



## **INFLUENCE OF FRICTION MODELS ON RESPONSE EVALUATION OF BUILDINGS WITH SLIDING ISOLATION DEVICES**

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### **SUMMARY**

The application of base isolation systems with energy dissipation for aseismic design of building is well established and has been implemented for a large number of buildings. Friction provides excellent mechanism for energy dissipation at the sliding surface. A wide variety of isolation devices using friction for dissipation of energy have been proposed. Their design requires numerical simulations that commonly model hysteretic behavior of friction dampers using Coulomb friction, where the coefficient of friction is constant. However, the basic laws for typical sliding materials and experimental investigations show complex behavior of friction force, in both sliding and transition phases, that include the non-linear relationship between friction and sliding velocity, and the effect of time history of response, stiction and sliding velocity. This paper investigates the behavior of framed structures with sliding isolation systems subjected to earthquake ground motions. The paper considers Coulomb friction model as well as other more complex models to assess the response of structures with sliding isolation systems. The analysis results show that stiction and Stribeck effect, which are often not considered in the design of buildings with friction based isolation systems, significantly influence the isolator behavior. It is concluded that simplified analysis using Coulomb friction model may significantly overestimate the effectiveness of sliding isolation devices.

### **INTRODUCTION**

Seismic isolation, commonly referred as base isolation, is based on the premise that structure response can be substantially decoupled from potentially damaging earthquake ground motions. Base isolation systems use a low-stiffness or sliding layer between the superstructure and its foundation. The sliding isolation systems (that permit sliding motion between the structure and its base at the isolator level) incorporate isolation and energy dissipation in one unit and have been found to be very effective in reducing the response of structures. During low intensity ground motions, the frictional resistance to sliding provides resistance to motion so that the structure substantially behaves as a fixed based structure. During large intensity ground motions, the structure slides at the isolator level, thereby decoupling its motion from that of its foundation. The sliding system has additional advantage of energy dissipation through friction, which reduces the energy transmitted to the superstructure (Mostaghel and Tanbakuchi [1]; and Constantinou et al. [2]).

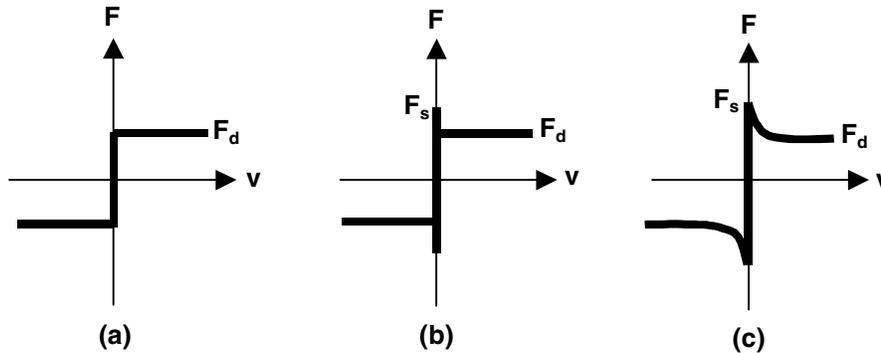
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The sliding isolation systems are required to be relatively maintenance free since earthquakes may occur infrequently and after long intervals. Most sliding isolation systems use dry friction at the sliding surface to reduce the maintenance that is otherwise necessary for lubricated sliding systems. Although the exact mechanism of dry friction is still not well understood, Coulomb friction model, in which the friction force is directly proportional to the force normal to direction of sliding, is commonly used for modeling of sliding isolation systems. The proportionality factor (or coefficient of friction) in Coulomb friction model is considered to be a constant. More comprehensive friction models, in which the influence of friction coefficient on slip velocity, normal load, and history of motion, has received considerable attention from researchers in the area of friction-induced vibration and control of machines with friction (Ibrahim [3]; Armstrong-Helouvry [4]; Armstrong-Helouvry et al. [5]; Olsson et al. [6]; Gaul and Nitsche [7]; Oden and Martins [8]; and Feeny et al. [9]). These and other investigations show that the coefficient of friction at stick stage is higher than the coefficient of friction at sliding stage, which leads to stick-slip effect or other unstable oscillations in the response behavior of structures.

In this paper, the response of structures isolated with sliding isolation systems has been investigated. The isolator has been modeled using different friction models including Coulomb model and other more complex models. Both single-degree-of-freedom and multi-degree-of-freedom structures have been considered. The following aspects have been examined in detail: (i) Response of sliding isolated structure considering three different friction models, (ii) Sensitivity of response of isolated structure to the coefficient of friction when using different friction models, and (iii) Importance of stick-slip and Stribeck effect on response behavior of sliding isolated structures.



**Figure 1. Friction force variation with sliding velocity for different friction models (a) Coulomb friction model (Model FM1), (b) Coulomb friction model with stiction effect (Model FM2) and (c) Coulomb friction model with Stribeck effect (Model FM3).**

## DRY FRICTION MODELS

The following friction models have been considered in the investigation of response behavior of sliding isolated structures (Fig. 1).

### Coulomb Friction Model (Model FM1)

This is the most frequently used model, proposed over 200 years ago and is represented in Fig. 1(a). In this model, the coefficient of friction remains constant and the friction force is expressed as

$$F_d = \mu F_N \text{sgn}(v) \quad (1)$$

where  $F_N$  is the normal load on the sliding surface,  $F_d$  is the frictional resistance at sliding stage,  $\mu$  is the coefficient of friction which same for both stick and sliding stages,  $v$  is relative sliding velocity, and  $\text{sgn}(v)$  is the signum function that assumes a value of +1 for positive sliding velocity and -1 for negative sliding velocity. This signum function determines the direction of sliding.

### Coulomb Model with Stiction Effect (Model FM2)

This is a slight modification to Coulomb friction model wherein different coefficients of friction are used for non-sliding (stick) and sliding stage and is represented in Fig. 1(b). The friction force at sliding ( $F_d$ ) and stick stage ( $F_s$ ) are given by

$$\begin{aligned} F_d &= \mu_d F_N \operatorname{sgn}(v) \\ F_s &= \mu_s F_N \end{aligned} \quad (2)$$

in which  $\mu_s$  and  $\mu_d$  are the coefficients of static and sliding friction, respectively. The coefficient of sliding friction is smaller than the coefficient of static or sticking friction. The influence of different coefficient of friction on the structure response is termed as stiction effect.

### Coulomb Model with Stiction and Stribeck Effect (Model FM3)

It has been observed through various experimental studies that friction force does not decrease abruptly at the initiation of sliding as assumed in model FM2. In this model, the transition of friction force from stick to sliding stage is considered to decay exponentially, and whose rate of decay is velocity dependent. The friction force during sliding,  $F$ , obeys the following exponential law:

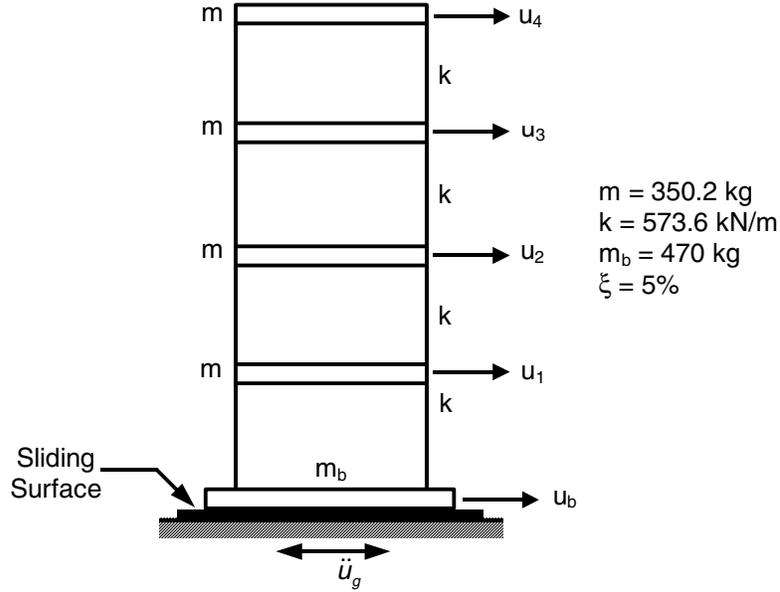
$$\begin{aligned} F &= \left( F_d + (F_s - F_d) \exp\left(-|v/v_s|^{\delta_s}\right) \right) \operatorname{sgn}(v) \\ F_d &= \mu_d F_N \\ F_s &= \mu_s F_N \end{aligned} \quad (3)$$

where  $v_s$  and  $\delta_s$  are empirical parameters. The parameter  $v_s$  is also known as Stribeck velocity whose value depends on response memory (history of response), material properties and surface finish. This constant can be regarded as the decay rate of the sliding friction coefficient with the sliding velocity of the friction surfaces. From Eq. (3) it is seen that  $F_d$  is the lower bound of the sliding frictional resistance and  $F_s$  is the upper bound. This phenomenon is known as Stribeck effect and the lower bound limit is called Stribeck friction. The actual values of the friction coefficients and empirical parameters depend on the conditions of the friction surfaces, and generally can be determined by experiments in which the friction force and sliding velocity are measured simultaneously. The friction force variation for Model FM3 is shown in Fig. 1(c).

From the friction models, it can be observed that all the three models have discontinuity at zero velocity. In model FM2, the coefficient of static friction is greater than the coefficient of sliding or kinetic friction. For systems following this model, stick-slip motion occurs where the sliding surfaces alternately switch between sticking and slipping in a more or less regular fashion. The sudden change of friction coefficient at the point of zero velocity causes discontinuous stick-slip effect on the response behavior. In model FM3, the coefficient of friction changes exponentially from upper bound limit to lower bound. The equation of motion of the structure using model FM3 is dependent on the direction of velocity. From the different friction models, it is also seen that the structure response is linear during sliding stage for FM1 and FM2 friction models. However, the exponential function in model FM3 makes sliding response nonlinear.

## RESPONSE ANALYSIS

The mathematical formulation of multi-degree-of-freedom (MDOF) systems with sliding isolation system shown in Fig. 2 has been presented below. The motion consists of two phases: (1) non-sliding or stick phase when the sliding friction resistance at base is not overcome and the structure behaves as conventional fixed-base structure, and (2) sliding or slip phase when relative motion across the sliding surface takes place. The overall response consists of series of sliding and non-sliding phases following one another.



**Figure 2. Schematic sketch of four-story isolated structure with friction isolator (Yang et al. [10]).**

#### Non-sliding or Stick Phase

During non-sliding phase, the structure behaves as a conventional fixed base system. The governing equation of motion at this phase can be expressed as:

$$\mathbf{M}_f \ddot{\mathbf{u}}_f + \mathbf{C}_f \dot{\mathbf{u}}_f + \mathbf{K}_f \mathbf{u}_f = -\mathbf{M}_f \mathbf{r}_f \ddot{u}_g \quad (4)$$

in which

$$\ddot{u}_b = \dot{u}_b = 0 \quad , \quad u_b = \text{constant} \quad (5)$$

and non-sliding ensures that

$$\left| \left( \sum_{i=1}^N m_i (\ddot{u}_i + \ddot{u}_g) + m_b \ddot{u}_g \right) \right| < F_s \quad (6)$$

In the above equations,  $\mathbf{M}_f$ ,  $\mathbf{C}_f$ , and  $\mathbf{K}_f$  are the  $N \times N$  mass, damping and stiffness matrices of the conventional fixed base structure,  $\mathbf{u}_f$  is the  $N$ -vector of relative displacements of the structure with respect to base,  $\mathbf{r}_f$  is the force influence vector of fixed base structure,  $u_b$  is the constant sliding displacement of the base,  $u_g$  is the ground displacement, and  $F_s$  is the maximum frictional resistance at stick stage. The over dots represent derivative with respect to time while  $N$  is the number of degree of freedom of the structure. The left-hand side of Eq. (6) is the absolute value of sum of the inertia forces at the isolator level.

Before the application of base excitation, the structure is at rest. Therefore, the response of the structure always starts in stick phase. This phase of response continues until the sliding force at the isolator level exceeds the frictional resistance, after which the structure begins to slide.

### Sliding Phase

When the inequality in the Eq. (6) is not satisfied the structure enters into sliding phase and the degree of freedom (DOF) corresponding to the base mass also becomes active. The corresponding equations of motion are given as:

$$\mathbf{M}_s \ddot{\mathbf{u}}_s + \mathbf{C}_s \dot{\mathbf{u}}_s + \mathbf{K}_s \mathbf{u}_s = -\mathbf{M}_s \mathbf{r}_s \ddot{u}_g - \mathbf{r}_s F \quad (7)$$

where  $\mathbf{M}_s$ ,  $\mathbf{C}_s$  and  $\mathbf{K}_s$  are the modified mass, damping and stiffness matrices of order  $N+1$ , and  $\mathbf{r}_s$  is the modified influence coefficient vector. The friction force at the sliding surface is represented as  $F$  whose expressions for different friction models are given in Eqs. (1-3). The force normal to the sliding surface, which governs the friction force, is equal to the total structure mass ( $m_t$ ), which consists of the sum of base mass and all story masses. In terms of the system matrices of the fixed base structure, the system matrices during sliding phase can be expressed as:

$$\mathbf{M}_s = \begin{bmatrix} \mathbf{M}_f & \mathbf{M}_f \mathbf{r}_f \\ [\mathbf{M}_f \mathbf{r}_f]^T & m_t \end{bmatrix}, \mathbf{C}_s = \begin{bmatrix} \mathbf{C}_f & 0 \\ 0 & 0 \end{bmatrix}, \mathbf{K}_s = \begin{bmatrix} \mathbf{K}_f & 0 \\ 0 & 0 \end{bmatrix}, \mathbf{u}_s = \begin{Bmatrix} \mathbf{u}_f \\ u_b \end{Bmatrix}, \mathbf{r}_s = \begin{Bmatrix} \mathbf{0} \\ 1 \end{Bmatrix} \quad (8)$$

The direction of sliding is governed by the signum function in Eqs. (1-3). For a multi-degree-of-freedom system, the direction of base sliding can be expressed as:

$$\text{sgn}(\dot{u}_b) = - \frac{\left[ \left( \sum_{i=1}^N m_i (\ddot{u}_i + \ddot{u}_g) + m_b \ddot{u}_g \right) \right]}{\left| \left( \sum_{i=1}^N m_i (\ddot{u}_i + \ddot{u}_g) + m_b \ddot{u}_g \right) \right|} \quad (9)$$

### Transition Phase

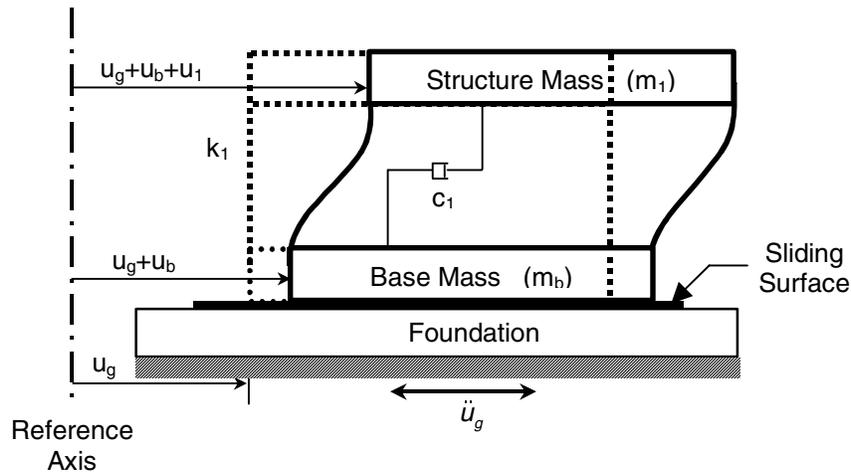
When the sliding velocity during motion becomes zero, the structure may enter a non-sliding phase or reverse its direction of sliding or have a momentary halt and continue in the same direction. The status of motion during transition phase can be evaluated by checking the validity of Eq. (6) and then solving the appropriate equations of non-sliding or sliding motion during the next time-step. Since a particular phase of response may be arbitrarily short, the response at the end of the previous phase greatly influences the overall response of the structure. The response evaluation is therefore extremely sensitive to accurate estimation of the initial conditions at the start of any phase of response. During the numerical simulations, determination of transition point with the accuracy of  $10^{-10}$  seconds has been found to be adequate.

The response at non-sliding and sliding phase can be determined using standard numerical techniques such as Newmark's average acceleration method. In sliding phase with Model FM3, a method for solving systems of non-linear simultaneous equations (Aluffi-Pentini et al. [11]) has been used. The solution at any phase uses the response at the end of the previous phase as the initial value. At start of a non-sliding phase the base acceleration and base velocity are zero, and the base displacement has a constant value depending on the extent of sliding in the previous phase.

## EXAMPLE SYSTEM

### Response of Single-Degree-of-Freedom Structure (SDOF)

A single story (Fig 3) shear structure isolated by sliding system has been considered for evaluating the influence of different friction models. For the example system, the mass of the structure and the base are taken as equal. The structural damping ratio has been taken as 5 percent of critical damping. The structure is represented as a lumped mass model with equal lumped masses of 25000 kg, while the story stiffness has been calculated such that it results in the required time period of fixed-base structure. Different structures with fixed-base time period between 0.03-2.0 s (0.5-33 Hz) have been considered in this investigation to cover the full range of typical structure time periods.

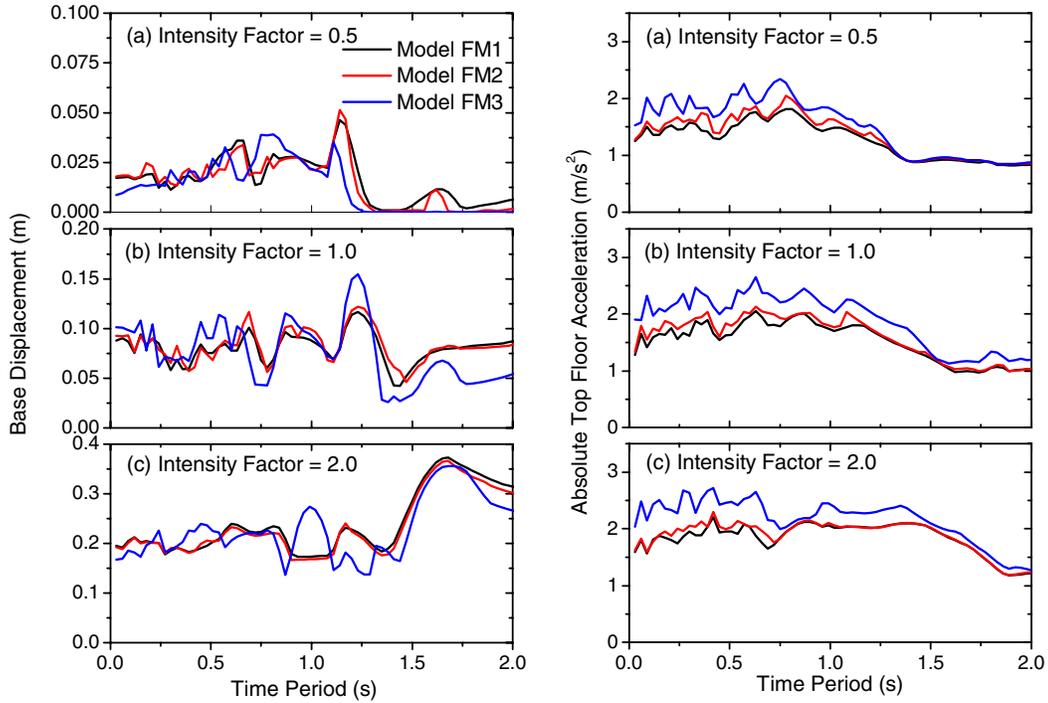


**Figure 3. Schematic sketch of SDOF isolated structure with friction isolator (solid lines: deformed position, dotted lines: original position).**

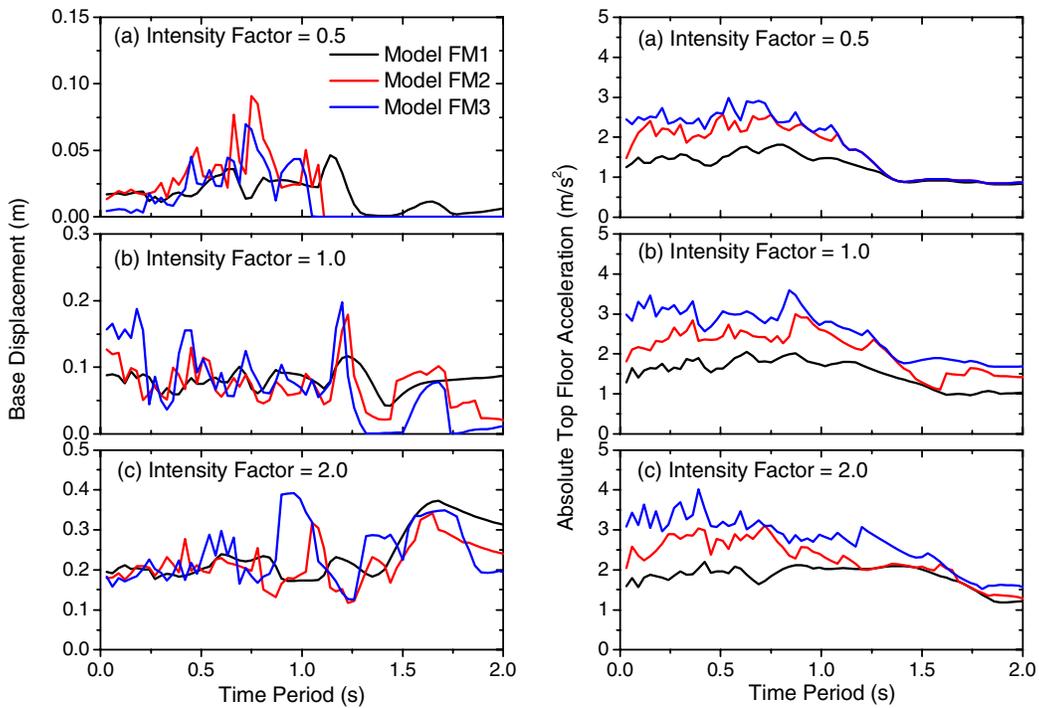
Different parametric studies have been carried out for the structure subjected to 1940 El Centro (NS) ground motions (Peak ground acceleration,  $PGA = 0.348g$ ) over the duration of 30 seconds. To simulate the response to different intensities of ground motions, the El Centro ground motions have been scaled by three different factors. Ground motions with intensity factor of 0.5, corresponding to  $PGA = 0.174g$  has been used to simulate the behavior under small earthquake; intensity factor of 1.0, corresponding to  $PGA = 0.348g$  has been used to simulate the behavior under moderate earthquake and intensity factor of 2.0, corresponding to  $PGA = 0.696g$  has been used to simulate the behavior under large earthquake.

The properties of sliding surface and model parameters based on Constantinou et al. [2] have been considered. Two different values of static to sliding friction coefficient ( $\mu_s / \mu_d = 1.5$  and  $\mu_s / \mu_d = 2.5$ ) have been considered. The corresponding Stribeck velocity in model FM3 have been taken as  $v_s = 0.08$  m/s and  $v_s = 0.0423$  m/s, respectively. The value of empirical parameter,  $\delta_s$ , in model FM3 has been taken 1.0 (Constantinou et al. [2]). The sliding coefficient of friction  $\mu_d = 0.05$  has been considered for all friction models.

Since the primary objective of base isolation is to reduce the peak responses, the investigations of maximum responses enable one to evaluate the influence of different friction models. The spectra of maximum base displacement and absolute floor acceleration for structures with different time periods for stiction ratios 1.5 and 2.5 have been shown in Figs. 4 and 5. These results show that the friction model has tremendous influence on the predicted maximum response of the structure. It is observed that both models FM2 (Coulomb friction with stiction) and FM3 (Coulomb friction with stiction and Stribeck effect) result in higher response than that predicted by Coulomb friction alone. This result has great significance in the practical design of base isolation systems, since the prevalent practice is based on the use of Coulomb friction to determine the isolation system parameters. It has been found that the other maximum responses such as relative story displacement and residual displacement are also much greater in friction model FM3 compared to Coulomb friction model (Patro and Sinha [12]). It is seen that the maximum absolute floor acceleration is consistently underestimated by not including stiction and Stribeck effect. This may adversely affect performance of secondary system mounted on the structure. The maximum base displacement estimate based on Coulomb friction model FM1 may also have large errors resulting in underestimation of building displacement.



**Figure 4. Peak responses of SDOF example structure subjected to different intensities of El Centro (1940) ground motions ( $\xi = 5\%$ ,  $\mu_d = 0.05$ ,  $\mu_s/\mu_d = 1.5$ ) (a) Intensity Factor = 0.5, (b) Intensity Factor = 1.0, (c) Intensity Factor = 2.0.**



**Figure 5. Peak responses of SDOF example structure subjected to different intensities of El Centro (1940) ground motions ( $\xi = 5\%$ ,  $\mu_d = 0.05$ ,  $\mu_s/\mu_d = 2.5$ ) (a) Intensity Factor = 0.5, (b) Intensity Factor = 1.0, (c) Intensity Factor = 2.0.**

### Response of Multi-Degree-of-Freedom Structure (MDOF)

The results from response of the SDOF example system shows that the friction model has substantial influence on the predicted maximum response of the structure over the entire range of structural time periods. The influence of friction model is further considered by evaluating the response of a four-story shear building (Fig. 2) isolated by sliding system. This system was earlier studied by Yang et al. [10] in which only Coulomb friction model was considered. The effect of different coefficients of sliding friction in each friction model has been presented. The damping ratio has been taken as 5 percent of critical in all the modes. For this system, the base mass is equal to 470 kg while the lumped story-mass is equal to 75% of the base mass for all floors ( $m_1 = m_2 = m_3 = m_4 = 350.2$  kg). The story stiffness are each equal to 573.6 kN/m ( $k_1 = k_2 = k_3 = k_4$ ).

The time history responses of this structure have been evaluated for three different coefficients of sliding friction: 0.02, 0.05 and 0.10 for each friction model. The results are presented for low, medium and high intensities of El Centro 1940 NS ground motion (corresponding to PGA = 0.174g, 0.348g and 0.696g), for different ratio of breakaway to sliding friction. The Stribeck velocities,  $v_s$ , in model FM3 have been taken as 0.08 m/s, 0.0423 m/s and 0.0423 m/s, with corresponding ratio of breakaway to sliding friction as 1.5, 2.0 and 2.5, respectively. The value of empirical parameter  $\delta_s$  in model FM3 is taken 1.0 (Constantinou et al. [2]).

The maximum values of base displacement, relative displacement of top floor, and the absolute top floor acceleration for the 4-story structure with Coulomb friction isolator are presented in Table 1. The normalized peak responses of example system have been presented in Tables 2-4. The peak responses of structure are normalized to corresponding peak responses from Coulomb friction model (FM1). The peak responses of structure for Coulomb friction model (FM1) are presented in Table 1. The investigation has been carried out for selected range of breakaway to sliding friction ratio, and for different intensities of ground excitation for each sliding coefficient.

Typical time-history response of base displacement, top floor displacement and absolute top floor acceleration are shown in Figs. 6-8. This investigation has been carried out for all three (low to high) intensities of ground motions considered for the SDOF system. The time-history results are presented for isolator with  $\mu = 0.05$  and the ratio of breakaway to sliding coefficient of 2.0, and the Stribeck velocity is 0.0423 m/s. The peak response evaluation and time history plots clearly show the vastly different response characteristics when Stribeck and stiction effects are considered (Model FM3) compared to classical Coulomb model, and the model where only stiction has been considered.

**Table 1. Peak responses of MDOF structure for Coulomb friction model (Model FM1) subjected to different intensities of El Centro (1940) ground motions.**

Intensity factor	Coefficient of sliding friction ( $\mu_d$ )	Base displ. (m)	Top floor displ. rel. to base (m)	Abs. top floor acc. ( $m/s^2$ )
0.5	0.02	0.0372	0.0037	1.1814
	0.05	0.0208	0.0067	2.1393
	0.10	0.0211	0.0119	3.3431
1.0	0.02	0.1445	0.0043	1.3354
	0.05	0.0586	0.0080	2.7630
	0.10	0.0416	0.0134	4.2785
2.0	0.02	0.5623	0.0052	1.5958
	0.05	0.1986	0.0104	3.4432
	0.10	0.1172	0.0160	5.5261

**Table 2. Normalized peak responses of 4-story example structure with  $\mu_d = 0.02$  subjected to El Centro (1940) ground motions (IF: Excitation intensity factor, BD: Base displacement, FD: Top floor relative displacement, FA: Top floor absolute acceleration).**

IF	Friction model	$(\mu_s/\mu_d) = 1.5$			$(\mu_s/\mu_d) = 2.0$			$(\mu_s/\mu_d) = 2.5$		
		BD	FD	FA	BD	FD	FA	BD	FD	FA
0.5	FM1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	FM2	1.068	0.970	1.050	0.987	1.196	1.288	1.023	1.348	1.376
	FM3	0.814	1.250	1.322	0.880	1.583	1.485	0.924	1.833	1.599
	Fixed Base	-	6.577	3.960	-	6.577	3.960	-	6.577	3.960
1.0	FM1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	FM2	1.050	1.039	1.030	1.117	1.283	1.150	1.079	1.501	1.391
	FM3	0.710	1.318	1.423	0.906	1.573	1.684	1.046	1.711	1.847
	Fixed Base	-	11.30	7.008	-	11.30	7.008	-	11.30	7.008
2.0	FM1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	FM2	1.011	1.028	1.016	1.080	1.168	1.099	1.102	1.254	1.259
	FM3	1.107	1.348	1.307	1.114	1.567	1.484	1.173	1.667	1.759
	Fixed Base	-	18.46	11.73	-	18.46	11.73	-	18.46	11.73

**Table 3. Normalized peak responses of 4-story example structure with  $\mu_d = 0.05$  subjected to El Centro (1940) ground motions (IF: Excitation intensity factor, BD: Base displacement, FD: Top floor relative displacement, FA: Top floor absolute acceleration).**

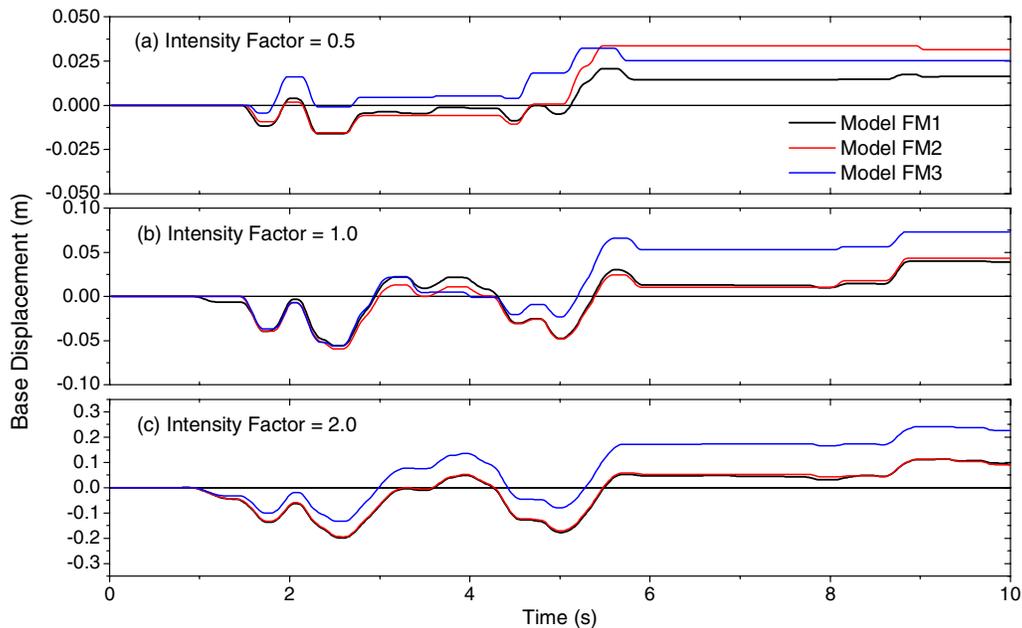
IF	Friction model	$(\mu_s/\mu_d) = 1.5$			$(\mu_s/\mu_d) = 2.0$			$(\mu_s/\mu_d) = 2.5$		
		BD	FD	FA	BD	FD	FA	BD	FD	FA
0.5	FM1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	FM2	0.985	1.059	1.013	1.640	1.430	1.216	2.565	1.694	1.478
	FM3	1.525	1.339	1.264	1.655	1.594	1.412	1.704	1.931	1.705
	Fixed Base	-	3.597	2.187	-	3.597	2.187	-	3.597	2.187
1.0	FM1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	FM2	0.985	1.054	1.060	1.017	1.264	1.216	1.385	1.606	1.296
	FM3	1.288	1.375	1.198	1.305	1.624	1.342	1.052	1.864	1.400
	Fixed Base	-	6.018	3.387	-	6.018	3.387	-	6.018	3.387
2.0	FM1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	FM2	1.025	1.043	0.998	0.979	1.264	1.318	1.406	1.383	1.406
	FM3	1.402	1.309	1.296	1.577	1.415	1.351	1.786	1.700	1.383
	Fixed Base	-	9.301	5.436	-	9.301	5.436	-	9.301	5.436

The results clearly show that the analysis procedures incorporating more complex friction models lead to substantially higher responses for all stiction ratios. Since the stiction ratio for dry friction is always greater than unity, these results indicate that the maximum response quantities for MDOF structures may be significantly underestimated by ignoring stiction effect. The results further show that analysis using friction model FM3, in which both stiction and Stribeck effect are included give higher responses than the other two models. This indicates that model FM2, in which only stiction is included does not provide adequate accuracy and more complex models that include the effect of response memory, material properties and surface finish are required.

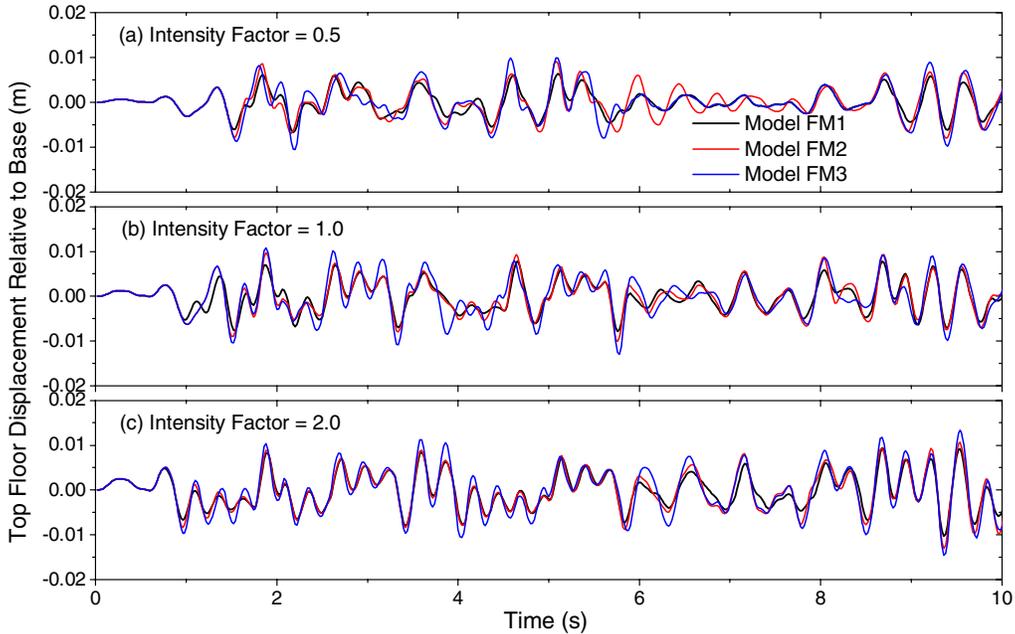
**Table 4. Normalized peak responses of 4-story example structure with  $\mu_d = 0.10$  subjected to El Centro (1940) ground motions (IF: Excitation intensity factor, BD: Base displacement, FD: Top floor relative displacement, FA: Top floor absolute acceleration).**

IF	Friction model	$(\mu_s/\mu_d) = 1.5$			$(\mu_s/\mu_d) = 2.0$			$(\mu_s/\mu_d) = 2.5$		
		BD	FD	FA	BD	FD	FA	BD	FD	FA
0.5	FM1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	FM2	1.279	1.127	1.022	0.698	1.378	1.079	0.906	1.705	1.244
	FM3	1.112	1.298	1.045	0.710	1.484	1.127	0.633	1.813	1.294
	Fixed Base	-	2.035	1.399	-	2.035	1.399	-	2.035	1.399
1.0	FM1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	FM2	0.9853	1.058	1.013	1.640	1.429	1.216	2.565	1.693	1.478
	FM3	1.595	1.303	1.229	2.190	1.463	1.364	2.178	1.965	1.674
	Fixed Base	-	3.594	2.187	-	3.594	2.187	-	3.594	2.187
2.0	FM1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	FM2	0.9851	1.054	1.060	1.017	1.264	1.216	1.385	1.606	1.296
	FM3	1.199	1.274	1.173	1.141	1.531	1.272	1.957	1.635	1.232
	Fixed Base	-	6.023	3.387	-	6.023	3.387	-	6.023	3.387

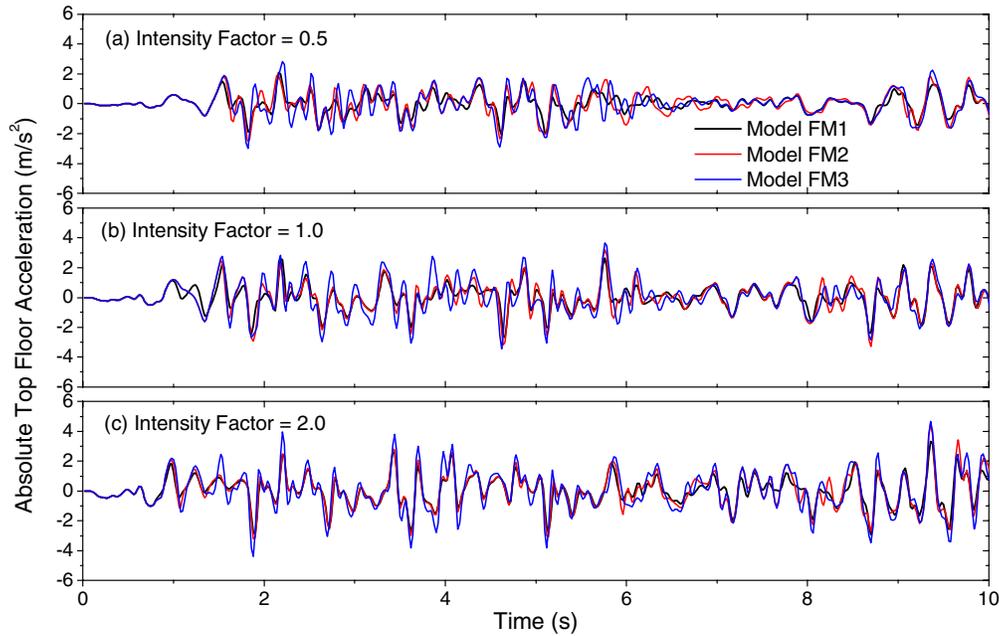
As the coefficient of sliding friction increases from 0.02 to 0.10, the total sliding time progressively decreases. The maximum responses presented in Tables 2-4 show that the error reduce for systems with higher coefficient of friction. However, higher coefficient of friction and the resulting reduction in sliding time is a manifestation of reduced effectiveness of the isolation system. For effective isolation, where the structure is expected to slide for a substantial proportion of excitation duration, the results of Tables 2 and 3 clearly illustrate the extent of error when Coulomb friction model is used.



**Figure 6. Base displacement of MDOF structure subjected to different intensities of El Centro (1940) ground motions ( $\xi = 5\%$ ,  $\mu_d = 0.05$ ,  $\mu_s/\mu_d = 2.0$ ) (a) Intensity Factor = 0.5, (b) Intensity Factor = 1.0, (c) Intensity Factor = 2.0.**



**Figure 7. Top floor relative displacement of MDOF structure subjected to different intensities of El Centro (1940) ground motions ( $\xi = 5\%$ ,  $\mu_d = 0.05$ ,  $\mu_s/\mu_d = 2.0$ ) (a) Intensity Factor = 0.5, (b) Intensity Factor = 1.0, (c) Intensity Factor = 2.0.**



**Figure 8. Top floor absolute acceleration of MDOF structure subjected to different intensities of El Centro (1940) ground motions ( $\xi = 5\%$ ,  $\mu_d = 0.05$ ,  $\mu_s/\mu_d = 2.0$ ) (a) Intensity Factor = 0.5, (b) Intensity Factor = 1.0, (c) Intensity Factor = 2.0.**

It is interesting to note that the maximum base displacement may also increase in models FM2 and FM3 compared to FM1 model. Since both these models include stiction, they provide higher resistance to initial sliding (transition from stick to slide). However the results of maximum response indicates that the maximum response using these models may be larger due to the highly nonlinear nature of the response.

These results demonstrate that all response quantities from numerical simulations have large errors when the isolator is modeled using Coulomb friction model. It is further seen that the structure response may be significantly underestimated if the effect of stiction and Stribeck are not considered. The numerical simulation of isolation system response considering Coulomb friction may therefore result in improper design of isolated buildings.

## DISCUSSIONS AND CONCLUSIONS

The influence of friction model on the effectiveness of sliding isolation systems has been investigated in this paper for both SDOF and MDOF structure. Three different friction models have been considered. The Coulomb friction model is the classical model. More comprehensive models that include the stiction effect and Stribeck effect have also been considered. The investigations have been carried out for coefficient of friction in the range of 0.02 to 0.10. The three different intensities (PGA = 0.174g, 0.348g and 0.696g) of El Centro ground motion have been considered in this investigation. Three different stiction ratios (1.5, 2.0 and 2.5) to cover the range of typical materials used for sliding isolation systems have been considered.

Based on the investigations presented in this paper, the following main conclusions can be drawn:

1. Most sliding isolation systems are designed based on Coulomb friction; however, behavior of actual friction force is more complex and includes stiction and Stribeck effect.
2. The structure response in numerical simulation is greatly influenced by the friction model. The use of more complex model results in much higher structure response, and may indicate the inadequacy of prevalent design procedure based on simplified friction model.

## REFERENCES

1. Mostaghel N, Tanbakuchi J. "Response of sliding structures to earthquake support motion." *Earthquake Engineering and Structural Dynamics* 1983; 11: 729-748.
2. Constantinou M, Mokha A, Reinhorn A. "Teflon bearings in base isolation, II : Modeling." *Journal of Structural Engineering* 1990; 116(2): 455-474.
3. Ibrahim RA. "Friction-induced vibration, chatter, squeal, and chaos; Part 1: Mechanics of contact and friction, Part 2: Dynamics and modeling." *Applied Mechanics Review* 1994; 47(7): 209-274.
4. Armstrong-Helouvry B. "Control of Machines with Friction." Kluwer Academic Publishers, Boston, 1991.
5. Armstrong-Helouvry B, Dupont P, Canudas WC. "A survey of models, analysis tools and compensation methods for the control of machines with friction." *Automatica* 1994; 30(7): 1083-1138.
6. Olsson H, Astrom KJ, Canudas WC, Gafvert M, Lischinsky P. "Friction models and friction compensation." *European Journal of Control* 1998; 4: 176-195.
7. Gaul L, Nitsche R. "The role of friction in mechanical joints." *Applied Mechanics Review* 2001; 54(2): 93-107.
8. Oden JT, Martins JAC. "Models and computational methods for dynamic friction phenomena." *Computer Methods in Applied Mechanics and Engineering* 1985; 52: 527-634.
9. Feeny B, Guran A, Hinrichs N, Popp K. "A historical review on dry friction and stick-slip phenomena." *Applied Mechanics Review* 1998; 51(5): 321-341.
10. Yang YB, Lee TY, Tsai IC. "Response of multi-degree-of-freedom structures with sliding supports." *Earthquake Engineering and Structural Dynamics* 1990; 19: 739-752.
11. Aluffi PF, Parisi V, Zirilli F. "A Differential-Equations Algorithm for Nonlinear Equations." *ACM Transactions on Mathematical Software* 1984; 10(3): 299-316.
12. Patro SK, Sinha R. "Influence of friction models on response of sliding-isolated structures." *Proceedings of National Seminar on Seismic Design of Nuclear Power Plants* 2003; 367-377.