal. But the method of building equivalent linear system with the trilinear force-deformation element modal has not been found. So bilinear force-deformation element modal without the grade of strength degeneration is used in building demand curve, but the model cannot reflect the actual working mechanism of the structure systems.

(2) The pattern of lateral loads

The pattern of lateral loads is very important for the analyses of nonlinear static for the structures. The equivalent lateral load pattern can be classified as two categories. The first is invariableness pattern, which keeps an invariable figure when the lateral load is increasing, such as the uniform pattern, the triangular pattern. The second is compatible pattern. This kind of equivalent lateral load pattern can be adjusted with the structure stepping into different nonlinear grade. It seems that the second one is more reasonable. But no more results of institution about them show that. On the contrary, selecting two invariable lateral load patterns may help us get the field of the results from different lateral load patterns (Fig.5).

(3) The problems about crossing angles

The crossing angles between the demand curves and the capacity spectrum are different with the different characters of structures. If the analyzed structures have stronger ductility than other structures, their demand curve may be cross with the capacity spectrum by more obtuse angles. Then, the different intersection points between the one demand curve with the different capacity spectrum will have larger X-coordinate distance than in other conditions. On the other hand, if the demand curve is steeper when it crosses with the capacity spectrum, the intersection points' X-coordinate distance will be smaller than that in other conditions (Fig.6).

(4) The method of building equivalent linear structural system modal

The different methods of building equivalent linear structural system modal affect the results of the analyses. Based on the relationship between the strength degrading factor  $R$  and the ductility factor  $\mu$ , Newmark, Hall, Miranda and Bertero gave their own method of building equivalent linear structural system modal. But these different methods have their own practical conditions and the results of analysis may be not enough exact if proper method cannot be applied.

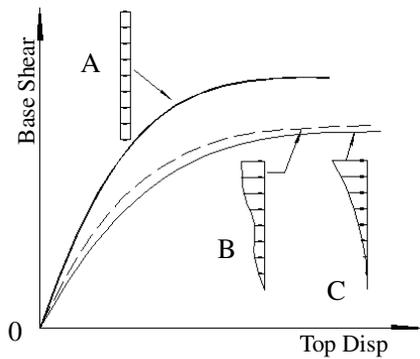
## **IMPROVED CAPACITY-DEMAND-DIAGRAM METHOD**

For the deficiencies discussed above, some important improvements of capacity-demand-diagram method are made in the paper.

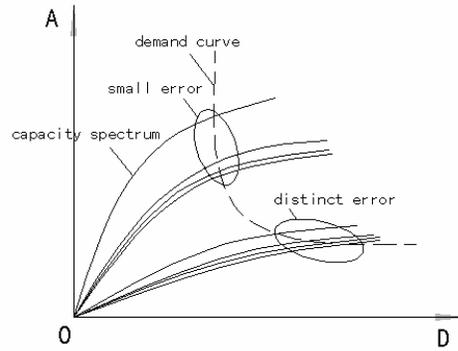
(1) Replace the simplified bi-linear capacity spectrum by the original pushover curve resulting from the pushover analyses of the structure; then convert the pushover curve to capacity spectra.

(2) Consider the uncertainty of the lateral load distribution pattern by double lateral load patterns. Some analyses indicate that: though there are many factors affecting the original pushover curve, the pattern of the lateral load distribution is the most significant factor. Different lateral load distribution patterns result in different pushover curves in the pushover analysis to a certain structure. Compared with other lateral load patterns (i.e. the converse triangle pattern and adaptive pattern), the lateral resultant force corresponding to the uniform lateral load distribution pattern locates at the lowest position on the structure and that of the parabolic lateral load distribution pattern locates at the highest position on the structure. It is easy to understand that the uniform pattern and the parabolic pattern will compose the envelope of the pushover curves cluster. So it is suggested that the two special lateral load distribution patterns be both used for a certain structural pushover analysis.

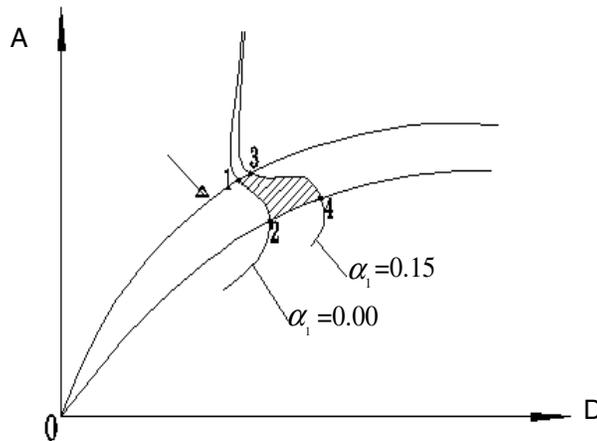
(3) Consider the effect coming from uncertainty of the strain-hardening factor  $\alpha$  by double envelope demand curves. Different simplification methods will result to different values of the strain-hardening factor  $\alpha$  if the degradation range of the pushover curve is ignored. Some researches [4] about the reasonable values of  $\alpha$  show that the excessively large value of  $\alpha$  will lead that the tilted up high ductility range of the demand curve cannot intersect with the capacity spectra. Based on the researching of recent years, the value of  $\alpha$  should be controlled in the range as 0.00~0.15. Then the demand curves cluster has a reasonable demand envelop.



**Fig. 5 The different lateral load patterns**



**Fig. 6 The different crossing angles**

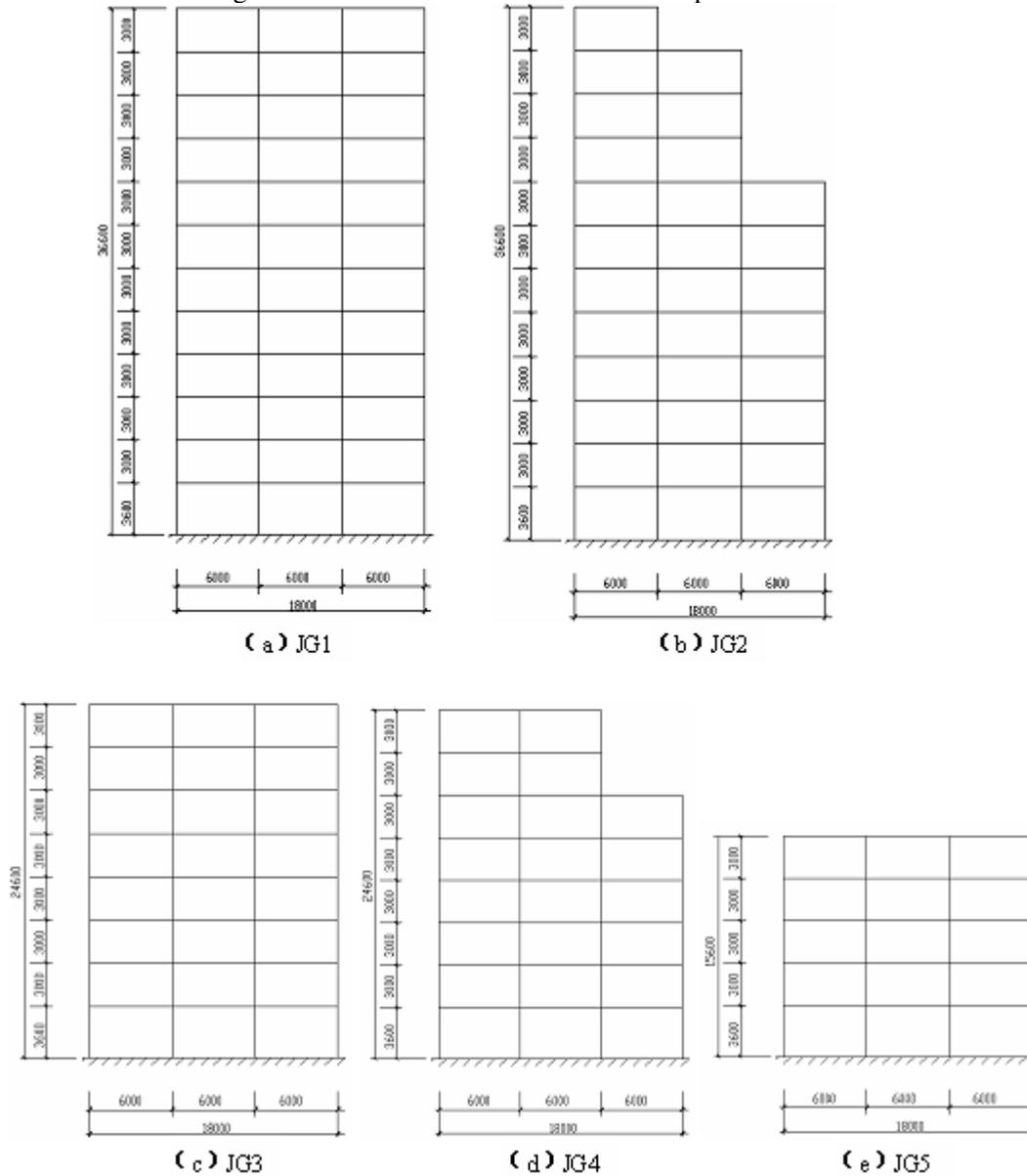


**Fig. 7 The discrepancy between different pattern of lateral load distribution and different values of  $\alpha$**   
 After the improvements are made, the envelopes composed of two capacity spectra and demand curves close a zone on the A-D format. So the uncertainty of selecting demand curve or capacity spectra can be avoided and the errors may be reduced with this method. But it is unknown that which kind of statistic value should be drawn from values in the zone as the result of analysis. Comparing the analysis results between the methods of capacity-demand-diagram and time history analysis may solve the problem.

## ANALYSIS OF PRACTICAL MODELS

### Typical building considered in evaluation studies

A set of concrete frame structures is designed according to fortification intensity 7 (Fig.8). The parameters of the set of structures' design are showed in Table 1. The character period of the site is 0.30s.



**Fig. 8 Sketch of the structures analyzed**

The selected seismic input includes three strong ground motions records. They are the N-S component of the El Centro station field motion recorded during the 1940 Imperial Valley earthquake, the N-S component of the Saticoy-St station field motion recorded during the 1994 Northridge earthquake, the E-W component of the Qian-An station field motion recorded during the 1976 Tang Shan earthquake. At last 20 artificial strong ground motions are fitted into standard response spectra. The modal of Takeda is selected as the hysteretic model of the elements belonged to the structures analyzed. But the degeneration of stiffness caused by crack is ignored for the consistence between the improved methods of capacity-demand-diagram and the time history analysis.

**Table 1 The data of the frame concrete structures analyzed**

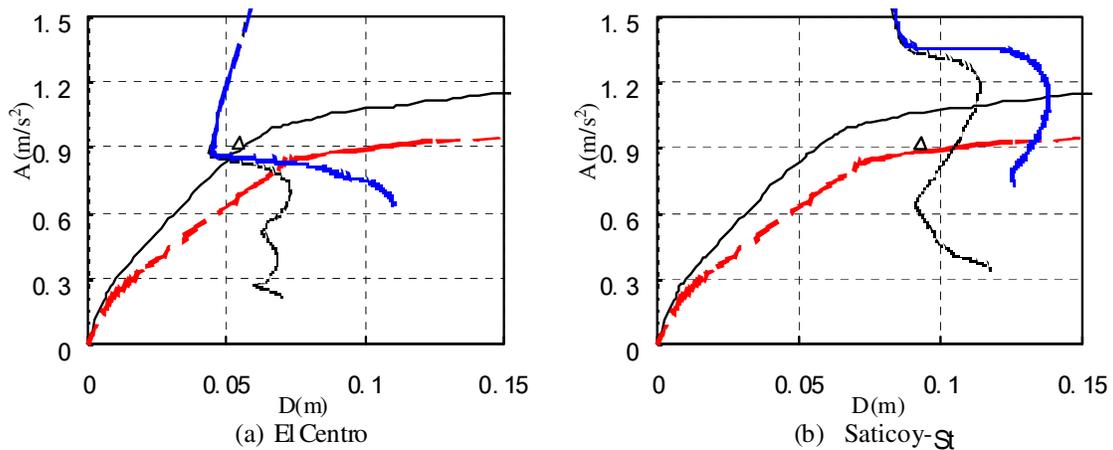
No.	Total stories	Total height (m)	Dim. of column (mm)	Stories	Dim. of beam (mm)	Grade of concrete	Dead load on beams (kN/m)	Live load on beams (kN/m)	Natural vibration of period(s)	The character period (s)
JG1	12	36.6	750×750 650×650 550×550	1~4 5~8 9~12	250×550	Beam C30 Column C40	33.0	9.0	1.472	0.3
JG2	12	36.6	750×750 650×650 550×550	1~4 5~8 9~12	250×550	Beam C30 Column C40	33.0	9.0	1.241	0.3
JG3	8	24.6	550×550 450×450	1~4 5~8	250×550	C30	33.0	9.0	1.196	0.3
JG4	8	24.6	550×550 450×450	1~4 5~8	250×550	C30	33.0	9.0	1.147	0.3
JG5	5	15.6	450×450	1~5	250×550	C30	33.0	9.0	0.809	0.3

By the way, in the inelastic time history analysis of the structures, it is difficult to see that the structural peak top displacement and the peak base shear appear at the same time and it is almost impossible for the buildings with unsymmetrical or irregular shape and many floors.

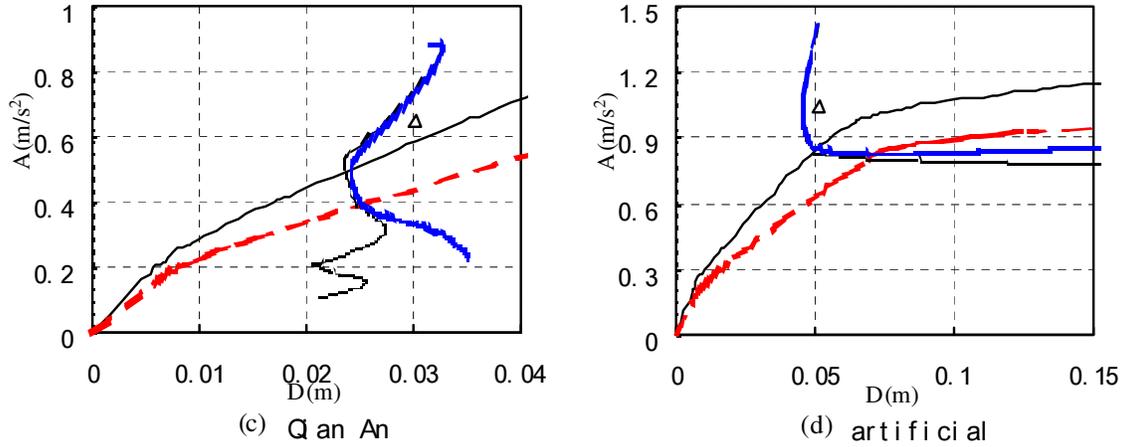
The most important use of the capacity-demand-diagram methods is estimating or appraising the seismic performance of the buildings. The two methods of the structural analysis will be compared according the peak top displacement of the same analyzed building.

### The Results of Analyses

Fig. 9 shows the results of the comparison between the two methods for the model of JG5. The average displacements corresponding to the intersections between the envelope capacity spectra and demand curves are shown in Table2, the results of inelastic time history are also shown in the table for comparisons. It is seemed that only the discrepancy of the analysis to the JG1 corresponding to the artificial ground motions input is larger than others. The other discrepancies are all less than 33% and the 10 of them are less than 20%. The precision of them is better than that of the Chopra's [2] and acceptable for practical purpose.



**Fig. 9 Comparisons between the capacity-demand-diagram methods and the THA (JG5)**



**Fig. 9 Comparisons between the capacity-demand-diagram methods and the THA (JG5)**

**Table 2 Comparisons between the capacity-demand-diagram methods and the THA (Dim.: m)**

structures	Seismic input	X1	X2	X3	X4	$\bar{X}$	$\Delta$	Discrepancy(%)
JG1	El Centro	0.0828	0.0737	0.0889	0.0854	0.0827	0.0610	26.24
	Saticoy-St	0.1100	0.0982	0.1190	0.1050	0.1081	0.1120	-3.66
	Qian An	0.0300	0.0239	0.0312	0.0251	0.0276	0.0273	-0.91
	artificial	0.0993	0.2480	0.1020	0.2970	0.1866	0.0860	53.91
JG2	El Centro	0.0682	0.0736	0.0724	0.0872	0.0754	0.0506	32.85
	Saticoy-St	0.1140	0.1030	0.1350	0.1290	0.1203	0.0800	33.47
	Qian An	0.0252	0.0275	0.0256	0.0292	0.0269	0.0237	11.81
	artificial	0.0709	0.0903	0.0726	0.0956	0.0824	0.0689	16.33
JG3	El Centro	0.0522	0.0674	0.0517	0.0727	0.0607	0.0552	9.02
	Saticoy-St	0.1120	0.1040	0.1380	0.1330	0.1218	0.0925	24.02
	Qian An	0.0236	0.025	0.0241	0.0252	0.0245	0.0302	-23.39
	artificial	0.0498	0.070	0.0512	0.0711	0.0605	0.0520	14.05
JG4	El Centro	0.0522	0.0650	0.0532	0.0694	0.0599	0.0523	12.72
	Saticoy-St	0.1100	0.1060	0.1490	0.1480	0.1283	0.0966	24.68
	Qian An	0.0235	0.0255	0.0240	0.0258	0.0247	0.0246	0.40
	artificial	0.0502	0.0654	0.0514	0.0685	0.0589	0.0527	10.49
JG5	El Centro	0.0348	0.0340	0.0371	0.0367	0.0357	0.0426	-19.50
	Saticoy-St	0.0765	0.0823	0.0838	0.0904	0.0833	0.0855	-2.70
	Qian An	0.0208	0.0199	0.0216	0.0209	0.0208	0.0245	-17.79
	artificial	0.0305	0.0338	0.0313	0.0347	0.0326	0.0275	15.58

## CONCLUSIONS

The capacity-demand-diagram method is improved in three aspects. The first one is to substitute the simplified bi-linear capacity spectrum with the actual curve obtained by POA. The second one is to use two capacity curves to consider the uncertainties of the lateral loading modes. The third one is to use two demand curves to consider the uncertainties of the nonlinear hysteretic models. The results obtained by the improved method are compared with those of the time history analysis method, and an expected value of the results is given. The improved method may reduce the uncertainties in the determination of capacity spectra and demand curves and is more suitable than the original method.

## ACKNOWLEDGEMENT

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## REFERENCES

1. Ye Lieping, Li Qi. "The capacity-demand-diagram methods for the performance/displacement based design." (in Chinese) *The Evolvement of Modern Earthquake Engineering*. Nanjing: Southeast University Press, 2003, 430~435.
2. Chopra AK, Goel RK. "Capacity-demand-diagram methods for estimating seismic deformation of inelastic structures: SDF system." PEER, Univ. of California, Berkeley, CA, 2000.
3. Bracci JM, Kunnath SK, Reinhorn AM. "Seismic performance and retrofit evaluation of reinforced concrete structures." *Journal of Structural Engineering*, 1997.
4. Yang Cheng. "Improved capacity-demand-diagram methods for estimating seismic deformation of inelastic structures." (in Chinese) Thesis for master degree, Chongqing University, Chongqing, China. 2003.
5. GB50010-2002. "Code for design of concrete structures." (in Chinese) Beijing: China Architecture & Building Press, 2002.
6. GB50010-2001. "Code for Seismic Design of Building." (in Chinese) Beijing: China Architecture & Building Press, 2001.