13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 1475

EFFECTS OF COUPLED VIBRATIONS ON THE DYNAMIC PROPERTIES OF SANDS

J.X. ZHANG[1] and M. Sherif AGGOUR[2]

SUMMARY

Experiments were conducted on Ottawa sand using three types of loading: sinusoidal, random and impulse. Soil specimens were subjected to longitudinal and torsional excitation both separately and in combination using one of the three types of loading. Each soil specimen was tested under three confining pressures. In the sinusoidal loading tests, the excitation signals were generated by a variable frequency sine-wave oscillator. In the random loading tests, the input signals were generated by a white-noise generator and filtered by a two-channel variable cut-off frequency filter. A pulse signal generator was used in the impulse loading tests. The input and output signals were analyzed by a FFT analyzer in the random and impulse loading tests. The strains were evaluated using the random vibration theory under random and impulse loading. From the measured resonant frequencies and responses, the shear modulus and damping of the sand specimens subjected to coupled vibrations under these different types of loading at different shear strain levels were obtained.

It was found that under each type of loading the normalized shear modulus with the initial maximum shear modulus of each test could be unified by using shear strain normalized by the octahedral shear strain. It was also found that the shear modulus and damping were significantly affected by the application of combined longitudinal and torsional excitation. In combined excitation, the modulus was reduced and the damping was higher than for the case of uncoupled excitation.

INTRODUCTION

Shear modulus and damping ratio of soils are considered to be the primary parameters of dynamic properties of soils. These dynamic properties can be evaluated from either laboratory or field testing. Research has been conducted over the past several decades to study the dynamic properties of soils under various conditions. Most of the research, especially research with resonant column techniques, was limited to the dynamic properties of soils under sinusoidal loadings. The soil specimens were normally excited in the torsional direction, and the dynamic properties of the soils under various conditions were obtained.

[1] Geotechnical Engineer, Tidewater Skanska Inc., Virginia Beach, VA, USA
[2] Professor, Civil and Environmental Engineering Department, University of Maryland, College Park, MD, USA, msaggour@eng.umd.edu

Earthquakes, wind, ocean waves, and certain man-made forces, however, do not provide a sinusoidal pattern of sinusoidal excitation. To establish meaningful results that represent field conditions, a nonperiodic loading should be used in laboratory testing. It was determined from available research using random and impulse loadings Al-Sanad [1], Amini [2], and Tawfiq [3] that loading type indeed has some effect on the dynamic properties of soils. On the other hand, due to the complexities of field conditions, such as topographies, soil stratums and soils anisotropies, and vibration sources, wave motions at any point on the ground surface or under the ground would be very complicated combinations of different types of wave forms, such as compressive waves, shear waves, etc., with different magnitudes of displacement, direction of incident, and phase of transmission. Therefore, different types of wave forms are always coupled in the field. However, the research on the dynamic properties of soils under coupled vibrations are very limited Griffin [4] and Tawfiq [3].

The purpose of this research is to study the effect of coupled vibrations of compressive waves and shear waves on the dynamic properties with a resonant column device. This research also includes the study of loading type effect. A series of tests were conducted with air-dry Ottawa 20-30 sand specimens. Sinusoidal, random and impulse excitations were employed. Shear modulus and damping ratio of the specimens at each condition under each type of loadings were determined.

TEST APPARATUS AND TEST PROCEDURES

The resonant column device used in this research was a Drnevich "fixed-free" type with solid cylindrical specimens. The specimens were fixed at the base with excitation forces applied to the top. The dimensions of solid cylindrical specimens were 7.5 cm in length and 3.6 cm in diameter. Air-dry Ottawa 20-30 sand was used in the specimen preparation, and all specimens were prepared in four layers to a relative density of approximately 78% using a dry tapping method. Then a vacuum pressure of around 15 psi was connected to the specimens for the assembly of the top platen, LVDT and pressure chamber. After the assembly, a confining pressure of about 3 psi was applied to the specimens and the vacuum pressure was disconnected. Then the confining pressure was increased to the pressure at which the specimen would be tested. To ensure the vacuum pressure completely released, a confinement period of around 30 minutes was maintained before the predetermined test sequence was started.

In this research, each specimen was excited in both longitudinal and torsional directions simultaneously, under either sinusoidal, random or impulse loadings, and its responses in both directions were measured. Two independent systems of signal input and response output were used, one for the longitudinal direction and the other one for the torsional direction, as illustrated in Figure 1. Each signal input system consisted of a signal generator and a power amplifier, and each response output system consisted of an acceleration transducer and a charge amplifier. For each direction, the excitation signals generated by the signal generator were amplified by a power amplifier and then sent to the exciting coil to vibrate soil specimens. The vibration responses of the specimens were picked up by a transducer and then sent to a charger amplifier. In the tests, the excitation and response signals for both directions of the specimens were measured.

Sinusoidal excitation is most commonly used in soil dynamics with the resonant column technique. In this research, two variable frequency sine-wave generators were used. The excitation and response signals of each direction were connected to an X-Y oscilloscope for monitoring the resonance condition of each direction. By adjusting the frequency of excitation, a resonance condition was established when a vertical ellipse was observed on the X-Y oscilloscope. The frequency of the excitation and vibration response of the direction of the specimen are called resonant frequency and resonant response of the specimen in the direction. The amplitudes of the excitation and resonant response of each direction were measured with a voltmeter. The resonant frequency was measured with a digital frequency meter.

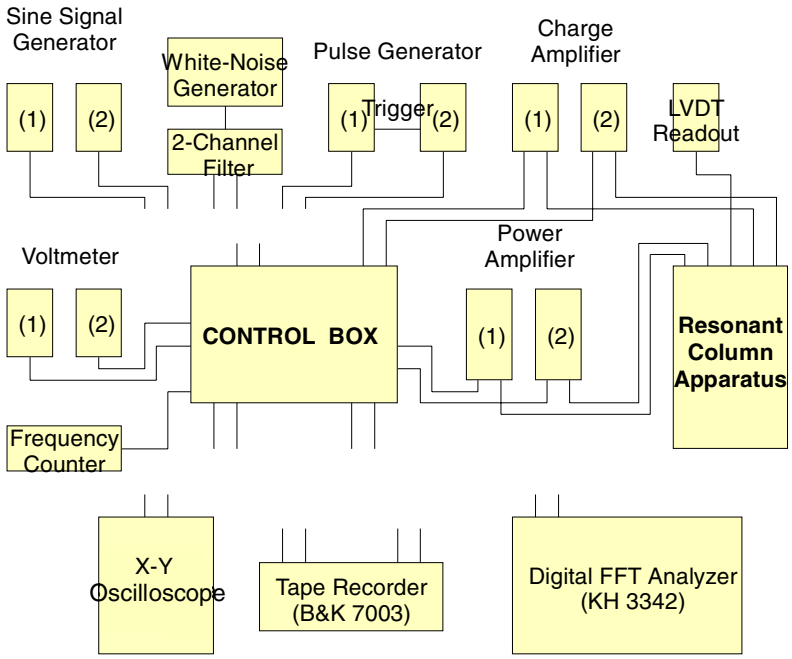

FIG. 1. A Schematic Diagram of the Resonant Column Testing System

The random excitation signals used in the research were generated by a white-noise generator and filtered through a 2-channel variable cut-off frequency filter according to the required frequency contents. The filtered random signals were then sent to the corresponding power amplifiers, one for the longitudinal direction and the other one for the torsional direction. The random excitation signals and random response signals were recorded with a B&K 4-channel FM analog tape recorder. The recorded signals were then replayed and sent to a Rockland digital signal analyzer (Model 5830B) for various spectrum analyses.

In the tests with impulse excitations, two pulse signal generators were used, one for the longitudinal direction, and the other one for the torsional direction. The two pulse generators were set at different pulse widths in order to provide different frequency ranges for different directions because the resonant frequencies of the specimens in the longitudinal and torsional directions are different. The width of a pulse signal produced by the generators was adjusted by changing the pulse frequency on the generators. A higher pulse frequency produced a pulse signal of a narrower width and a higher frequency range. A pulse frequency of 1000 Hz was used for the longitudinal direction, and pulse frequencies of 30, 40 and 60 Hz were used for the torsional direction at confining pressures of 5, 10 and 40 psi, respectively. In order to synchronize these two pulse signal generators for the tests of coupled motions, the pulse signal of the generator for the longitudinal direction was used as the trigger signal of the pulse generator for the torsional direction. The time between two pulse signals was set at one second. The pulse signals from each generator were amplified by the corresponding power amplifier. The impulse excitation signal and the impulse response signal of the specimens in each direction were sent to the FFT analyzer and analyzed for the spectrum and transfer functions.

In the research, many soil specimens were tested and each specimen was tested under one of the confining pressures of 5, 10 and 40 psi and one of the three types of loadings. On each specimen, the tests consisted of the measurements of amplitudes of excitation inputs and response outputs, and resonant frequencies of the longitudinal and torsional directions under coupled vibrations, and the measurements of amplitudes of

excitation input and response output, and resonant frequencies of the two directions under low amplitude excitations before and after the coupled vibrations. As illustrated in Figure 2, each specimen was first confined under a specified confining pressure for about 20 minutes, and then the initial low-amplitude Young's and shear moduli were measured. At a confinement period of 30 minutes, the first stage of coupled vibration testing on the specimen was started. On each specimen, the amplitude of excitation in the longitudinal direction was maintained at a constant, and the excitation amplitude in the torsional direction, on the other hand, varied from low to high from stage to stage. At each stage of coupled vibration testing, the specimen was excited for a duration of 2 minutes (T_V), and the Young's modulus, shear modulus, damping ratios, axial and shear strain amplitudes of the specimen under the coupled vibrations were measured. After each stage of coupled vibrations, the specimen rested for a period of 8 minutes (T_S). During this period, the residual axial deformation of the specimen was measured with an LVDT, and the low-amplitude Young's and shear moduli of the specimen were measured to evaluate the effect of the coupled vibrations of the previous testing stage. The test sequence on each specimen was completed when the testing stage at the highest excitation level in the torsional direction was finished. For another excitation level in the longitudinal direction, another specimen was constructed and the same test procedures were followed.

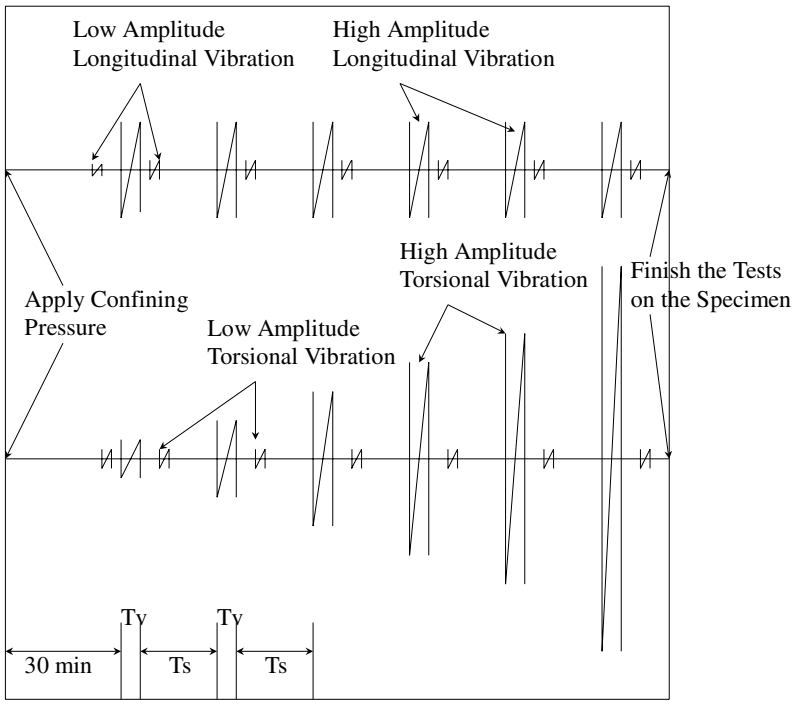

FIG. 2. Test Sequence of Coupled Vibration Tests

PARAMETER DETERMINATIONS

In coupled vibration tests with sinusoidal excitations, the resonant frequencies of the longitudinal and torsional directions of specimens were directly obtained from a digital frequency meter. The sinusoidal excitation and response amplitudes were measured with digital voltmeters in root-mean-square (rms). In coupled vibration tests with random and impulse excitations, the resonant frequencies of the longitudinal and torsional directions were obtained from the transfer functions. The excitation and response amplitudes were measured on the excitation power spectrum and response power spectrum functions by

taking the average values around the resonant frequencies. The values of the transfer functions at resonant frequencies were directly taken from the transfer functions.

With the values of resonant frequencies, excitation amplitudes, response amplitudes and transfer functions at the resonant frequencies of the longitudinal and torsional directions of specimens under coupled vibrations of each type of loadings, Young's modulus, longitudinal damping ratio, and shear modulus and torsional damping ratio of soil specimens can be determined. In the paper, only the results of shear modulus and torsional damping ratio of sand specimens under coupled vibrations of sinusoidal, random and impulse loadings will be presented.

Wave propagation theory was used to develop the equation of shear modulus of soil specimens. The shear modulus equation was obtained as:

$$G = \rho(2\pi L)^2 (f_n / F_T)^2 \qquad (\mathrm{kN/m^2}) \qquad (1)$$

In which ρ is the density of the specimen in gram/cm^3; L is the length of the specimen in cm; f_n is the resonant frequency of the torsional direction in Hz; and F_T is a non-dimensionless frequency factor and is defined as:

$$F_T = \frac{1}{\sqrt{0.405 + \frac{J_T}{J_S}}}$$

and J_S and J_T are the polar mass moment of soil specimen and the top platen, respectively.

There are many methods available for the damping ratio determination from resonant column testing. Through this research it was found that the damping calibration factor method is the most effective method for all three types of loadings. Damping ratio of soil specimens is determined using the damping calibration factor method from the following equation:

$$D = \frac{1}{2} \frac{DCF}{\sqrt{H_n^2 - 2(DCF)^2}} \qquad (2)$$

in which, H_n is the transfer function value at resonant frequency f_n; and DCF is the Damping Calibration Factor, and is defined as the transfer function value at a frequency of $f_n/\sqrt{2}$, or half of the transfer function value at a frequency of $\sqrt{2}f_n$. DCF is a calibration factor of the testing system and is relatively constant. It slightly decreases with the increase of mean effective principal stress σ_o in specimens. For the testing system used in this research, the correlation of DCF of the torsional direction was found by regression as follows:

$$DCF = 0.045 - 0.00015 \bullet \sigma_o \qquad (3)$$

According to Drnevich [5] the average shear strain in a solid cylindrical specimen approximately equals the shear strain at 2/3 of the radius from the center as follows:

$$\gamma = \frac{d}{3L} \bullet RCF \bullet y_{rms} \qquad \text{(rms)} \qquad (4)$$

in which, d and L are the diameter and length of specimens in cm; RCF is the rotational motion transducer calibration factor in rad/V; and y_{rms} is the output of the torsional transducer in rms.

In the tests with sinusoidal loading, the y_{rms} values were read directly from the voltmeter, and then average shear strain amplitudes in rms were determined. In the tests with random or impulse loadings, the strain amplitude induced in a specimen was not a constant. Based on random vibration theory, the following equation was derived for the determination of the rms values of the output of transducers:

$$y_{rms} = \frac{G_{Fn}}{J \bullet \sqrt{(4\pi f_n)^3 D}} \qquad (5)$$

in which, G_{Fn} is the average value of excitation spectrum function around the resonant frequency; J is the total polar mass moment of inertia of the system including the soil specimen and top platen in kg-cm^2; and D is the damping ratio in the torsional direction.

In a solid cylindrical soil specimen under coupled vibrations, based on the elasticity theory the average octahedral shear strain γ_{oct} induced in the specimen can be determined from the following equation:

$$\gamma_{oct} = \frac{\sqrt{2}}{3} \sqrt{4(1+\nu)^2 \varepsilon^2 + 3\gamma^2} \qquad \text{(rms)} \qquad (6)$$

in which, ν is the Poisson's ratio of the soil specimen; ε and γ are the average axial and shear strain amplitudes in rms induced in the soil specimen by longitudinal and torsional vibrations, respectively.

More detailed equation derivations and discussions regarding the determinations of damping ratio and strain amplitude are given in reference [6].

RESULTS AND DISCUSSION

As described in the section of test apparatus and procedures of this paper, each sand specimen was vibrated in both longitudinal and torsional directions simultaneously under one of the three types of loadings. The longitudinal excitation level was maintained at a constant and the torsional excitation level varied from low to high. The shear modulus and damping ratio of the specimens at each combination of axial and shear strains were determined from the measured excitation amplitudes, resonant frequencies and responses according to the procedures and equations described in the previous section. The shear modulus and damping ratio from coupled sinusoidal vibrations are the only ones presented in this paper.

Figure 3 presents the shear modulus of sand specimens under coupled sinusoidal vibrations at different shear strain amplitudes and different confining pressures. Different lines represent the test results of different specimens on which different constant longitudinal excitation levels were applied and torsional excitation levels were varied from low to high. It can be seen from the figure that shear modulus increased with the increase of confining pressure and decreased with the increase of shear strain

amplitude, as generally concluded in the published results in the past. Considering the effect of prestraining, due to which the properties of soil specimens are changed when the specimens are subjected to vibrations of amplitudes higher than a certain level, the maximum shear modulus of the specimens at each high amplitude vibration stage should be used to normalize the shear modulus obtained from the corresponding stage, as G/G_{max}, in order to compare the test results under different conditions. As illustrated in Figure 2, low-amplitude moduli of the specimens were measured after each stage of high amplitude vibrations.

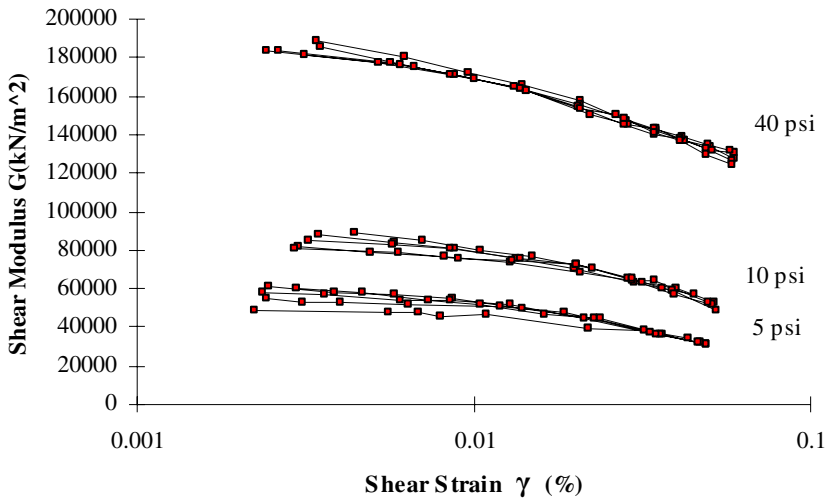

FIG. 3. Shear Modulus G vs. Shear Strain γ(%) (rms) from Coupled Sinusoidal Vibration Tests at different Confining Pressures

By using the corresponding maximum shear modulus obtained from the low-amplitude shear modulus Zhang [7], the normalized shear modulus, G/G_{max}, are compared in Figure 4. In an uncoupled study, it is expected that the shear modulus of soil specimens at low shear strains is close to the maximum shear modulus of the specimens and the normalized shear modulus will be one in the low shear strain range. In Figure 4, however, some values of G/G_{max} as low as 0.65 were obtained in the low shear strain range. This implies that the shear modulus of the specimens were affected by some other factors besides shear strain and confining pressure. On the other hand, the normalized shear modulus of soil specimens subjected to vibration in the uncoupled torsional direction could be unified by using the normalized shear strain as γ/γ_r, where γ_r is called a reference shear strain and for cohesionless soils is defined as:

$$\gamma_r = \frac{\sigma_o \sin \phi}{G_{max}} \tag{7}$$

in which σ_o and ϕ are the mean principal stress in a soil specimen and the internal friction angle of the soil at the tested conditions, respectively. In Figure 5, however, the normalized shear modulus, G/G_{max}, of the specimens under coupled sinusoidal vibrations could not be unified with the normalized shear strain, γ/γ_r. Larger scatters of G/G_{max} exist in the range of lower γ/γ_r.

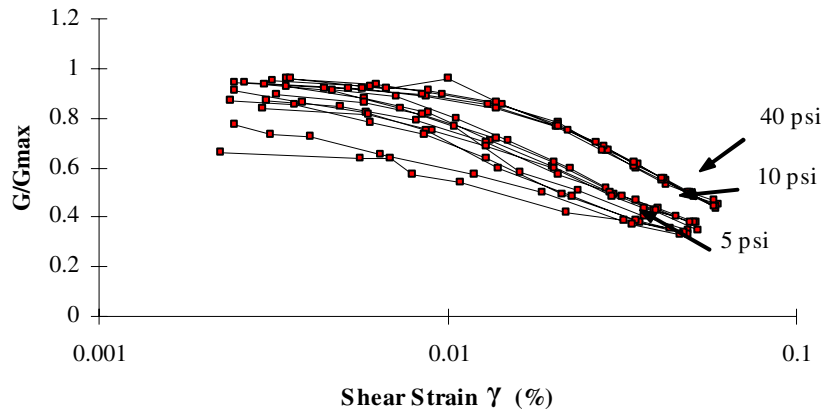

FIG. 4. Normalized Shear Modulus G/G_{max} vs. Shear Strain γ(%) (Coupled Sinusoidal Vibrations)

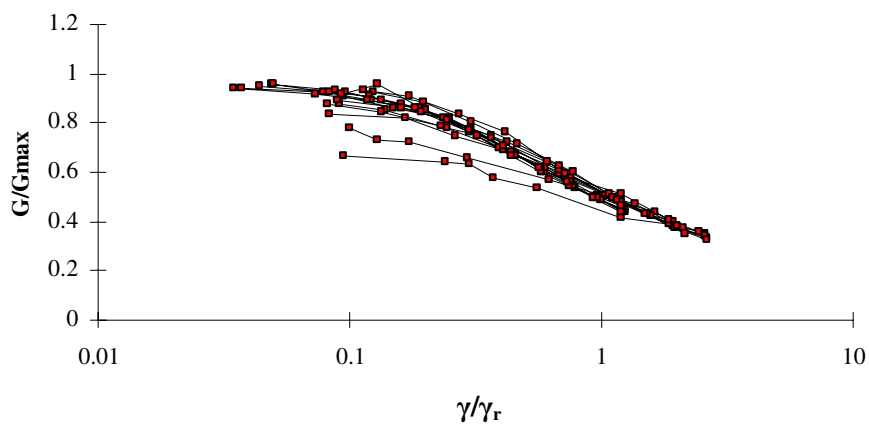

FIG. 5. G/G_{max} vs. γ/γ_r (Coupled Sinusoidal Vibrations)

This indicates that the shear modulus of soil specimens under coupled vibrations was affected not only by the shear strains but, also by the axial strains induced in the specimens. Therefore, axial strain should also be included for the unification of shear modulus of soil specimens under coupled vibrations. From this research, it was found that the octahedral shear strain, γ_{oct}, which includes shear strain and axial strain as given in equation (6), is the strain to be used to unify the shear modulus of soil specimens under coupled vibrations. In Figure 6, G/G_{max} of specimens under coupled sinusoidal vibrations and under different confining pressures are unified under the normalized octahedral shear strain, γ_{oct}/γ_r.

The unified results in Figure 6 imply that the normalized octahedral shear strain, γ_{oct}/γ_r, is the controlling factor of shear modulus of soils. With γ_{oct}/γ_r, the effects of axial and shear strains, and confining pressure are included, and G/G_{max} of soil specimens under coupled vibrations in the longitudinal and

torsional directions and under various confining pressures could be unified. The unified G/G_{max} monotonically decreases with the increase of γ_{oct}/γ_r.

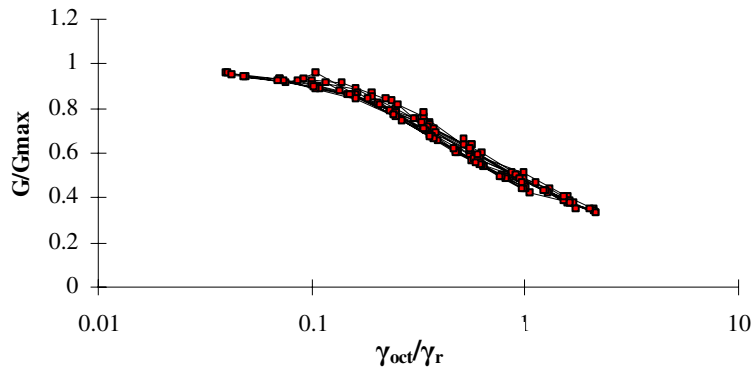

FIG. 6. G/G_{max} vs. γ_{oct}/γ_r (Coupled Sinusoidal Vibrations)

Shear damping ratio of the sand specimens under coupled sinusoidal vibrations increased with the increase of shear strain amplitude, as shown in Figure 7, and decreased with the increase of confining pressure. Similarly, the axial strain induced in the specimens under the coupled vibrations also had some effects on the damping ratio. These effects can be understood from the unification of the damping ratio of the specimens under the normalized octahedral shear strain, γ_{oct}/γ_r, induced in the specimens, as shown in Figure 8.

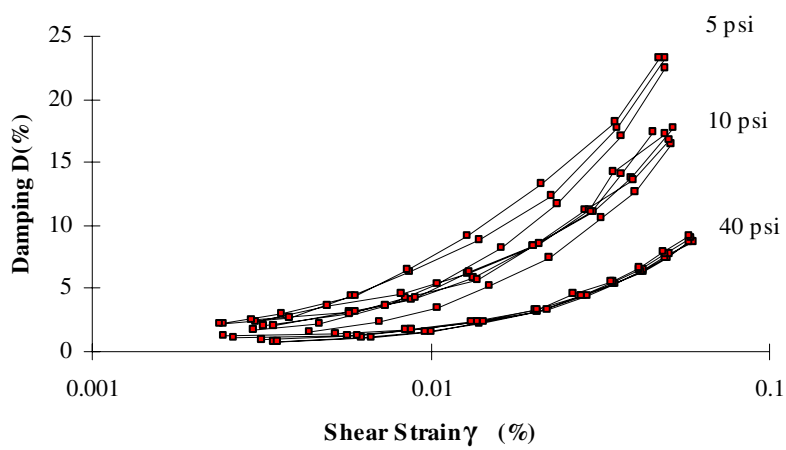

FIG. 7. Shear Damping Ratio D(%) vs. Shear Strain γ(%) at Different Confining Pressures (Coupled Sinusoidal Vibrations)

CONCLUSION

A series of tests were performed on solid cylindrical specimens of air-dry Ottawa 20-30 sand with a Drnevich's "fixed-free" resonant column device to study the effect of coupled motions of longitudinal and torsional directions on the dynamic properties of sands. Three types of excitations: sinusoidal, random and impulse, were used in this research. At each testing stage a sand specimen was vibrated simultaneously in the longitudinal and torsional directions under one of the three types of excitations, and

the excitation amplitudes, resonant frequencies and responses of both directions were measured. The shear modulus and damping ratio of the specimens were then determined. The effect of coupled vibrations from each type of the excitations was then determined.

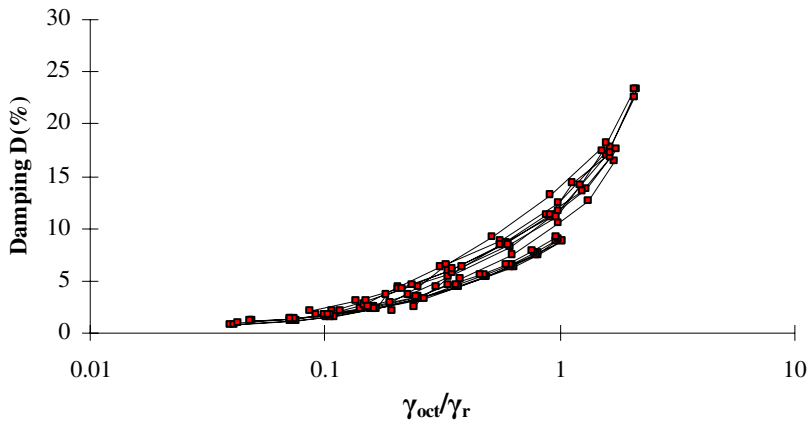

FIG. 8. Damping Ratio D(%) vs. γ_{oct}/γ_r (Coupled Sinusoidal Vibrations)

For uncoupled vibration the reference shear strain γ_r, is a very useful parameter for the unification of shear modulus and damping ratio of soils under various confining pressures, strain levels and different prestraining conditions.

Shear modulus and damping ratio of soils are indeed affected by the coupling of vibrations of longitudinal and torsional directions. Axial strain induces lower shear modulus and higher damping ratio. It was found that the octahedral shear strain γ_{oct}, which contains the axial strain and shear strain, controls the behavior of soils subjected to coupled vibrations. Under various coupled vibrations of axial strain and shear strain, the normalized shear modulus and damping ratio of soils can be unified by using the normalized octahedral shear strain, γ_{oct}/γ_r. The unification of the shear modulus and damping ratio with the normalized octahedral shear strain was also applicable for the other types of loading, the random and impulse loading.

REFERENCES

1. Al-Sanad, H., Aggour, M.S., and Yang, J.C.S. "Dynamic shear modulus and damping ratio from random loading tests." *Geotech. Testing Journal*, ASTM, 1983:6(2):120-127.
2. Amini, F. "Dynamic behavior under random excitation conditions." Ph. D. Dissertation, Univ. of Maryland, 1986.
3. Tawfiq, K.S. "Effect of time and anisotropy on dynamic properties of cohesive soils." Ph.D. Dissertation, Univ. of Maryland, 1986.
4. Griffin, P.M., and Houston, W.N. "Interaction effects of simultaneous torsional and compressional cyclic loading of sand." Report No. UCB/EERC-79/34, Univ. of California, Berkeley, 1979.
5. Drnevich, V.P., Hardin, B.O., and Shippy, D.J. "Modulus and damping of soils by the resonant-column method." *Dynamic Geotech. Testing*, ASTM STP 654, 91-125, 1978.
6. Zhang, X.J. "Effect of coupled motions on the dynamic properties of sands." Ph.D. Dissertation, Univ. of Maryland, 1994.

7. Zhang, X.J., Aggour, M.S. "Maximum moduli of sands under various loadings." 3rd International Conf. on Recent Advances in Geotech. Earthquake Engg. and Soil Dynamics, St. Louis, Missouri, 1995.