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SIMULATION OF GROUND MOTIONS USING TIME VARYING VECTOR ARMA MODEL

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SUMMARY

This paper aims at simulating a set of earthquake ground motions with horizontal and vertical components which are temporally and spatially correlated. Present simulation methods mainly deal with the problem of simulating ground motions with single (horizontal or vertical) component and cannot take the effect of spatial correlation into account. The method based on time varying vector ARMA model is proposed for this purpose which can simulate a set of ground motions with considering the temporal and spatial correlation both in time and frequency domain. By recasting the temporally and spatially correlated ground motions into state-space equations using time varying vector ARMA model and applying Kalman filter to estimate the time vary parameters of the ARMA model, a sample of large population with the same statistic characteristics is generated by changing the seeds of random numbers. The comparison between the simulated ground motions and the actual ones shows the method proposed in this paper is satisfactory and feasible.

INTRODUCTION

For complex civil engineering structures (e.g. high-rise or severely torsional irregular buildings) seismic design, dynamic time history analysis, as a necessary enhancement for other analysis methods, is recommended with considering two or three dimensional seismic action. In order to provide more reasonable inputs for structural time history analysis, artificial ground motions are usually adopted as an important input form, especially in case the actual records of ground motions are scarce or they are not suitable for the sites where the structures will be erected. Though a lot of methods for simulation of earthquake ground motions have been proposed, many of them are concern with the problem of simulating ground motions with single (horizontal or vertical) component and cannot take the effect of spatial correlation into account. From the point of view of the stochastic process, earthquake ground motions, as a realization of a nonstationary stochastic process, are temporally and spatially correlated and this property has notable influence on structural dynamic response. For this purpose, the time varying Vector Auto Regressive Moving Average (ARMAV) model is applied to the simulation of multidimensional earthquake ground motions, which can not only describe the properties of spatiality but also the nonstationarity of intensity and frequency. A Kalman filter is used to estimate the time varying

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coefficients of the ARMAV model which is proved to be very efficient and precise. It has been shown from the simulation results presented in the end of the paper that the ARMAV model is quite suitable for the simulation of multidimensional earthquake ground motions and the precision is also satisfactory.

ARMAV MODELING OF MULTIDIMENSIONAL EARTHQUAKE GROUND MOTIONS

ARMAV modeling of earthquake ground motions

It has been shown, Conte et al. [1], that the earthquake ground motions as a realization of a stochastic process, can be modeled the response of a time varying linear system loaded by a white noise. Let $A(k)$ be the discrete (sampled) representation of the continuous earthquake process A_c with m components (dimension), i.e. $A(k)=A_c(k\Delta t)$, where Δt is the sampling interval and $A(k)=\{a_i(k),\ i=1,2,\ldots,m\}^T$. The discrete ARMAV($p$,$q$) model corresponding to $A(k)$ is then given by

$$A(k)=\sum_{i=1}^{p}\Phi_i(k)A(k-i)-\sum_{i=1}^{q}\Theta_i(k)e(k-i)+e(k) \tag{1}$$

where the AR coefficients $\Phi_i(k)$ and the MA coefficients $\Theta_i(k)$ are full $m\times m$ matrices and the noise time series $e(k)=\{e_i(k),\ i=1,2,\ldots,m\}^T$, and following assumption are made

$$E\left[e_i(k)\right]=0 \tag{2}$$

$$Cov\left[e_i(k),e_j(k)\right]=R(k)\cdot\delta_{ij} \tag{3}$$

Where the covariance matrix $R(k)$ is $m\times m$.

In order to estimate the modal parameters, the ARMAV(p,q) in Eq. (1) is formulated to the corresponding discrete time state space model, see Xie [2],

$$X(k)=\Phi(k)X(k-1)+\Theta(k)V(k) \tag{4}$$

where

$$X(k)=\begin{bmatrix}A(k)\\A(k-1)\\A(k-2)\\\vdots\end{bmatrix};\quad V(k)=\begin{bmatrix}e(k)\\e(k-1)\\e(k-2)\\\vdots\end{bmatrix} \tag{5}$$

and the Auto Regressive matrix and the Moving Average matrix of the state space model is given by

$$\Phi(k)=\begin{bmatrix}\Phi_1(k) & \Phi_2(k) & \cdots & \Phi_p(k)\\ I & 0 & \cdots & 0\\ 0 & I & & 0\\ \vdots & \vdots & & \vdots\\ 0 & \cdots & I & 0\end{bmatrix};\quad \Theta(k)=\begin{bmatrix}1 & -\Theta_1(k) & \cdots & -\Theta_q(k)\\ 0 & 0 & \cdots & 0\\ 0 & 0 & & 0\\ \vdots & \vdots & & \vdots\\ 0 & \cdots & 0 & 0\end{bmatrix} \tag{6}$$

Based on the state space model, a Kalman filter is applied to estimate time varying parameters of the ARMAV(p,q) model to minimize the determinant of the covariance matrix of the noise time series $e(k)$.

Order selection of ARMAV model

Though a lot of order selection approaches has been proposed and used widely, Xie [2], the theoretical frequency resolving power of ARMA spectrum is often ignored. It has been shown, Kang et al. [3], the efficient frequency range for ARMA spectrum to resolve the peak of the spectrum is [$f/2n$, $f/2-f/2n$], n is sum of order of the ARMA model and $f=1/\Delta t$ is the sampling frequency, e.g. for a ARMA(p,q) model, $n=p+q$ and $f=50$ when Δt is 0.02. It can be proved that for ARMAV(p,q) model in Eq. (1), the efficient frequency range should be [$f/2m(p+q)$, $f/2-f/2m(p+q)$]. The efficient frequency range is considered in this

paper in the process of order selection of ARMAV model parallel with the conventional order selection criteria.

SIMULATION OF MULTIDIMENSIONAL EARTHQUAKE GROUND MOTIONS

Once the time varying parameters of the ARMAV model is estimated, it is very convenient to generate a sample of large population with the same statistical properties by changing noise time series $e(k)$. In this paper, the noise time series $e(k)$ is generated by modulating the white noise of different random seeds with the square root of $R(k)$ and then the simulated earthquake ground motions can be generated according to Eq. (1).

AN EXAMPLE OF SIMULATION

Taking the actual earthquake records of the 1940 Imperial Valley earthquake recorded at El Centro, California, USA, as an example and three components (i. e. N-S, E-W and Vertical component) are used as simulation targets. A sample with a population of 100 is generated based on ARMAV(2,1) model. For the practical purpose, the property in frequency domain of earthquake ground motions is usually concerned in structural dynamic time history analysis and they are regarded as one of the essential properties of earthquake ground motions. In this paper the Fourier spectrum and the acceleration response spectrum of simulated ground motions and actual ones are compared and the results are represented as below.

The actual and simulated ground acceleration time histories corresponding to the three (N-S, E-W and Vertical) components of El Centro record are shown in Fig. 1~6, and it can be seen from them that the shape of each actual component is quite similar to the simulated one and the value and occur time of the Peak Ground Acceleration is also compatible.

The simulated and actual Fourier amplitude spectra of each component are shown in Fig. 7~12, and it can be seen that the frequency contents distribution of them is coincident within the major frequency range.

Fig. 13~15 show the statistical characteristics of acceleration response spectra of the sample of population of 100 and it can be seen the simulation precision is satisfactory.

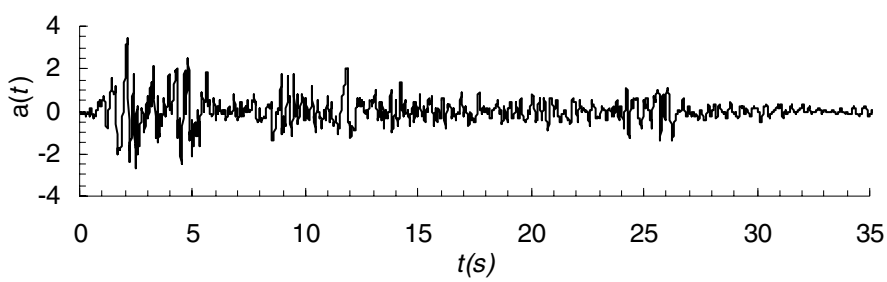

Fig. 1 Ground acceleration time history of El Centro (1940, N-S)

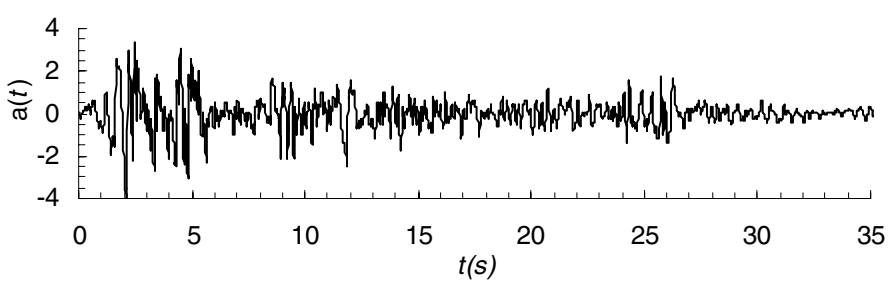

Fig. 2 Simulated ground acceleration time history of El Centro (1940, N-S)

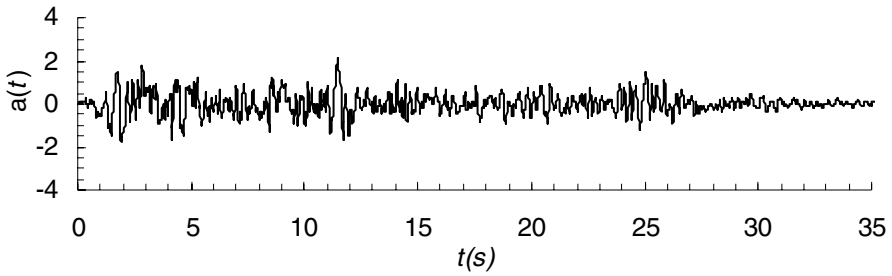

Fig. 3 Ground acceleration time history of El Centro (1940, E-W)

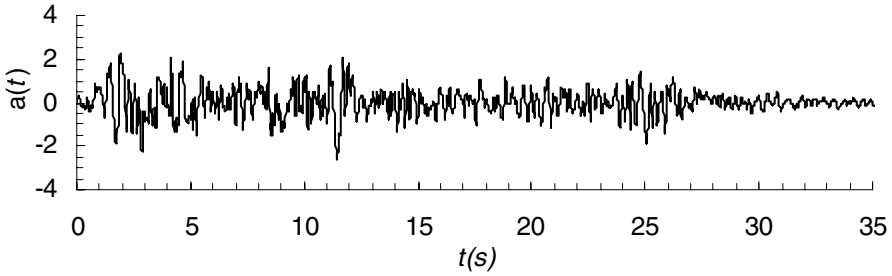

Fig. 4 Simulated ground acceleration time history of El Centro (1940, E-W)

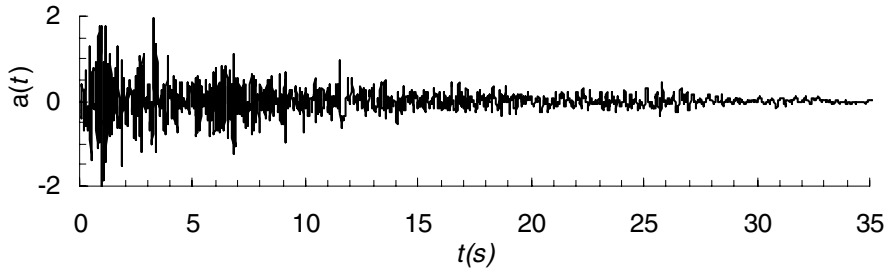

Fig. 5 Ground acceleration time history of El Centro (1940, Vertical)

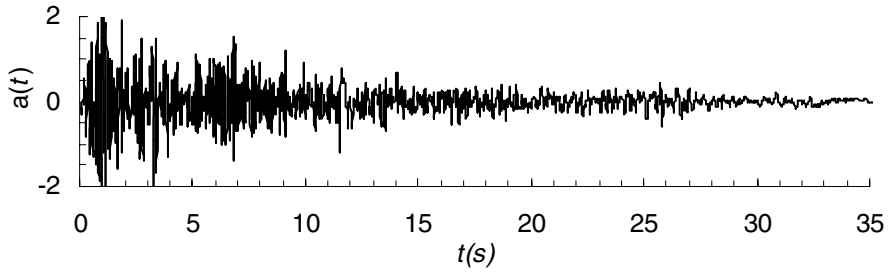

Fig. 6 Simulated ground acceleration time history of El Centro (1940, Vertical)

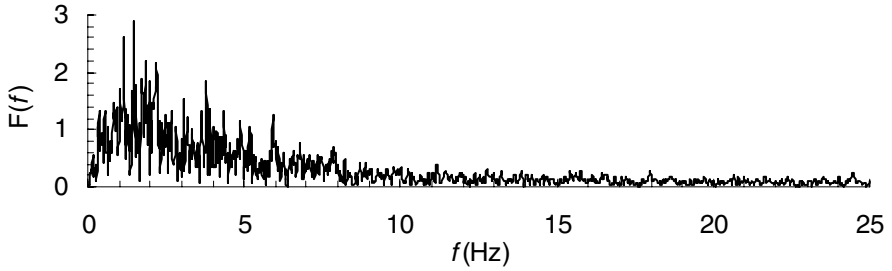

Fig. 7 Ground acceleration Fourier amplitude spectra of El Centro (1940, N-S)

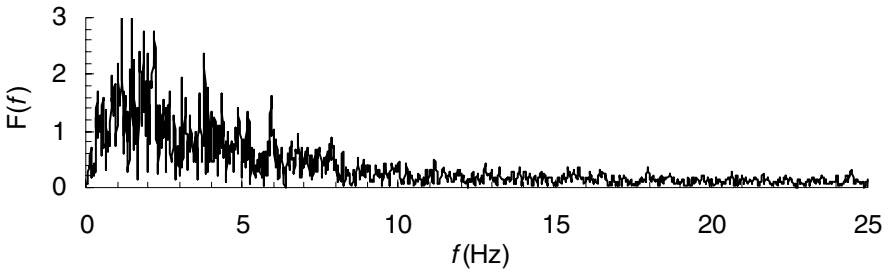

Fig. 8 Simulated ground acceleration Fourier amplitude spectra of El Centro (1940, N-S)

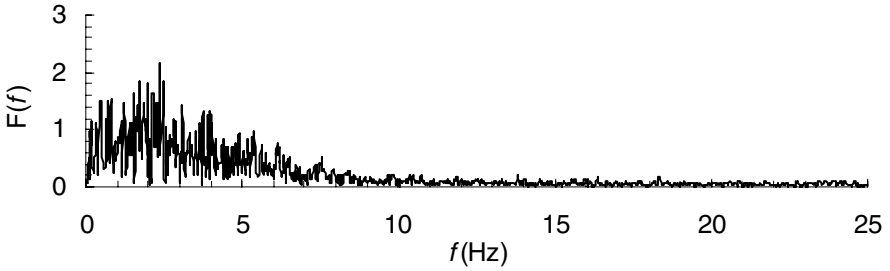

Fig. 9 Ground acceleration Fourier amplitude spectra of El Centro (1940, E-W)

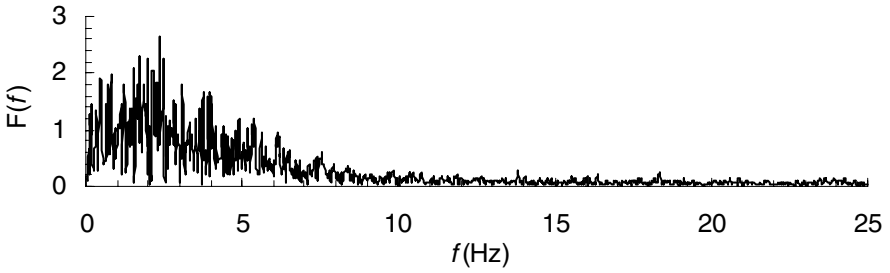

Fig. 10 Simulated ground acceleration Fourier amplitude spectra of El Centro (1940, E-W)

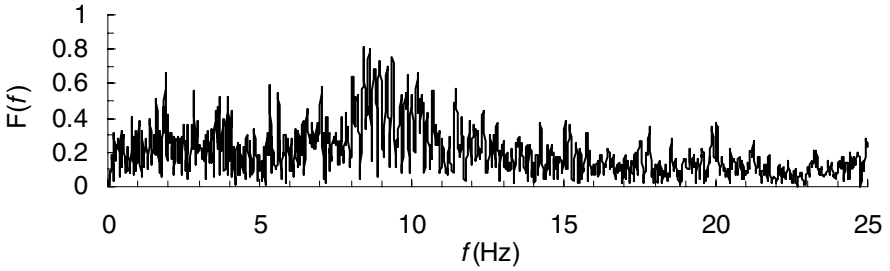

Fig. 11 Ground acceleration Fourier amplitude spectra of El Centro (1940, Vertical)

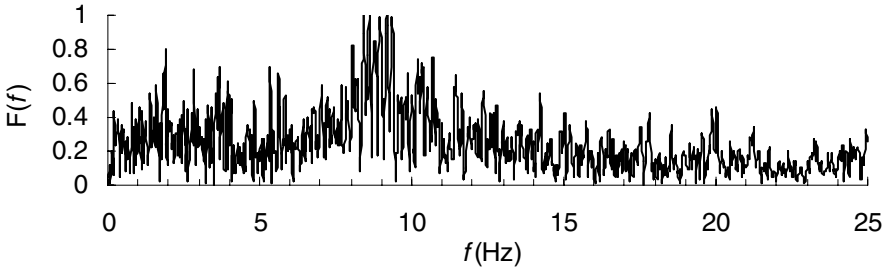

Fig. 12 Simulated ground acceleration Fourier amplitude spectra of El Centro (1940, Vertical)

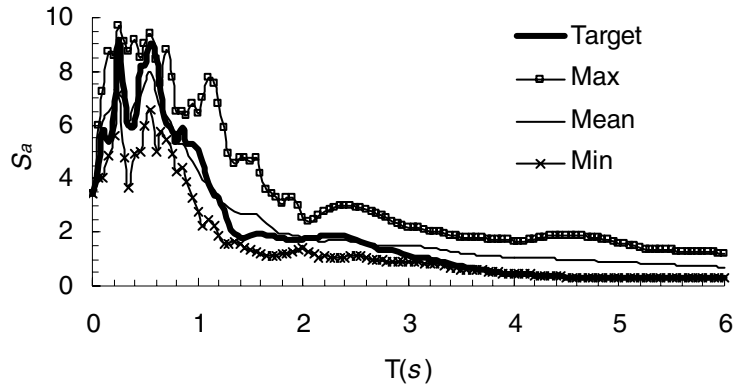

Fig. 13 Simulated acceleration response spectra of El Centro (1940, N-S)

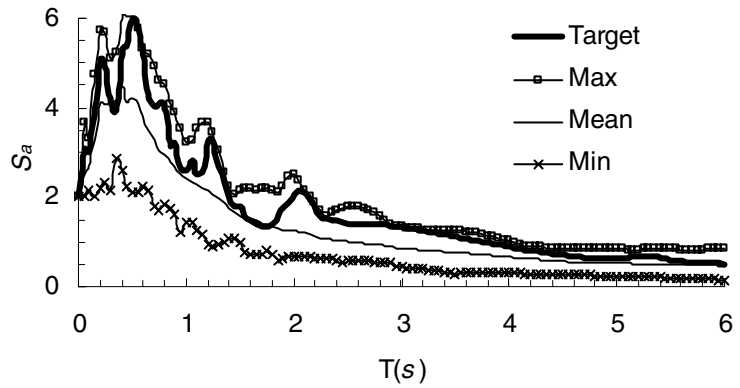

Fig. 14 Simulated acceleration response spectra of El Centro (1940, E-W)

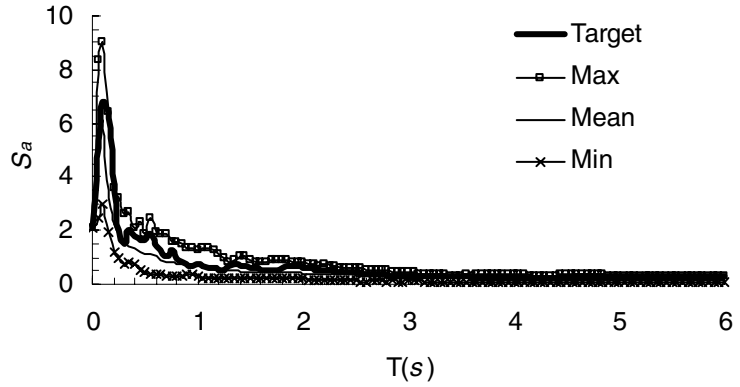

Fig. 15 Simulated acceleration response spectra of El Centro (1940, Vertical)

CONCLUSIONS

The discrete time varying Vector Auto Regressive Moving Average (ARMAV) model is applied to the simulation of multidimensional earthquake ground motions, which can not only describe the properties of spatiality but also the nonstationarity of intensity and frequency. A Kalman filter is used to estimate the time varying coefficients of the ARMAV model which is proved to be very efficient and precise. It has been shown from the simulation results presented in the end of the paper that the ARMAV model is quite suitable for the simulation of multidimensional earthquake ground motions and the precision is also satisfactory.

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