NONLINEAR ANALYSIS OF FRAME SYSTEMS BY STATE SPACE APPROACH (SSA)

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SUMMARY

This paper presents an alternative approach to the formulation and solution of frame structures involving inelastic-nonlinear distributed-parameter structural systems. The response of the structure, which is spatially discretized, is completely characterized by a set of state variables which represent global nodal displacements and velocities, and local element quantities such as forces and strains at seleced sections used as the integration points. The evolution of the global state variables is governed by physical principles, such as momentum balance or dynamic equilibrium. The evolution of the local variables is governed by constitutive behavior. The essence of the proposed approach is to solve the two sets of evolution equations simultaneously in time using direct numerical methods, in general as a system of differential-algebraic equations. This is in contrast to the common approach of formulating the equations of motion and the constitutive equations in an incremental form and solving them separately using finite-difference methods with iterative correction. The proposed methodology results in a more consistent formulation with a clear distinction between spatial and temporal discretization. A nonlinear beam element based on force interpolation functions and a constitutive macro-model is developed and presented in this framework. The state-space formulation and the nonlinear bending element, in particular, are compared witht benchmark solutions using commonly used approaches.

INTRODUCTION

The most commonly used strategy for nonlinear analysis is based on formulating the equations of motion and the constitutive equations in incremental form and using finite difference methods for integration in time. The governing differential equations are linearized over a time step by using the tangent properties of the system. Methods such Newmark's are used to obtain the equivalent tangent properties for dynamic systems. Iteration within an analysis increment is necessary to reduce the error introduced by the use of the tangent properties.

Another approach is to solve simultaneously the equations of motion and the constitutive equations. For a dynamic problem involving elements whose nodal force-deformations relationships are known directly, separating the linear and nonlinear components of the resistance force and introducing velocities as additional unknowns results in a set of explicit first-order ordinary differential equations (ODE). The system can be solved using any appropriate numerical method. This approach has been extensively employed in the solution of purely dynamic linear and non-linear problems especially in structural control and non-deterministic analysis. However, in the general case, when there are quasi-static degrees of freedom (i.e., the mass matrix is singular) and elements for which only stress-strain (and not force-displacement) relations are known, the resulting system of equations not only consists of explicit ODE's but also contains implicit ODE's and algebraic equations. The numerical solution of such systems of Differential-Algebraic Equations (DAE) is more complex than the solution of ODE's and reliable methods for this purpose have been developed only recently (Brenen et al. (1996). The state-space approach (SSA) involving DAE's has been used extensively in multi-body dynamics of aerospace and mechanical assemblies. A detailed review of the previous applications is presented by Simeonov

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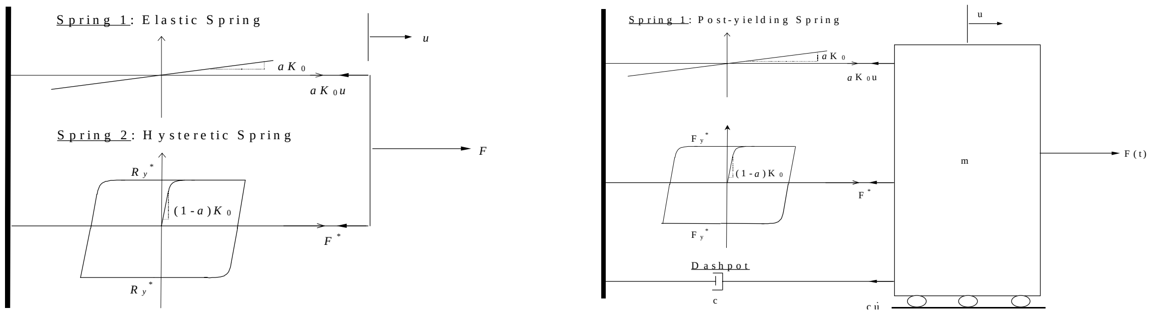
(1999). The objective of this paper is to formalize this approach using state variables for complex inelastic structures subjected to earthquakes and other dynamic and static

CONSTITUTIVE MODELS FOR STATE-SPACE ANALYSIS

Numerous constitutive models have been developed based on the internal-variable theory of inelastic material behavior Sivaselvan et al (1999). One such model developed originally by Bouc and Wen, subsequently modified by Sivaselvan et al. (1999) is used here for its simplicity and numerical tractability. The generalized resistance force R is modeled as a combination of elastic and hysteretic components.

$$\dot{R} = \left[a\,K_0 + (1-a)\,K_H \right]\dot{u}\,; \qquad K_H = K_0 \left\{ 1 - \left| \frac{R^*}{R_y^*} \right|^n \left[\eta_1\, sgn\left(R^*\,\dot{u}\right) + \eta_2 \right] \right\} \tag{1}$$

where K_0 = initial stiffness; a = ratio of post-yield to elastic stiffness; K_H = hysteretic stiffness; u = total generalized displacement, and where $R^* = R - a\,K_0\,u$ is the force in the hysteretic spring; $R_y^* = (1-a)R_y$ is the yield force of the hysteretic spring; R_y = total yield force; n = parameter controlling the transition between the elastic and plastic range; η_1 and η_2 = parameters controlling the shape of the hysteretic loop, which must fulfil the condition $\eta_1 + \eta_2 = 1$.



(a) Schematic Representation of Constitutive Model (b) Dynamic System with Hysteretic Restoring Force

Fig. 1 - Inelastic Nonlinear Systems

STATE VARIABLES AND EQUATIONS OF A SINGLE-DEGREE-OF-FREEDOM SYSTEM

A nonlinear single-degree-of-freedom (SDOF) system, subjected to dynamic and quasi-static forces, will be used to illustrate the state-space formulation. The model in Fig. 1(a) has three variables therfore three state equations:

$$y_1 = u \;\;,\;\; y_2 = \dot{u} \;\;,\;\; y_3 = R \tag{2a}$$

Then, for the dynamic problem (Fig (1b)) the response is described by,

$$m\,\dot{y}_2 + c\,\dot{y}_1 + y_3 - F = 0\,; \qquad y_2 - \dot{y}_1 = 0\,; \qquad \dot{y}_3 - \left[a\,K_0 + (1-a)\,K_H \right]\dot{y}_1 = 0 \tag{2b}$$

For this system, y_3 is the local and y_1 and y_2 are the global state variables. Correspondingly Eq. (2b) provides global and the local state equation. The hysteretic stiffness is obtained by reordering Eq (1):

$$R^* = y_3 - a\,K_0\,y_1; \qquad K_H = K_0 \left\{ 1 - \left| \frac{y_3 - a\,K_0\,y_1}{R_y^*} \right|^n \left[\eta_1\, sgn\left(y_3\,y_2 - a\,K_0\,y_1\,y_2 \right) + \eta_2 \right] \right\} \tag{2c}$$

In contrast with the dynamic system Eq. (2b) a quasi-static system subjected to identical force history, has only two state variables, hence, two state equations.

Then, the response of quasi-static SDOF system is described by:

$$y_1 = u \;,\ldots\, y_2 = R\,; \qquad y_2 - F = 0\,; \qquad \dot{y}_2 - \left[a\,K_0 + (1-a)\,K_H \right]\dot{y}_1 = 0 \tag{3}$$

In this case, the constitutive Eq.(3) has an implicit differential equation (latter) with equation of equilibrium (the formar) being algebraic. Therefore, a set of differential-algebraic equations must be solved to obtain the quasi-static response of SDOF system with nonlinear restoring force.

DIFFERENTIAL-ALGEBRAIC SYSTEMS (DAS)

A DAS is a coupled system of *N* ordinary differential and algebraic equations, which can be written in the form:

$$\mathbf{\Phi}(t,\mathbf{y},\dot{\mathbf{y}})=\mathbf{0} \tag{4}$$

where $\mathbf{\Phi}$, $\mathbf{y}$ and $\dot{\mathbf{y}}$ are *N*-dimensional vectors; *t* is the independent variable; $\mathbf{y}$ and $\dot{\mathbf{y}}$ are the dependent variables and their derivatives with respect to *t*.

Some of the equations in (4) however, may not have a corresponding component of $\dot{\mathbf{y}}$. Consequently, the matrix

$$\frac{\partial \mathbf{\Phi}}{\partial \dot{\mathbf{y}}}=\left[\frac{\partial \Phi_i}{\partial \dot{y}_j}\right] \tag{5}$$

may be singular. A measure of the singularity is the *index* (Brenan, 1996)]. This, is equal (for simplicity) to the minimum number of times Eq. (4) must be differentiated to determine $\dot{\mathbf{y}}$ explicitly as functions of $\mathbf{y}$ and *t*.

The explicit ODE system , $\dot{\mathbf{y}}=\mathbf{g}(t,\mathbf{y})$, has an index of 0. For example,Eq. (2b) can be converted to the standard form without additional differentiation.

$$\dot{y}_2=\frac{F-c\,y_2-y_3}{m}\,;\,\dot{y}_1=y_2\,;\ldots\ldots\ldots\ldots..$$

$$\ldots.\,\dot{y}_3=a\,K_0\,y_2+(1-a)K_0\left\{1-\left|\frac{y_3-a\,K_0\,y_1}{R_y^*}\right|^n\left[\eta_1\,sgn(y_3\,y_2-a\,K_0\,y_1\,y_2)+\eta_2\right]\right\}y_2 \tag{6}$$

Eq. (3) modeling the quasi-static response of SDOF system, however, is index 1, because the algebraic equation must be differentiated *once* to obtain:

$$\dot{y}_2=\dot{F}\,;\qquad \dot{y}_1=\frac{\dot{F}}{a\,K_0+(1-a)K_0\left\{1-\left|\frac{y_2-a\,K_0\,y_1}{R_y^*}\right|^n\left[\eta_1\,sgn(y_2\,\dot{y}_1-a\,K_0\,y_1\,\dot{y}_1)+\eta_2\right]\right\}} \tag{7}$$

The numerical solution of DAE is more involved than the solution of ODE. A summary of the integration method in comercial libraries such as DASSL is provided by Simeonov (1999)

STATE VARIABLES AND EQUATIONS OF A MULTI-DEGREE-OF-FREEDOM SYSTEM

The equations of equilibrium of a multi-degree-of-freedom (MDOF) system can be written as:

$$\mathbf{M}\,\ddot{\mathbf{u}}(t)+\mathbf{C}\,\dot{\mathbf{u}}(t)+\mathbf{R}(t)=\mathbf{F}(t) \tag{8}$$

where, $\mathbf{M}$ = mass matrix; $\mathbf{u}(t)$ = displacement vector; $\mathbf{C}$ = damping matrix; $\mathbf{R}(t)$ = resistance force vector; $\mathbf{F}(t)$ = forcing vector and '·' denotes time derivative.

Global State Variables: In the general case, the set of global state variables of the system consists of three parts: (i) Generalized displacements along all *free* nodal degrees of freedom. Displacements along constrained generalized coordinates are excluded by virtue of imposing boundary conditions; (ii) Generalized displacements along degrees of freedom with imposed displacement histories. This occurs, for example, when support displacements due to settlement or earthquake motion are prescribed and in displacement-controlled laboratory testing; (iii) Velocities along mass degrees of freedom. The number of velocity state variables may be less than the number of displacement variables because often, rotational and even some translational mass components, are ignored if their effect is presumed negligible.

Local State Variables: The local state variables describe the evolution of individual elements. These consist of (i) Independent element end forces; (ii) Constitutive variables, such as stresses or strains at the integration points, which may be required to characterize inelasticity; (iii) Any other internal variable that may govern the behavior of the element (e.g. yield stresses, back-stress, etc.)

Force Vectors: The element end forces, which are state variables, are transformed to global coordinates by regular local-to-global transformations to obtain their contributions to the global resistance vector. This is assembled by using node-number and element connectivity information. The forcing vector is assembled from the applied nodal forces. In this context, the formation of a global stiffness matrix becomes unnecessary.

State Equations The three sets of global state equations can be summarized as follows:

$$\mathbf{M}\,\dot{\mathbf{y}}_{\mathbf{2}\,(1:ND)} + \mathbf{C}\,\dot{\mathbf{y}}_{\mathbf{1}\,(1:ND)} + \mathbf{R} - \mathbf{F} = \mathbf{0}\,;\, \mathbf{y}_{\mathbf{1}\,(ND+1:ND+NDH)} - \mathbf{d}_{(1:NDH)} = \mathbf{0}\,;\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots\,\mathbf{y}_{\mathbf{2}\,(ND+NDH+1:ND+NDH+NV)} - \dot{\mathbf{y}}_{\mathbf{1}\,(1:NV)} = \mathbf{0} \qquad (9)$$

where, $\mathbf{y_1} = \mathbf{u}(t)$, $\mathbf{y_2} = \dot{\mathbf{u}}(t)$, $\mathbf{d}$ = prescribed displacement history vector, *ND* = number of unconstrained DOF, *NDH* = number of DOF with specified displacement histories, *NV* = number of DOF with mass and *NDOF* = total number of DOF. The mass and damping matrices in Eq. (28) have been condensed from their original dimensions ($NDOF\times NDOF$) to ($ND\times ND$), while $\mathbf{y_2}$ has been expanded from *NV* to *ND*.

The state of each nonlinear element is defined by evolution equations involving the end forces, displacements and the internal variables used in the formulation of the element model. These equations are of the form:

$$\dot{\mathbf{R}}_\mathbf{e} = \mathbf{G}(\mathbf{R_e}, \mathbf{u_e}, \dot{\mathbf{u}}_\mathbf{e}, \mathbf{z_e}, \dot{\mathbf{z}}_\mathbf{e})\,; \qquad \dot{\mathbf{z}}_\mathbf{e} = \mathbf{H}(\mathbf{z_e}, \mathbf{R_e}, \dot{\mathbf{R}}_\mathbf{e}, \mathbf{u_e}, \dot{\mathbf{u}}_\mathbf{e}) \qquad (10)$$

where, $\mathbf{G}$ and $\mathbf{H}$ are nonlinear functions, $\mathbf{R_e}$ are the independent element end forces, $\mathbf{u_e}$ are the displacements of the element nodes and $\mathbf{z_e}$ are the internal variables. The formulation of these equations for a beam element is illustrated in the next section.

FORMULATION OF A FLEXIBILITY-BASED PLANAR BEAM ELEMENT

Constitutive Relations

Direct relationships between the end force and displacement rates are not typically available for line elements subjected to a combination of bending, shear and axial load. In the present formulation, the nonlinear *moment-curvature* relationships of cross sections at discrete locations along the element axis are chosen to represent constitutive laws for bending response. Eq. (1) is used to represent this constitutive macro-model. This model can simulate smooth transition from elastic to inelastic behavior due to distributed yielding of the section as well as cyclic degradation behavior Sivaselvan et al. (1999). Detailed treatment of axial force-bending moment interaction in 2D and biaxial moment interaction in 3D space is presented by Simeonov (1999)..

State Variables and State Equations

The beam element, like any other frame element, is internally statically determinate. Therefore, a flexibility formulation using force interpolation functions is utilized here. The displacement interpolation functions used in the regular stiffness-based formulations are exact only for elastic prismatic members. In contrast, the force interpolation functions, which are statements of equilibrium, are always exact. The state variables of the beam element are defined as the *independent end forces* and the *curvatures* at sections located at the quadrature points:

$$\mathbf{y}_{\mathbf{e}\,(1:3)} = \overline{\mathbf{R}}_\mathbf{e}\,; \qquad \mathbf{y_e}\,(4{:}3{+}\mathrm{NG}) = \boldsymbol{\varphi} \qquad (11)$$

where **ye** = element state vector, $\overline{\mathbf{R}}_\mathbf{e} = \{F_i\ M_i\ M_j\}^T$ = independent end forces, including the axial force at one end and the bending moments at both ends; $\boldsymbol{\varphi}$ = $\{\varphi_1\ \ \varphi_2\ \ldots\ \varphi_{NG}\}T$ = section curvatures at *NG* quadrature points.

The state equations define the constitutive laws of the element and the individual sections:

$$\mathbf{F_e}\,\dot{\overline{\mathbf{R}}}_\mathbf{e} = \dot{\overline{\mathbf{u}}}_\mathbf{e}\,; \qquad \dot{\mathbf{M}} = \left[\mathbf{a}\,\mathbf{K_0} + (\mathbf{I} - \mathbf{a})\,\mathbf{K_H}\right]\dot{\boldsymbol{\varphi}} \qquad (12)$$

where, $\mathbf{F_e}$ = element flexibility matrix of size 3×3, the derivation of which is described later,

$\bar{\mathbf{u}}_\mathbf{e} = \{\bar{u} \;\; \bar{\theta}_i \;\; \bar{\theta}_j\}^T$ = axial deformation and chord rotations at the ends, **I** = *NG*×*NG* identity matrix, **a** = $diag[a_1 \;\; a_2 \ldots a_{NG}]$ = ratio of post-yield to elastic stiffness, $\mathbf{K_0} = diag[K_{0,1} \;\; K_{0,2} \ldots K_{0,NG}]$ = elastic stiffness, $\mathbf{K_H} = diag[K_{H,1} \;\; K_{H,2} \ldots K_{H,NG}]$ = hysteretic stiffness and $\mathbf{M} = \{M_1 \;\; M_2 \ldots M_{NG}\}^T$ = total bending moments at the quadrature locations.

The latter are given by

$$\mathbf{b_G}\,\dot{\mathbf{M}}_\mathbf{e} = [\mathbf{a}\,\mathbf{K_0} + (\mathbf{I}-\mathbf{a})\,\mathbf{K_H}]\,\dot{\boldsymbol{\varphi}} \tag{13}$$

where $\mathbf{M_e} = \{M_i \;\; M_j\}^T$ = element end moments and $\mathbf{b_G}$ = moment interpolation matrix.

The element end displacements, which are global state variables, are transformed to the element deformations:

$$\dot{\bar{\mathbf{u}}}_\mathbf{e} = \mathbf{T_e}\mathbf{T_g}\dot{\mathbf{u}}_\mathbf{e}^\mathbf{g} \tag{14}$$

where $\mathbf{T_e}$, a 3x6, $\mathbf{T_g}$ = 6×6 global to local displacement transformation matrices and $\mathbf{u}_\mathbf{e}^\mathbf{g}$ = {*ui vi θi uj vj θj*}*T* = element end displacements in global coordinates.

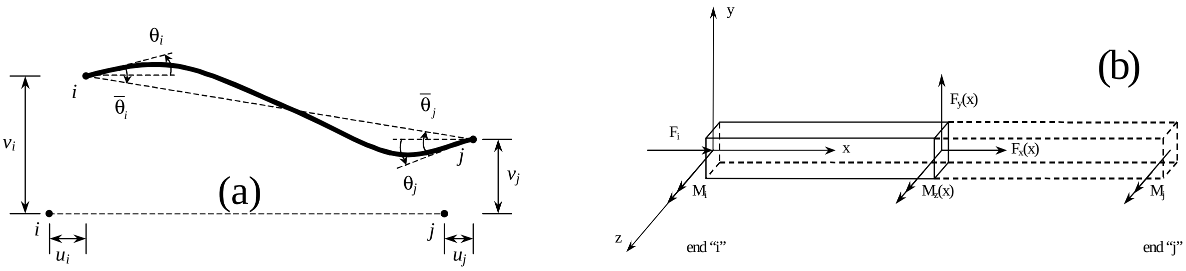


Fig. 2: (a) Element End Displacements and Deformation, (b) Element Coordinates

Combining Eqs. (12) and (14) renders the formulation of the element force-displacement relation:

$$\dot{\mathbf{y}}_{\mathbf{e}\,(1:3)} - \mathbf{K_e}\,\mathbf{T_e}\,\mathbf{T_g}\,\dot{\mathbf{u}}_\mathbf{e}^\mathbf{g} = \mathbf{0} \quad ; \quad \mathbf{b_G}\,\dot{\mathbf{y}}_{\mathbf{e}\,(2:3)} - [\mathbf{a}\,\mathbf{K_0} + (\mathbf{I}-\mathbf{a})\,\mathbf{K_H}]\dot{\mathbf{y}}_{\mathbf{e}\,(4:3+NG)} = \mathbf{0} \tag{15}$$

These equations establish one of the necessary links between local and the global state variables. The other link is the contribution of the element end forces to the global resistance vector. The full set of end forces, $\mathbf{R_e}$, is generated by equilibrium transformations of the independent forces $\bar{\mathbf{R}}_\mathbf{e} : \mathbf{R_e} = \mathbf{T}_\mathbf{e}^T\,\bar{\mathbf{R}}_\mathbf{e}$, where $\mathbf{R_e} = \{F_i \;\; V_i \;\; M_i \;\; F_j \;\; V_j \;\; M_j\}^T$. The element contribution $\mathbf{R}_\mathbf{e}^\mathbf{g}$ to the global restoring force vector **R** is obtained by local-to-global transformation of the end forces.

$$\mathbf{R}_\mathbf{e}^\mathbf{g} = \mathbf{T}_\mathbf{g}^T\,\mathbf{T}_\mathbf{e}^T\,\bar{\mathbf{R}}_\mathbf{e} \tag{16}$$

Element Flexibility Matrix

Compatibility of deformation within the element may be expressed in weak form using the principle of virtual forces as,

$$\delta\mathbf{R}_\mathbf{e}^T\,\dot{\mathbf{u}}_\mathbf{e} = \int_0^L \delta\mathbf{R}_\mathbf{e}(x)^T\,\dot{\boldsymbol{\varepsilon}}(x)\,dx \tag{17}$$

where, $\boldsymbol{\varepsilon}(x) = \{\varepsilon(x) \;\; \gamma(x) \;\; \phi(x)\}^T$ = section axial strain, shear strain and curvature and $\delta\mathbf{R}_\mathbf{e}$ = virtual forces. The forces at any section may be obtained from equilibrium using the force interpolation functions as,

$$\dot{\mathbf{R}}_\mathbf{e}(x) = \mathbf{b}(x)\,\dot{\bar{\mathbf{R}}}_\mathbf{e} \tag{18}$$

where, $\mathbf{R_e}(x) = \{F(x) \;\; V(x) \;\; M(x)\}^T$ = section forces and $\mathbf{b}$(x) = interpolation matrix.

The rates of deformation at a particular location x are related to the respective stress resultant rates through the matrix of section flexibility distributions $\mathbf{f}(x)$:

$$\dot{\boldsymbol{\varepsilon}}(x) = \mathbf{f}(x)\dot{\mathbf{R}}_{\mathbf{e}}(x) \tag{19}$$

where $\mathbf{f}(x) = diag[\frac{1}{EA}, \frac{1}{GA_S}, \frac{1}{a\,K_0 + (1-a)\,K_H}]$.

The flexibility matrix $\mathbf{F}$ of the element is derived from Eqs. (17), (18), (19) as:

$$\mathbf{F_e} = \int_0^L \mathbf{b}(x)^T\,\mathbf{f}(x)\mathbf{b}(x)dx \tag{20}$$

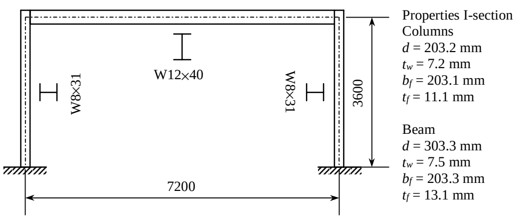
Gauss-Lobatto quadrature is used to integrate eq. (20) numerically, rather than the commonly used Gauss-Legendre type rules because the former has integration points at the ends of the interval and hence can detect yielding immediately. A Lobatto rule with n integration points integrates exactly a polynomial of order ($2n$-3).

NUMERICAL EXAMPLES

To illustrate the method, the system of state equations of an simple portal frame structure is solved quasi-static and dynamic excitations. The response of the state space model is ompared with solutions of ANSYS (1992), which uses a conventional incremental algorithm and stiffness-based beam elements.

Model Description

The example structure is shown in Fig. (3). The stress-strain curve of the material is assumed bilinear : E = 199955 kN/mm^2, σ_y = 248.2 kN/mm^2. The section constitutive model of Eqs. (1) and (2) require definition of four parameters: (i) the initial bending rigidity K_0, (ii) the post-yield bending rigidity aK_0, (iii) the parameter n controlling the smoothness of transition and (iv) a discrete yield point M_y. These are listed in Fig 3.

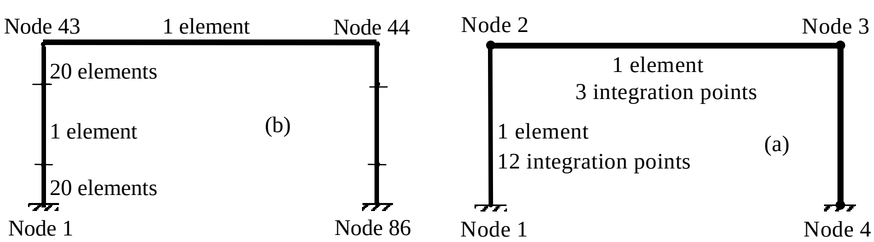


	A [cm2]	I [cm4]	$\alpha \approx$ ET/E [%]	n -	M_y [kNm]
Columns	57.99	4504.98	3	8	117.05
Beam	73.96	12535.21	3	8	215.67

Fig. 3: Portal frame and properties of sections

State Space Macro-Model The macro-element model for the proposed state-space solution consists of three elements. Results of analyses with gradually increasing number of quadrature points reveal a consistent trend of convergence. Finally, the columns are assigned 12 integration points (Fig. 4(a))

ANSYS Finite Element Model The finite-element model in ANSYS (Fig 4 (b) was created using the plastic beam element BEAM 24 (ANSYS, 1992). The cross sections of the frame members were divided into 10 fibers.. All analyses were performed using a 1% relative tolerance for force unbalance and a line search technique.



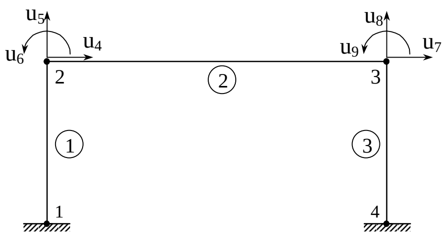
(b) "Beam 24" Element of ANSYS (a) State-Space Approach Element

Fig 4: Differences between Beam Element Formulations

State Variables and Equations

Quasi-static Analysis with Displacement Input

The structure is subjected to displacement-controlled loading (Fig.6) applied at node 2. Fig (5) maps: (*i*) the nodal displacements of the structure into the global state variables and (*ii*) the independent end forces and curvatures at the quadrature points of each element into the local state variables. The indices indicate position in the state vector.



Global	u_4	u_5	u_6	u_7	u_8	u_9
State Variable	y_1	y_2	y_3	y_4	y_5	y_6
Element 1	F_I	M_i	M_j	ϕ_1	…	ϕ_{12}
State Variable	y_7	y_8	y_9	y_{10}	…	y_{21}
Element 2	F_I	M_i	M_j	ϕ_1	…	ϕ_{12}
State Variable	y_{22}	y_{23}	y_{24}	y_{25}	…	y_{36}
Element 3	F_I	M_i	M_j	ϕ_1	…	ϕ_{12}
State Variable	y_{37}	y_{38}	y_{39}	y_{40}	…	y_{51}

Fig 5 Quasi-Static Analysis - Global and Local State Variables (y_{11} to y_{20}, y_{26} to y_{35} and y_{41} to y_{50} represent curvatures of quadrature sections)

The global state equations involve only active DOF and DOF with known displacement histories. The local state equations of the three elements are listed below (Eqs. 21-26), with the parenthesized superscript referring to the element number. The response of the frame is shown in Fig 6.

$$\{\dot{y}_7 \quad \dot{y}_8 \quad \dot{y}_9\}^T - \left[\mathbf{F}_{\mathbf{e}}^{(1)}\right]^{-1} \mathbf{T}_{\mathbf{e}}^{(1)} \mathbf{T}_{\mathbf{g}}^{(1)} \{0 \quad 0 \quad 0 \quad \dot{y}_1 \quad \dot{y}_2 \quad \dot{y}_3\}^T = \mathbf{0} \tag{21}$$

$$\mathbf{b}_{\mathbf{G}}^{(1)} \{\dot{y}_8 \quad \dot{y}_9\}^T - \left[\mathbf{a}^{(1)} \mathbf{K}_{\mathbf{0}}^{(1)} + \mathbf{K}_{\mathbf{H}}^{(1)}\right] \{\dot{y}_{10} \quad \dot{y}_{11} \quad \dots \quad \dot{y}_{21}\}^T = \mathbf{0} \tag{22}$$

$$\{\dot{y}_{22} \quad \dot{y}_{23} \quad \dot{y}_{24}\}^T - \left[\mathbf{F}_{\mathbf{e}}^{(2)}\right]^{-1} \mathbf{T}_{\mathbf{e}}^{(2)} \mathbf{T}_{\mathbf{g}}^{(2)} \{\dot{y}_1 \quad \dot{y}_2 \quad \dot{y}_3 \quad \dot{y}_4 \quad \dot{y}_5 \quad \dot{y}_6\}^T = \mathbf{0} \tag{23}$$

$$\mathbf{b}_{\mathbf{G}}^{(2)} \{\dot{y}_{23} \quad \dot{y}_{24}\}^T - \left[\mathbf{a}^{(2)} \mathbf{K}_{\mathbf{0}}^{(2)} + \mathbf{K}_{\mathbf{H}}^{(2)}\right] \{\dot{y}_{25} \quad \dot{y}_{26} \quad \dots \quad \dot{y}_{36}\}^T = \mathbf{0} \tag{24}$$

$$\{\dot{y}_{37} \quad \dot{y}_{38} \quad \dot{y}_{39}\}^T - \left[\mathbf{F}_{\mathbf{e}}^{(3)}\right]^{-1} \mathbf{T}_{\mathbf{e}}^{(3)} \mathbf{T}_{\mathbf{g}}^{(3)} \{\dot{y}_4 \quad \dot{y}_5 \quad \dot{y}_6 \quad 0 \quad 0 \quad 0\}^T = \mathbf{0} \tag{25}$$

$$\mathbf{b}_{\mathbf{G}}^{(3)} \{\dot{y}_{38} \quad \dot{y}_{39}\}^T - \left[\mathbf{a}^{(3)} \mathbf{K}_{\mathbf{0}}^{(3)} + \mathbf{K}_{\mathbf{H}}^{(3)}\right] \{\dot{y}_{40} \quad \dot{y}_{41} \quad \dots \quad \dot{y}_{51}\}^T = \mathbf{0} \tag{26}$$

Dynamic Analysis with Ground Acceleration Input

A ground acceleration record from the 1994 Northridge earthquake (Fig. (6b)) was used as input for the nonlinear dynamic analysis. The masses, $m1 = m2 = 24.9626$ kN.s/m2, are lumped at the two joints of the frame and the horizontal velocities of nodes 2 and 3 are added to the global state variables. A mass-proportional damping coefficient, $\alpha = 0.8378$, is chosen to provide a 5% damping ratio. The resulting global state equations (28) and (29).have the state variables and equations the same as those in quasi-static case, Fig.5, but translated by: $y_{n+2}^{d} = y_{n}^{s}$, for $n \geq 7$. where, $\ddot{u}_g(t)$ is the base acceleration. The results are shown in Fig (6b).

$$\begin{Bmatrix} m_1\,\dot{y}_7 \\ 0 \\ 0 \\ m_2\,\dot{y}_8 \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} \alpha m_1\,\dot{y}_1 \\ 0 \\ 0 \\ \alpha m_2\,\dot{y}_4 \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} -(y_{10}+y_{11})/L_1 + y_{24} \\ -y_9 + (y_{25}+y_{26})/L_2 \\ y_{11}+y_{25} \\ -y_{24} + (y_{40}+y_{41})/L_3 \\ -(y_{25}+y_{26})/L_2 - y_{39} \\ y_{26}+y_{40} \end{Bmatrix} - \begin{Bmatrix} -m_1\,\ddot{u}_g \\ 0 \\ 0 \\ -m_2\,\ddot{u}_g \\ 0 \\ 0 \end{Bmatrix} = \mathbf{0} \qquad (27)$$

$$\{y_7 \quad y_8\}^T - \{\dot{y}_1 \quad \dot{y}_4\}^T = \mathbf{0} \qquad (28)$$

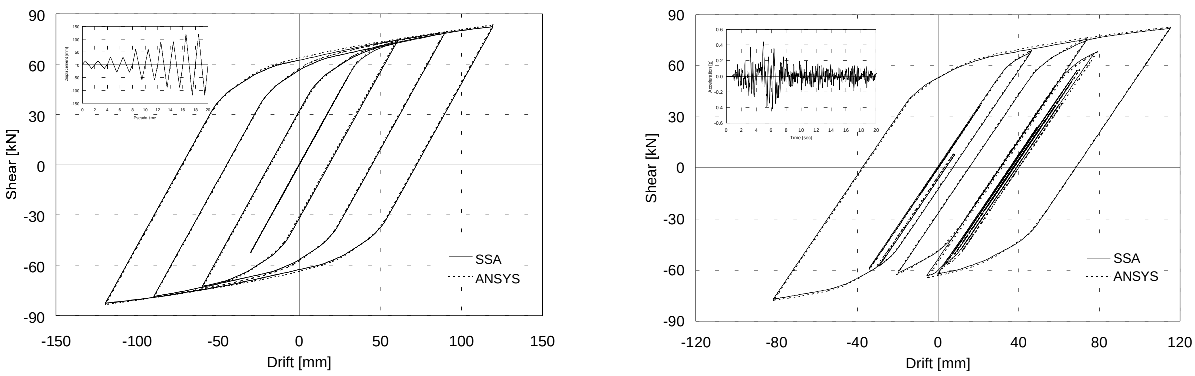


Fig. 6: Quasi-Static Analysis: Shear Force vs Displacement of Element 1

CONCLUSIONS

A general formulation for state-space analysis of frame structures has been applied to both quasi-static and dynamic problems. Using a macromodel approach with a flexibility-based nonlinear bending element the solution is obtained using iterative procedures applied to the differential-algebraic equations. The global state equations of equilibrium and the local constitutive state equations are solved simultaneously. The accuracy of this macro-element can be refined by increasing the number of quadrature points, at which the constitutive equations are monitored, in contrast to increasing the number of elements in conventional finite element analysis. The former is computationally more economical. The state-space approach has good correlation with results from a finite element program, using a conventional incremental solution with densely meshed beam elements.

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