



## JOINT PDF OF GROUND MOTION INTENSITY AND DURATION TIME BASED ON PSHA

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### SUMMARY

In this study, for more detailed probabilistic evaluation of the future ground motions, a method of bivariate probabilistic seismic hazard analysis which evaluates PGV and the duration time simultaneously is proposed through statistical analyses of dense strong motion records in Japan. This method was applied to the hazard analyses at some sites and the results were compared and discussed. It was concluded that the method makes it possible to estimate the probabilistic characteristics of future ground motions in more detail and has wider applications.

### INTRODUCTION

To accurately characterize seismic ground motions which are used for dynamic response analyses of buildings, it is essential to adequately represent the following four basic properties of earthquake ground motions; their maximum amplitude, spectral content, duration time and temporal change of envelop shape. It is possible to generate synthesized design ground motions by properly taking into the above basic properties. Although the current probabilistic seismic hazard analysis (PSHA) has given a design basis in terms of seismic hazard map or curves, it can estimate only PGA, PGV, spectral response, but not produce any information which is needed to capture all the above basic properties. Due to this shortcoming, design ground motions cannot be generated directly based on the results from the PSHA.

It is our goal to evaluate all of those four basic properties simultaneously based on the results from the PSHA. In the first step, it is shown in this paper that by focusing on PGV and the duration time of future ground motions, the joint probability density function (PDF) is formulated in the framework of the probabilistic seismic hazard analysis.

To evaluate the joint PDF, it is necessary to estimate each property correctly. So the estimate function of each property was studied by analyzing dense strong motion records of K-NET [1] and KiK-net observed in Japan [2] in this study. Usually, PGV and the duration time are estimated on the empirical basis that it is expressed as a function of earthquake magnitude, a distance between the earthquake source and the station and the soil condition of the station. Because the estimate equations have uncertainties, it is necessary to evaluate the variations of the equations correctly.

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Then, a method to carry out a bivariate PSHA which can predict the indices of PGV and the duration time simultaneously is suggested in this study. This method is applied to some sites and the numerical results are discussed. It was concluded that the bivariate PSHA of this study can play an essential role to predict future ground motions in more detail.

## EVALUATING MAXIMUM AMPLITUDE AND DURATION TIME

### Earthquake records for analyses

To evaluate the joint PDF of PGV and the duration time caused by an earthquake source, statistical analysis of dense strong motion records of K-NET and KiK-net observed in Japan is done herein. The earthquake records used for the analyses are indicated in Table 1.

A bandpass filter (0.2~10Hz) is applied to the acceleration records observed on the ground. Average S-wave velocity from the ground to 30 meters below the ground surface ( $AVS_{30}$ ) of each station was estimated from the soil information.

**Table 1 Earthquake records used for analyses**

No	Date	Source	$M_w$	$M_j$	Depth [km]	Source type	Number of Sites
1	1999/8/21	North Wakayama	5.6	5.5	70	Slab	125
2	2000/6/3	Northeast Chiba	6.1	5.8	50	Inter-plate	64
3	2000/7/21	Off Ibaraki	6.0	6.1	50	Inter-plate	155
4	2000/10/6	West Tottori	6.6	7.1	1	Crustal	188
5	2000/10/31	Central Mie	5.5	5.7	40	Slab	148
6	2001/3/24	Geiyo	6.8	6.4	40	Slab	229
7	2001/4/25	Hyuganada	5.7	5.6	30	Slab	166
8	2001/4/27	Southeast Off Nemuro Peninsula	6.0	5.9	80	Slab	57
9	2001/12/2	South Inland Iwate	6.5	6.3	130	Slab	141
10	2002/11/3	Off Miyagi	6.4	6.2	50	Inter-plate	134
11	2002/11/4	Hyuganada	5.7	5.6	30	Slab	170
12	2003/5/26	Off Miyagi	7.0	7.0	60	Slab	166
13	2003/7/26	North Miyagi	6.2	6.2	10	Crustal	183
14	2003/9/26	Off Tokachi	8.0	7.8	60	Inter-plate	190

### Index of maximum amplitude and its prediction

Peak ground velocity (PGV) was chosen as the index of maximum amplitude. The maximum value of two horizontal components is used for each station. An attenuation relationship of PGV on the engineering bedrock proposed by Midorikawa and Ohtake [3] is used for the prediction.

$$\log_{10} V = \begin{cases} b - \log_{10}(X + C) - kX & (D \leq 30\text{km}) \\ b + 0.6 \log_{10}(1.7 + C) - 1.6 \log_{10}(X + C) - kX & (D > 30\text{km}) \end{cases} \quad (1)$$

$$C = 0.0028 \times 10^{0.5M_w} \quad (2)$$

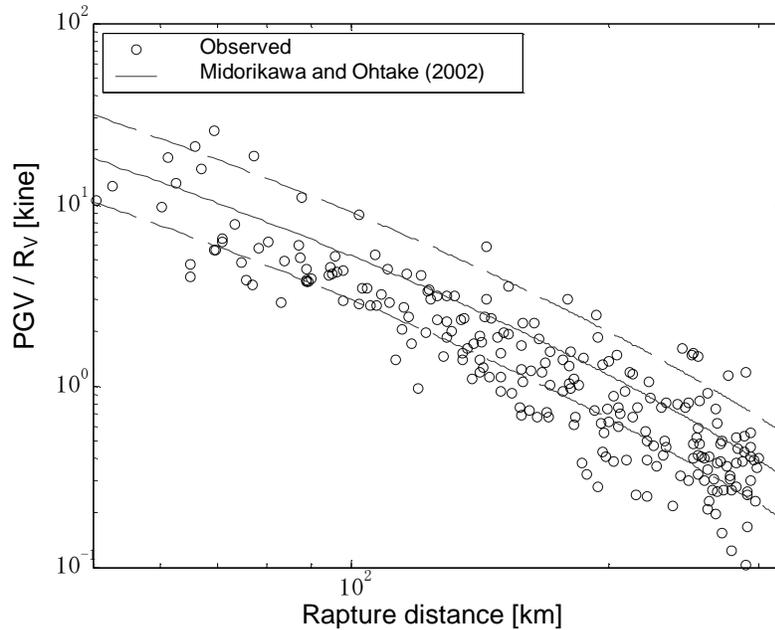
$$b = \begin{cases} 0.65M_w + 0.0024D - 1.77 & \text{(crustal earthquakes)} \\ 0.65M_w + 0.0024D - 1.72 & \text{(inter-plate earthquakes)} \\ 0.65M_w + 0.0024D - 1.62 & \text{(slab earthquakes)} \end{cases} \quad (3)$$

where  $V$  [kine] is the maximum velocity on the engineering bedrock,  $X$  [km] is the distance from the rupture surface,  $M_w$  is the moment magnitude,  $D$  [km] is the depth of the hypocenter, and  $k$  is a coefficient which represents the viscous damping effect. In this study,  $k$  of 0.002 is assumed according to Si and Midorikawa [4]. ‘The engineering bedrock’ means the hard soil whose average S-wave velocity is 600m/s here.

To convert a maximum velocity on the engineering bedrock into one on the ground surface, a relationship between the site amplification factor,  $R_v$ , and  $AVS_{30}$  (m/s) proposed by Midorikawa *et al.* [5] is used.  $AVS_{30}$  is the average S-wave velocity from the ground to 30 meters below the ground surface.

$$\log_{10} R_v = 1.83 - 0.66 \log_{10}(AVS_{30}) \quad (4)$$

This estimate method can produce the reasonable results to the observed values as shown in Figure 1. This figure demonstrates that the smaller PGV is observed, the longer the distance is.



**Figure 1 Relationship between the observed value and estimate equation of PGV. (2001 Geiyo Earthquake)**

### Index of duration time

There are some definitions of duration time of seismic wave. In this paper, a definition proposed by Satoh *et al.* [6] has been chosen. Satoh *et al.* have defined the duration time of the principal shock and of the decaying part as below.

First, for each  $t$  at intervals of 0.2 second, the maximum value of absolute amplitude between the period from  $t-0.2$ (s) to  $t+0.2$ (s) is extracted from an observed acceleration record.

Then a Jennings-type envelope function, eq (5), is fitted to the observed envelope.

$$E(t) = \begin{cases} 0 & 0 \leq t \leq t_a \\ A(t-t_a)^2 / (t_b-t_a)^2 & t_a \leq t \leq t_b \\ A & t_b \leq t \leq t_c \\ A \exp\{-B(t-t_c)\} & t_c \leq t \leq t_d \end{cases} \quad (5)$$

where  $t_a$ ,  $t_b$ ,  $t_c$  and  $t_d$  are the time parameters of Jennings-type envelope function.  $A$  is the amplitude of the principal shock, and  $B$  is the parameter of decaying characteristics.

In addition, following conditions have introduced by Kamada *et al.* [7]

$$t_b \leq t_{A_{\max}} \leq t_c \quad (6)$$

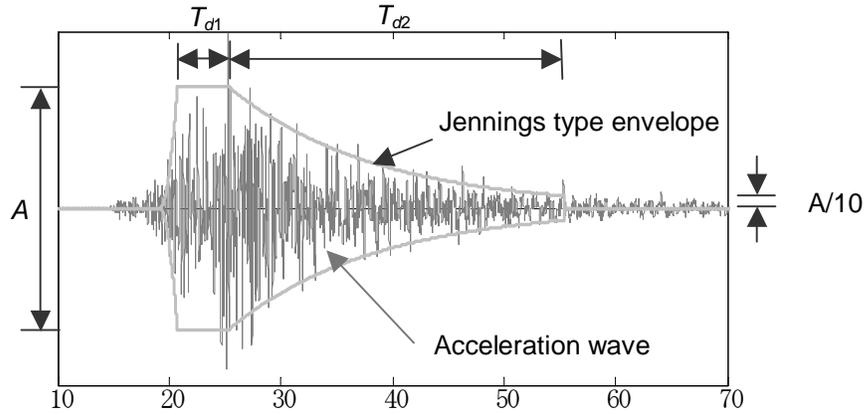
$$t_b - t_a = (t_c - t_b) / 3 \quad (7)$$

$$0.7A_{\max} \leq A \leq A_{\max} \quad (8)$$

where  $A_{\max}$  is the maximum value of the observed envelope and  $t_{A_{\max}}$  is the time instant of  $A_{\max}$ .

The parameters of Jennings-type envelope, which meet the conditions shown above and fit the observed envelope best, are identified by a non-linear least-square method.

Finally,  $t_c - t_a$  and  $t_d - t_c$  are defined as the duration time of principal shock,  $T_{d1}$ , and the duration time of decaying part,  $T_{d2}$ .



**Figure 2** Definition of duration time,  $T_{d1}$  and  $T_{d2}$ .

### Prediction for duration time

Estimate equations of duration time on the engineering bedrock proposed by Kamada *et al.* [7] are used for slab earthquakes and for inland earthquakes.

(for slab earthquakes)

$$\log_{10} T_{d1} = 0.236M_j + 0.931 \log_{10} X - 2.614 \quad (9)$$

$$\log_{10} T_{d2} = 0.059M_j + 1.080 \log_{10} X - 1.411 \quad (10)$$

(for inland earthquakes)

$$\log_{10} T_{d1} = 0.162M_j + 0.506\log_{10} X - 1.421 \quad (11)$$

$$\log_{10} T_{d2} = 0.092M_j + 0.607\log_{10} X - 0.610 \quad (12)$$

where  $M_j$  is the JMA magnitude and  $X$ [km] is the distance from a hypocenter.

For inter-plate earthquakes, the above equations proposed by Kamada *et al.* have a tendency to underestimate the duration time. So the equations to be used in this study are built up by modifying their equations as follows.

(for inter-plate earthquakes)

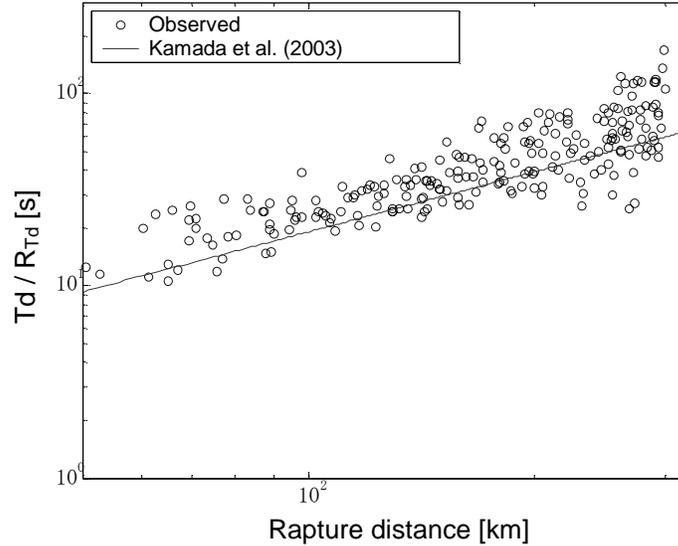
$$\log_{10} T_{d1} = 0.236M_j + 0.931\log_{10} X - 2.463 \quad (13)$$

$$\log_{10} T_{d2} = 0.059M_j + 1.080\log_{10} X - 1.260 \quad (14)$$

To convert the duration time on the engineering bedrock into that on the ground, a relationship between the site amplification factor,  $R_{Td}$ , and  $AVS_{30}$  was proposed in this study from a regression analysis of records.

$$R_{T_d} = -0.177 \times \ln(AVS_{30} / 600) \quad (15)$$

Hereafter, the duration time  $T_d$  represents the total duration time, i.e.  $T_{d1} + T_{d2}$ . Of course, each part of duration time can be evaluated separately in a similar way.



**Figure 3 Relationship between the observed value and prediction of the duration time (2001 Geiyo Earthquake)**

## SIMULTANEOUS EVALUATION OF MAXIMUM AMPLITUDE AND DURATION TIME

### Simultaneous evaluation model

To evaluate the joint PDF of PGV and duration time at a station from an earthquake source, the observed value of PGV,  $V_{observed}$ , and the observed value of the duration time,  $T_{d\ observed}$ , were modeled as follows.

$$V_{observed} = V(M, X) \times \mathcal{E}_{V_{INTER}} \times \mathcal{E}_{V_{INTRA}} \quad (16)$$

$$T_{d \text{ observed}} = T_d(M, X) \times \mathcal{E}_{T_d \text{ INTER}} \times \mathcal{E}_{T_d \text{ INTRA}} \quad (17)$$

where  $M$  is the magnitude of the source and  $X$  is the distance from the source.  $V(M, X)$  and  $T_d(M, X)$  are the values from the above equations at the station.  $\mathcal{E}_{V \text{ INTER}}$  and  $\mathcal{E}_{T_d \text{ INTER}}$  are the inter-event variation components which follow a bivariate lognormal distribution and have the inter-event correlation,  $\rho_{\text{INTER}}$ . In the same way,  $\mathcal{E}_{V \text{ INTRA}}$  and  $\mathcal{E}_{T_d \text{ INTRA}}$  are the intra-event variation components which follow a bivariate lognormal distribution and have the intra-event correlation,  $\rho_{\text{INTRA}}$ . It is assumed that the median value of each variation component is unity.

Finally, this model makes it possible to evaluate the joint PDF of PGV and duration time if only the standard deviation of each variation and inter-event and intra-event correlations above are given.

### Intra-event variation and correlation

For each earthquake record, an analysis about intra-event variations,  $\mathcal{E}_{V \text{ INTRA}}$  and  $\mathcal{E}_{T_d \text{ INTRA}}$ , is done in this section. The results of analyses are tabulated in Table 2.

**Table 2 Results of analyses on uncertainties**

Variation		Standard deviation (natural logarithm)	Coefficient of correlation
Intra-event	PGV $\mathcal{E}_{V \text{ INTRA}}$	0.5 ~ 0.6	$\rho_{\text{INTRA}} = -0.2 \sim -0.4$
	Duration $\mathcal{E}_{T_d \text{ INTRA}}$	0.3 ~ 0.4	
Inter-event	PGV $\mathcal{E}_{V \text{ INTER}}$	0.406	$\rho_{\text{INTER}} = -0.287$
	Duration $\mathcal{E}_{T_d \text{ INTER}}$	0.212	

Though the result varies earthquake by earthquake, it is obtained that the standard deviation of the intra-event variation of PGV  $\mathcal{E}_{V \text{ INTRA}}$  and the duration time  $\mathcal{E}_{T_d \text{ INTRA}}$  are about 0.5 ~ 0.6 and 0.3 ~ 0.4 and the correlation coefficient of the two intra-event variation components  $\rho_{\text{INTRA}}$  is about -0.2 ~ -0.4. For most of earthquakes, the correlation coefficient  $\rho_{\text{INTRA}}$  is not notable.

### Inter-event variation and correlation

Then an analysis about the inter-event variations is done. The result of analysis is also shown in Table 2. It is derived from the simultaneous evaluation model that for each earthquake and each index of ground motions, the mean value of the logarithm residual values of all observing stations can be treated as the component of the inter-event variation. The correlation coefficient for the two inter-event variation components  $\rho_{\text{INTER}}$  is -0.29, which is not notable either.

## METHOD OF BIVARIATE PSHA

### Outline of bivariate PSHA

An outline of the method of bivariate PSHA proposed in this study is shown in Table 3. This method is developed by adding the simultaneous evaluation of two indices of this study to the existing uni-variate evaluation method of invented by Cornell[8]. Earthquake sources are categorized into three source types:

active faults, inter-plate earthquakes and random earthquakes. The standard deviations of the estimate equations are set considering the result of this study and past literature.

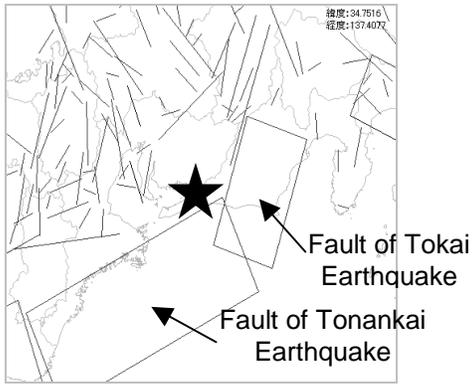
**Table 3. Outline of bivariate PSHA**

Source		Active faults	Inter-plate earthquakes	Random earthquakes
Dataset		Based on Matsuda <i>et al.</i> [9]	Off Miyagi, Nankai, Tounankai, Toukai and Kantou Earthquake	Based on Matsuda <i>et al.</i> [9]
Magnitude and frequency		Inherent magnitude and recurrence interval for each source		Gutenberg-Richter equation ( <i>b</i> -value model)
Process		A model concerning elapsed time from last activity or Stationary Poisson process	A model concerning elapsed time from last activity	Stationary Poisson process
Equation	PGV	Bedrock	Midorikawa and Ohtake [3]	
		Ground	Midorikawa <i>et al.</i> [5]	
		S.D.	0.41 (natural logarithm)	
	Duration Time	Bedrock	Kamada <i>et al.</i> [7]	
		Ground	This study: eq. (15)	
		S.D.	0.64 (natural logarithm)	

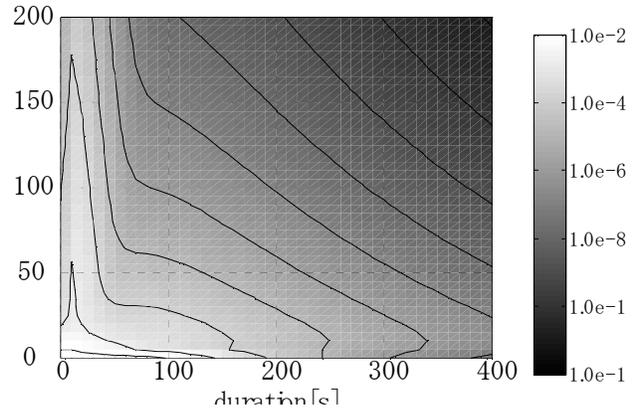
## RESULTS

### Comparison of source types

The bivariate PSHA is applied to the hazard analysis at Toyohashi, Aichi. It is assumed that two major inter-plate earthquake sources, Tokai Earthquake (M8.4) and Tonankai Earthquake (M8.1), exist around Toyohashi as indicated as a star symbol in Figure 4. The evaluated joint PDF of the duration time and PGV at Toyohashi is indicated in Figure 5. The evaluation period is 50 years.

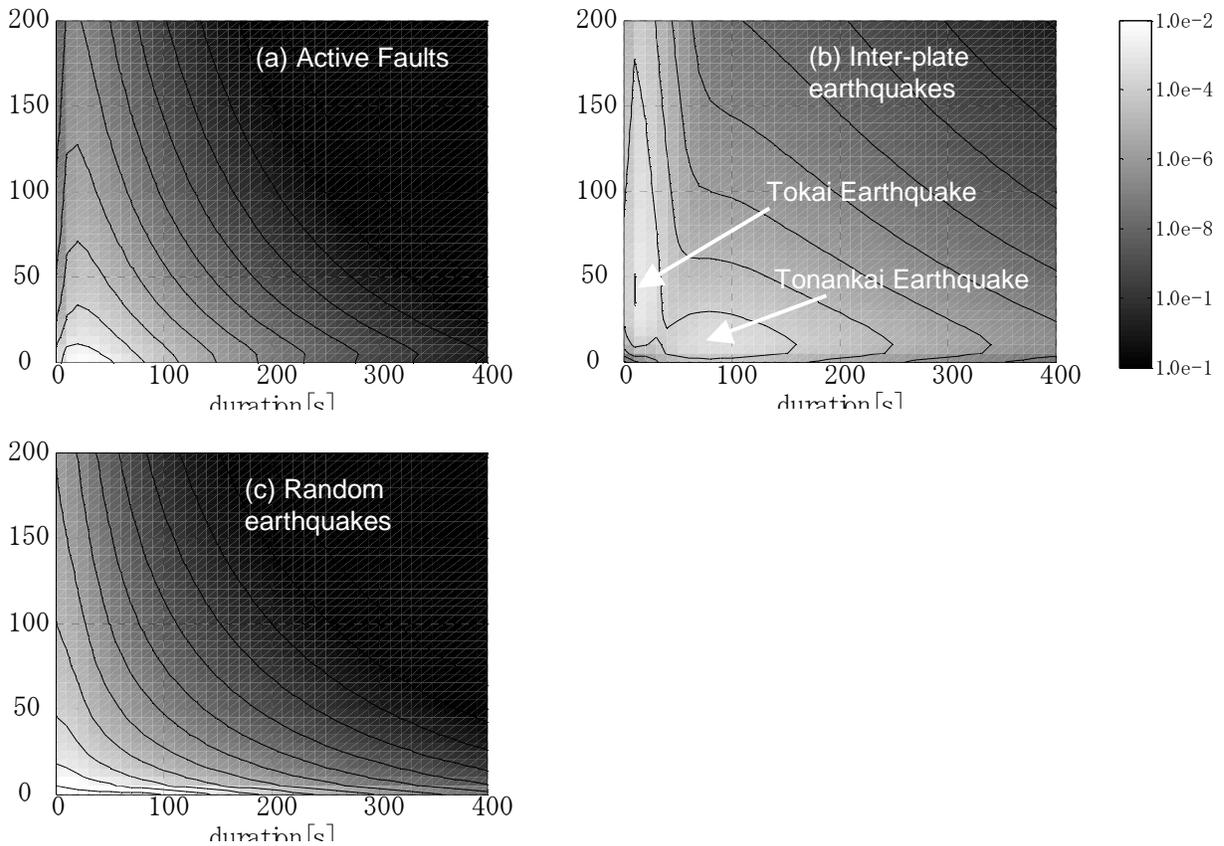


**Figure 4. Earthquake sources around Toyohashi, Aichi**



**Figure 5. Joint PDF at Toyohashi, Aichi (50 years)**

To discuss the difference between source types, the joint PDFs caused by the three source types were evaluated separately and shown in Figure 6.



**Figure 6. Joint PDFs of three source types (Aichi, 50 years)**

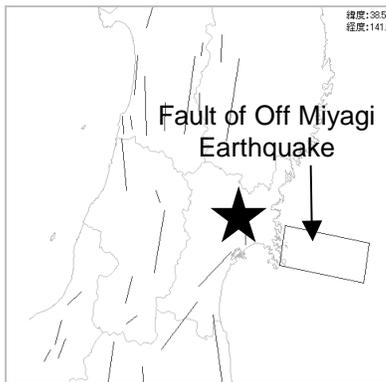
It is distinctive that the joint PDF caused by inter-plate earthquakes has two major peaks corresponding to the two major inter-plate earthquake sources, Tokai Earthquake and Tonankai Earthquake. It is clear that

the inter-plate earthquakes have the most significant impact on the total result PDF of three source types. The joint PDF from active faults have relatively high probability in the area which has short duration time and high PGV. On the other hand, the joint PDF from random earthquakes has relatively high probability in the area where long duration time and high PGV are observed.

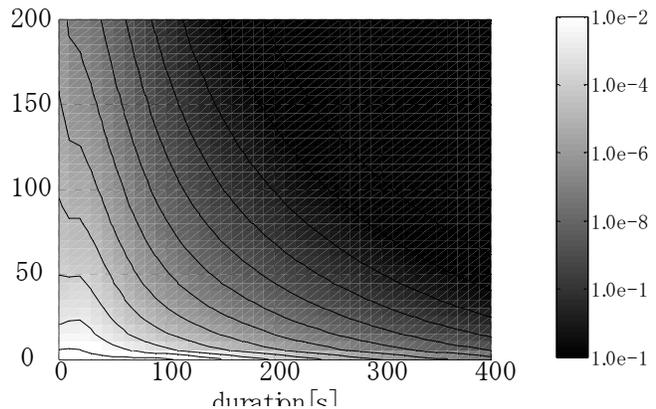
In this way, the joint PDF of PGV and duration time of future ground motions can be evaluated and the characteristics of probabilistic future ground motions can be discussed in more detail by using the bivariate PSHA method proposed in this study.

### Comparison of focused sites

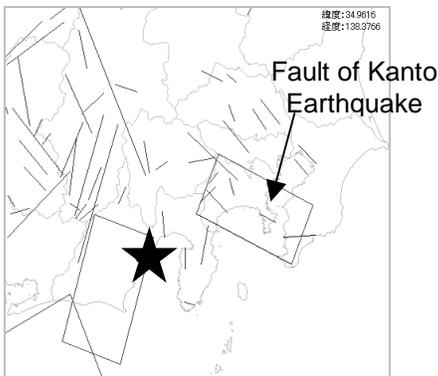
To discuss the difference between the results of different sites, the method is applied to the hazard analyses of some sites in addition to Toyohashi. The examples of evaluated sites and results are shown from Figures 7 to 12. Distinctive site-specific characteristics are shown.



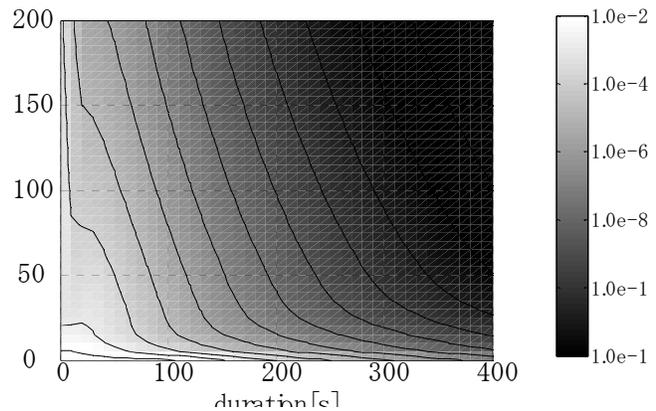
**Figure 7. Earthquake sources around Tajiri, Miyagi**



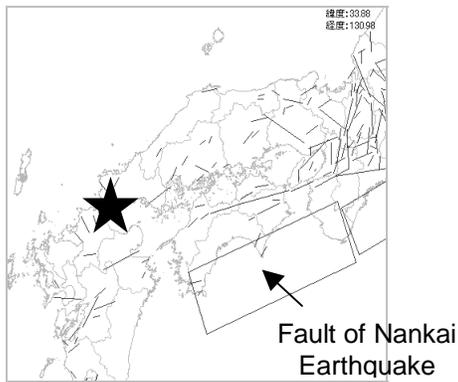
**Figure 8. Joint PDF at Tajiri, Miyagi (50years)**



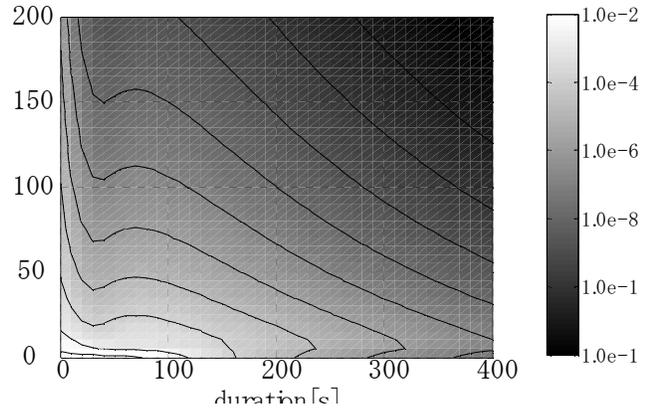
**Figure 9. Earthquake sources around Shizuoka, Shizuoka**



**Figure 10. Joint PDF at Shizuoka, Shizuoka (50years)**



**Figure 11. Earthquake sources around Kitakyushu, Fukuoka**



**Figure 12. Joint PDF at Kitakyushu, Fukuoka (50years)**

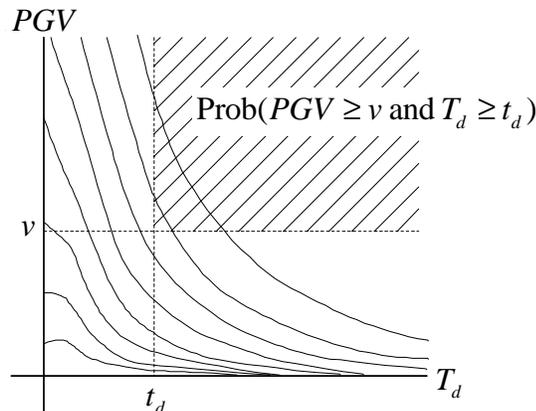
At Tajiri and Shizuoka, the joint PDFs have relatively high probability in the area which has short duration time and high PGV because of nearby major inter-plate earthquakes. On the other hand, the joint PDF has relatively high probability in the area which has long duration and low PGV at Kitakyushu because it is assumed that no major earthquake source exists nearby and that Nankai Earthquake exists far away from the site. At Toyohashi, the joint PDF has the combination characteristics of both sides. As observed above, the site-specific characteristics of the probabilistic future ground motions can be discussed in more detail by using the bivariate PSHA method of this study.

### How to use the joint PDF

Once the joint PDF of PGV and duration time is evaluated at a particular site, it can be conveniently used to characterize ground motions in a seismic resistant design.

Using specified probability that the PGV exceeds certain velocity  $v$  and the duration time exceed a certain time  $t_d$  as shown in **Figure 13**. Design parameters for PGV and duration time can be determined on the basis of probability.

Selection of the specified probability should be made from acceptable risk of facilities to be designed and is beyond the scope of this study. Furthermore, note that there are many combinations of  $v$  and  $t_d$  under the condition of the fixed exceedance probability. There may be possible methods for identifying the two parameters under condition. This will be reported elsewhere.



**Figure 13. Schematic illustration to estimate exceedance probability**

## CONCLUSION

In this study, for more detailed evaluation of the probabilistic future ground motions, a method of bivariate probabilistic seismic hazard analysis of PGV and the duration time was proposed through statistical analyses of dense strong motion records in Japan. Then this method was applied to the hazard analyses at some sites and the results were compared and discussed. As the results, following conclusions are obtained.

- 1) PGV and the duration time of the observed records can be estimated by existing prediction equations from the past literature with some modification.
- 2) The correlation between the variations of PGV and the duration time can be negligible.
- 3) The method of bivariate PSHA of this study makes it possible to evaluate the joint PDF of PGV and the duration time of the probabilistic future ground motions.

A multi variate PSHA like this has wide applications. For example, generating more detailed design ground motions for dynamic design of buildings, more detailed risk analyses which take into account the site-specific characteristics, and so on.

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