EXTREME RESPONSE OF LINEAR STRUCTURES TO NONSTATIONARY BASE EXCITATION

Shahram SARKANI[1], Loren D LUTES[2] And George MICHAELOV[3]

SUMMARY

The fractile levels of a random process are defined as the levels that have specific probability of not being exceeded by the process within a specified time interval. The study presents an efficient method for approximate computation of the fractile levels of the extreme response of a linear structure subjected to nonstationary Gaussian excitation. The approximate procedure can significantly facilitate the utilization of nonstationary models, since it avoids the computational difficulty associated with direct application of extreme value theory. The method is based on the approximation of the cumulative distribution function (CDF) of the extreme value of a nonstationary process by the CDF of a corresponding "equivalent" stationary process. Approximate procedures are developed for both the Poisson and Vanmarcke approaches to the extreme value problem, and numerical results are obtained for an example problem. These results demonstrate that the present method agrees quite well with the direct application of extreme value theory, while avoiding the solution of nonlinear equations containing complicated time integrals.

INTRODUCTION

This study presents the development of approximate solutions for random vibration analysis of building structures subjected to earthquake loadings, in the case when both the loading and the response are modeled as nonstationary random processes. Regardless of the modeling of the earthquake (deterministic or random), the maximum of the structural response has to be quantified, in order to make sure that the appropriate safety criteria are met. In the random response case, the maximum response value is a random variable, and its observed value is different for each possible realization of the random loading. As a random variable, the extreme response is completely described by its cumulative distribution function (CDF). Accordingly, an appropriate "design level" (i.e., a response value that the structure is designed to sustain without failure) can be derived from the CDF in terms of the response level that has a specified probability of not being exceeded for the considered earthquake loading event. This quantity, which is essentially the inverse of the CDF, will be referred to hereafter as the "fractile level," although in the literature it can also be found as "percentile level" or "quantile level."

The major difficulty encountered in computing the fractile levels is that a convenient mathematical form of the exact CDF of the extreme value of a random process does not exist in general. A number of approximations have been shown to agree well with statistical simulation results. In particular, the common Poisson approximation results in the simplest CDF expressions, however, it is often regarded as being too conservative, particularly for structures with very small damping. In the latter cases, the so-called Vanmarcke approximation is usually found more appropriate and commonly used in applications of the extreme value theory. In the nonstationary random models either the Poisson or Vanmarcke approximation results in a CDF that includes time integrals, which does not allow for simple solution for the fractile levels as can be derived in the case of stationary models.

[1] *School of Engineering and Applied Science, The George Washington University, Washington, DC 20052, sarkani@seas.gwu.edu*
[2] *Department of Civil Engineering, Texas A&M University, College Station, TX 77098, L-Lutes@tamu.edu*
[3] *School of Engineering and Applied Science, The George Washington University, Washington, georgem@seas.gwu.edu*

In this study, an approximation technique is presented, which approximates the nonstationary extreme value CDF by a stationary CDF for both the Poisson and Vanmarcke approximations of the extreme value theory. This approximation is based on the introduction of an equivalent stationary process, which has nearly the same extreme value CDF as the nonstationary one in the neighborhood of the fractile level of interest. For the Poisson extreme value approach, the parameters that govern the approximation are the variance and the duration of this equivalent stationary process. In addition, an equivalent bandwidth factor is introduced for the Vanmarcke extreme value approach. The method is somewhat similar to the one based on the empirical Gumbel approximation introduced by Michaelov et al. (1996) but is more precise in identifying the Gumbel distribution parameters as the variance and duration of an equivalent stationary process and in providing a systematic procedure for their computation.

The focus of the extreme value analysis in this study is the nonstationary dynamic response of a simple oscillator – a linear, single-degree-of-freedom (SDOF) system – with the excitation and the response of the oscillator both taken to be of the so-called "evolutionary" class of nonstationary processes (Priestley, 1965). In fact, all derivations in this study assume evolutionary processes, although some definitions may also be applicable to general nonstationary processes. It should be noted that the response fractile level for a simple oscillator, when viewed as a function of the oscillator's natural frequency and damping ratio, represents a response spectrum. Thus, the approximate formulas presented in this study may be viewed as an efficient way of computing response spectra based on a nonstationary stochastic process modeling of the excitation. Thus, the formulas developed in this study could also be used in analyzing the extreme response of a multi-degree-of-freedom (MDOF) system through modal analysis and modal combination rules such as the square root of the sum of the squares (SRSS) method or the complete quadratic combination (CQC) method (Wilson et al. 1981).

RESPONSE OF THE SIMPLE OSCILLATOR TO NONSTATIONARY RANDOM EXCITATION

Let $X(t)$ denote the response of a simple oscillator to random excitation $F(t)$. Assuming unit mass of the oscillator for simplicity, the differential equation relating $X(t)$ to $F(t)$ can be written as:

$$\ddot{X}(t) + 2\zeta\omega_0\dot{X}(t) + \omega_0^2 X(t) = F(t) \tag{1}$$

in which ω_0 and ζ are the natural frequency and the damping ratio of the system. Zero initial conditions are assumed; that is, $X(0)=0$ and $\dot{X}(0)=0$ with probability one. To apply the approximations presented in this study, the excitation $F(t)$ may be modeled as a zero-mean, Gaussian, evolutionary process. For reasons of simplicity it will be taken as a special case of an evolutionary process, a modulated white noise in the form of

$$F(t) = A(t)W(t) \tag{2}$$

in which $W(t)$ represents a stationary white noise with spectral density equal of S_0 and $A(t)$ is a slowly varying deterministic modulating function.

For the extreme value analysis, the following four characteristics of the $X(t)$ are needed: the variances $\sigma_{\dot{X}}^2(t)$, $\sigma_X^2(t)$, the correlation coefficient $\rho_{X\dot{X}}(t)$, and the bandwidth factor $q_X(t)$. As discussed by Michaelov, et al., (1999a) for the case of modulated white noise excitation these parameters can be obtained as

$$\sigma_X^2(t) = 2\pi S_0\int_0^t h^2(t-\tau)A^2(\tau)\,d\tau \qquad \sigma_{\dot{X}}^2(t) = 2\pi S_0\int_0^t \dot{h}^2(t-\tau)A^2(\tau)d\tau$$
$$\rho_{X\dot{X}}(t) = -\frac{\mathrm{Im}[c_{01}(t)]}{\sigma_X(t)c_{\dot{X}}(t)} \qquad q_X(t) = \sqrt{1-\left\{\frac{\mathrm{Re}[c_{01}(t)]}{\sigma_X(t)c_{\dot{X}}(t)}\right\}^2} \tag{3}$$

in which

$$\mathrm{Im}[c_{01}(t)] = -2\pi S_0\int_0^t h(t-\tau)\dot{h}(t-\tau)A^2(\tau)\,d\tau \tag{4}$$

and

$$\mathrm{Re}[c_{01}(t)] = 2S_0 \int_0^t \frac{1}{u}\left\{\int_u^t \left[h(t-\tau+u)\dot{h}(t-\tau) - h(t-\tau)\dot{h}(t-\tau+u)\right]A(\tau-u)A(\tau)d\tau\right\} du \tag{5}$$

In the above equations, $h(t)$ denotes the impulse response function $h(t) = [\exp(-\zeta\omega_0 t)/\omega_d]\sin(\omega_d t)$ in which $\omega_d = \omega_0\sqrt{1-\zeta^2}$ is the damped frequency of the oscillator. Note that the bandwidth factor definition in (3) is not the same as the classical definition based on spectral moments. This definition is more appropriate for evolutionary processes, as discussed by Michaelov et al. (1999a), and unlike the definition based on spectral

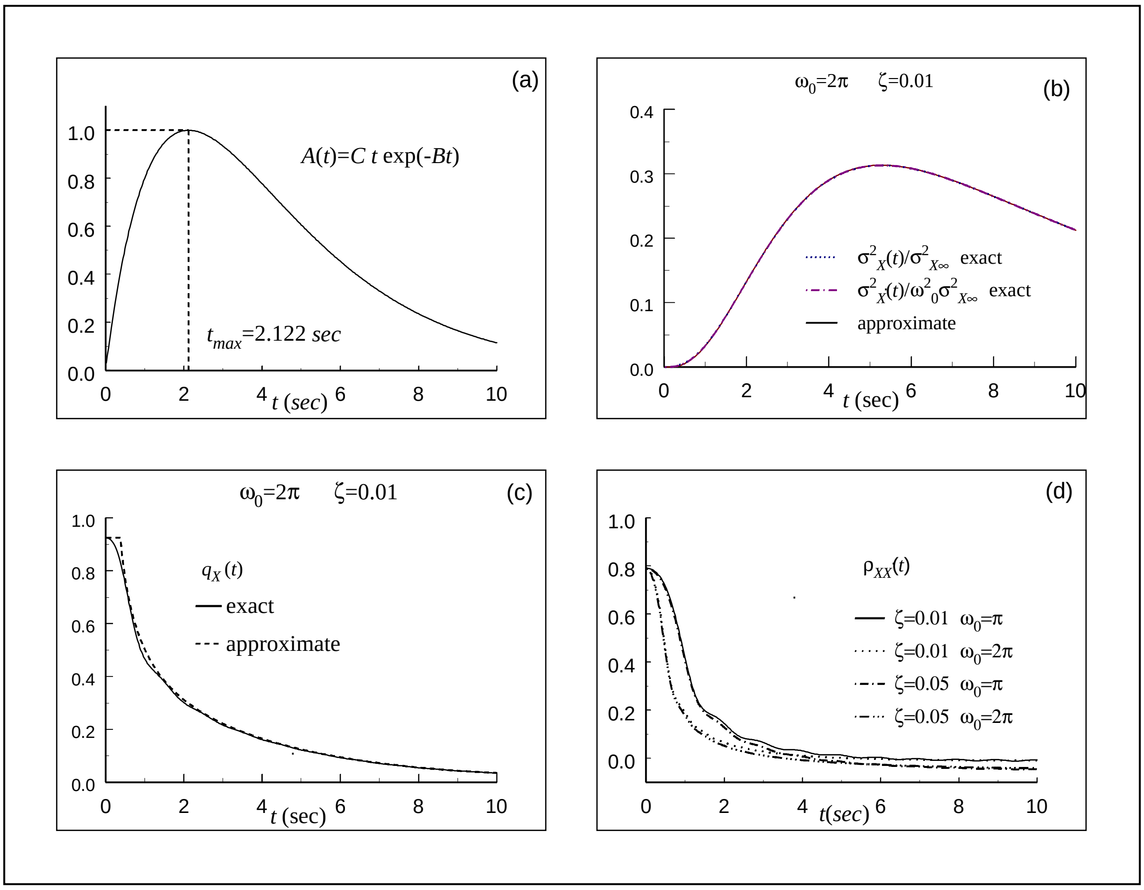

Figure 1. Modulating function of the excitation and corresponding characteristics of the response

moments, does not create integration difficulties since the bandwidth factor is finite as long as $\sigma_X(t)$ and $\sigma_{\dot X}(t)$ are also finite.

To demonstrate the application of the results obtained in this study for the extreme value characteristics of the nonstationary $X(t)$, the modulating function $A(t)$ will be taken as the product of a linear and exponential terms which can be expressed as

$$A(t) = C\,t\exp(-Bt) \quad \text{for} \quad t \geq 0 \quad B > 0 \tag{6}$$

The parameter C is chosen so that the maximum of the modulating function is unity and the intensity of the excitation $F(t)$ is governed by only the spectral density of the white noise S_0. In this study it is assumed that S_0 =1 and B=0.15π which results in C=1.281. The modulating function is shown in Fig. 1a. For this particular choice of $A(t)$, Michaelov et al. (1999a) developed the following approximations of $\sigma_X^2(t)$, $\sigma_{\dot X}^2(t)$, and $q_X(t)$:

$$\sigma_X^2(t) \approx \frac{\sigma_{\dot X}^2(t)}{\omega_0^2} \approx \sigma_{X\infty}^2 C^2 \frac{2\zeta\omega_0}{\gamma}\left\{\left[t^2 - \frac{2t}{\gamma} + \frac{2}{\gamma^2}\right]\exp(-2Bt) - \frac{2}{\gamma^2}\exp(-2\zeta\omega_0 t)\right\} \tag{7}$$

and

$$q_X(t) \approx 2\sqrt{\frac{\zeta}{\pi}}\frac{\sigma_{X\infty}C}{\sigma_X(t)}\left[t^2\exp(-2Bt)+\frac{2\exp(-2\zeta\omega_0 t)}{3\omega_0^2}\right] \le 0.926 \quad \text{otherwise} \quad q_X(t)=0.926 \tag{8}$$

in which $\gamma = 2\zeta\omega_0 - 2B$ and $\sigma^2_{X\infty} = (S_0\pi)/(2\zeta\omega_0^3)$. The values of $\sigma_X^2(t), \sigma_{\dot{X}}^2(t)$, and $q_X(t)$ computed from (7), and (8) are shown in Figs. 1b and 1c along with the corresponding exact values for ζ=0.01 and ω_0=2π. Figure 1d shows the exact values of the transient correlation coefficient $\rho_{X\dot{X}}(t)$, which suggests that $\rho_{X\dot{X}}(t)$ is only significant within a short time interval at the beginning of the response. These plots seem to justify the approximation $\rho_{X\dot{X}}(t) \approx 0$, and this will be used in the subsequent extreme value developments.

APPROXIMATIONS OF THE PROBABILITY DISTRIBUTION OF THE EXTREME VALUE

The extreme value $X_m(T)$ of a random process $X(t)$ over a time interval $[0,T]$ is defined as

$$X_m(T) = \max_{0\le t\le T}\, [|\,X(t)\,|] \tag{9}$$

The extreme value CDF, $F_{X_m}(x,T)$, which represents the probability that the extreme value $X_m(T)$ is less than or equal to x within the time interval $[0,T]$, is commonly expressed as

$$F_{X_m}(x,T) = \exp[-\int_0^T \eta_X(x,t)\,dt] = \exp[-N_X(x,T)] \tag{10}$$

in which $\eta_X(x,t)$ and $N_X(x,T)$ respectively denote the rate and the total number of occurrences of the event of upcrossing of level x by $|X(t)|$ at time t, given that such an upcrossing has not occurred prior to t. The above equation assumes that not exceeding the level x at time t=0 is a certain event given that $X(0)$=0. Only for a stationary process can $\eta_X(x,t)$ ever become independent of time.

Although the exact analytical form of $\eta_X(x,t)$ is just as difficult to obtain as the CDF of $X_m(T)$, the notion of upcrossing rate is very useful because it points to a number of possible approximations. The simplest choice is to assume that the upcrossings of level x occur independently and thus constitute a Poisson process. The resulting Poisson approximation of $\eta_X(x,t)$ is equal to the expected unconditional rate of upcrossings of level x by $|X(t)|$ and for a zero- mean Gaussian process is given by

$$\eta_X^P(x,t) = \frac{1}{\pi}\sqrt{1-\rho^2_{X\dot{X}}(t)}\,\frac{\sigma_{\dot{X}}(t)}{\sigma_X(t)}\,\exp\left(-\frac{1}{2}\frac{x^2}{\sigma_X^2(t)}\right)\Psi\left(\frac{\rho_{X\dot{X}}(t)\,x}{\sqrt{1-\rho^2_{X\dot{X}}(t)}\,\sigma_X(t)}\right) \tag{11}$$

in which $\Psi(x) = \exp(-x^2/2) + \sqrt{2\pi}x\Phi(x)$ and $\Phi(x)$ denotes the cumulative normal distribution function. For large levels of x, the assumption of independent upcrossings is generally quite reasonable. The Poisson approximation has been criticized, though, for being too conservative when the time period of interest, T, is quite short and/or when the process is narrowband.

The approximation that has gained the most recognition for adequately addressing the narrowband situations is the so-called "two-state Markov process assumption," otherwise known as the "Vanmarcke" approximation. Vanmarcke (1969, 1975) proposed that the independence assumption be applied to the so-called "qualified" crossings of the envelope process. For a nonstationary Gaussian process, the Vanmarcke upcrossing rate $\eta_X^V(x,t)$ is usually written as (Corotis, et al. 1972).

$$\eta_X^V(x,t) = \frac{1}{\pi}\frac{\sigma_{\dot{X}}(t)}{\sigma_X(t)}\left[1-\exp\left(-\sqrt{\frac{\pi}{2}}\,q_X^{1.2}(t)\frac{x}{\sigma_X(t)}\right)\right]\Big/\left[\exp\left(\frac{x^2}{2\sigma_X^2(t)}\right)-1\right] \tag{12}$$

in which $q_X(t)$ is the bandwidth factor defined from spectral moments. The above expression is a direct extension of the formula for stationary processes obtained by simply replacing the stationary parameters with their corresponding nonstationary counterparts. As shown by Michaelov et al. (1999b), the correct expression for $\eta_X^V(x,t)$ resulting from the application of Vanmarcke's approach to evolutionary processes is

$$\eta_X^V(x,t) = \frac{1}{\pi}\sqrt{1-\rho_{X\dot{X}}^2(t)}\,\frac{\sigma_{\dot{X}}(t)}{\sigma_X(t)}\,\frac{1-\exp\left[-\sqrt{\frac{\pi}{2}}\,\frac{x}{\sigma_X(t)}\sqrt{\frac{q_X^2(t)-\rho_{X\dot{X}}^2(t)}{1-\rho_{X\dot{X}}^2(t)}}\,\Psi\left(\frac{\rho_{X\dot{X}}(t)x}{\sqrt{q_X^2(t)-\rho_{X\dot{X}}^2(t)}\,\sigma_X(t)}\right)\right]}{\exp\left[\frac{x^2}{2\sigma_X^2(t)}\right]-1} \tag{13}$$

in which $q_X(t)$ is computed from (3). It can be seen that apart from the empirical exponent 1.2, (12) and (13) will give the same result if the correlation coefficient $\rho_{X\dot{X}}(t)$ is neglected [i.e., if $\rho_{X\dot{X}}(t)=0$].

For given values of T and P, the fractile level, $\hat{x}$, having probability P of not being exceeded in the fixed interval $[0,T]$ is obtained by solving the equation

$$P = F_{X_m}(\hat{x},T) \tag{14}$$

Solutions for $\hat{x}$ are relatively easy to obtain when $X(t)$ is a stationary processes in which case $\eta_X(x,t)$ and its approximations are independent of t. For the Poisson approximation case the resulting fractile level solution is exact. For the Vanmarcke approximation case the resulting equation (14) is nonlinear but an approximate solution was derived by Michaelov et al. (1996). Similar solutions cannot be obtained directly for the nonstationary Poisson and Vanmarcke fractile levels, because a substitution of (11), (13), or (14) into (10) results in a nonlinear equation in which the fractile level is in the middle of time integrals. The following sections show how the nonstationary fractile levels can be evaluated approximately.

APPROXIMATION OF THE POISSON FRACTILE LEVELS

Consider the approximation of the Poisson extreme value CDF resulting from (13) with $\rho_{X\dot{X}}(t)=0$. The expression of the exponent of the CDF can be written as

$$N_X^P(x,T) = 2\nu_0\int_0^T \exp\left(-\frac{x^2}{2\sigma_X^2(t)}\right)dt \tag{15}$$

in which $\nu_0 = \nu_0(t) = \eta_X^P(0,t)/2 \approx \omega_0/2\pi$ reflects the approximate formulas (7). To overcome the time integral, which is the main obstacle to solving the fractile level equation, an approximation of $N_X^P(x,T)$ is sought in the form of

$$N_X^P(x,T) = -2\nu_0 T_{eq}\exp\left(-\frac{x^2}{2\sigma_{eq}^2}\right) \tag{16}$$

in which σ_{eq} and T_{eq} are parameters to be found to best fit (15) and (16). It can be shown that such a match is equivalent to finding a stationary process with variance σ_{eq}^2 so that its total number of upcrossings of the Poisson fractile level in the interval [0, T_{eq}] is approximately equal to the total number of upcrossings of the nonstationary process, $N_X^P(x,T)$, in the interval $[0,T]$. It is also evident that this "equivalent" stationary process creates the same extreme value probability as the original nonstationary process $X(t)$. Therefore, if the equivalent stationary process parameters σ_{eq} and T_{eq} were known, the nonstationary Poisson fractile levels, $\hat{x}^P(T,P)$, could be computed from (16), (10), and (14) as

$$\hat{x}^P(T,P) = \sigma_{eq}\sqrt{2\ln\left(-\frac{2\nu_0 T_{eq}}{\ln(P)}\right)} \tag{17}$$

The issue now is how to determine the parameters σ_{eq} and T_{eq} so that (16) best matches (15), particularly in the neighborhood of some particular fractile level. To this end, consider the following two functions:

$$f_1(y) = \int_0^T \exp\left(-\frac{y}{2\sigma_X^2(t)}\right)dt \qquad \text{and} \qquad f_2(y) = T_{eq}\exp\left(-\frac{y}{2\sigma_{eq}^2}\right) \qquad y \geq 0 \tag{18}$$

which are proportional to the right-hand sides of (15) and (16), respectively, but in which x^2 has been replaced by y for convenience. It can be seen that both functions have a similar behavior: both are monotonically decreasing, both have maximum at y=0 and both asymptotically approach zero as $y \to \infty$. The parameters σ_{eq} and T_{eq} can be viewed as characteristics of the geometric shapes formed by the area under the $f_2(y)$ function. By taking different values of σ_{eq} and T_{eq}, the shape of $f_2(y)$ can be matched closely (although not completely) to the shape of $f_1(y)$. One way to match the two shapes is to impose specific geometric conditions, such as that the two shapes should have the same total area, the same first or second moment, the same maximum, or the same centroid. It is more appropriate, however, to fit two other functions, $g_1(y)$ and $g_2(y)$, defined as

$$g_1(y) = y^{n-1} f_1(y) \qquad \text{and} \qquad g_2(y) = y^{n-1} f_2(y) \tag{19}$$

rather than directly fitting $f_2(y)$ to $f_1(y)$. Both functions in (19) have a single maximum in the interval $[0,\infty]$. The role of the term y^{n-1} is to shift this maximum to the large y region by choosing n sufficiently large. In particular, it can be seen that the maximum of $g_2(y)$ takes place at $y = (n-1)2\sigma_{eq}^2$. Accordingly, the geometric characteristics of $g_1(y)$ and $g_2(y)$ are much more influenced by the important larger values of y than the geometric characteristics of the original $f_1(y)$ and $f_2(y)$.

Let $E_1(m)$ and $E_2(m)$ denote the mth moment of $g_1(y)$ and $g_2(y)$, respectively. These moments can be found as

$$E_1(m) = \int_0^\infty y^m g_1(y)dy = 2^{n+m}\Gamma(n+m)I(n+m) \quad E_2(m) = \int_0^\infty y^m g_2(y)dy = T_{eq}2^{n+m}\Gamma(n+m)\sigma_{eq}^{2(n+m)} \tag{20}$$

in which $I(n)$ denotes the following integral on the variance of the process:

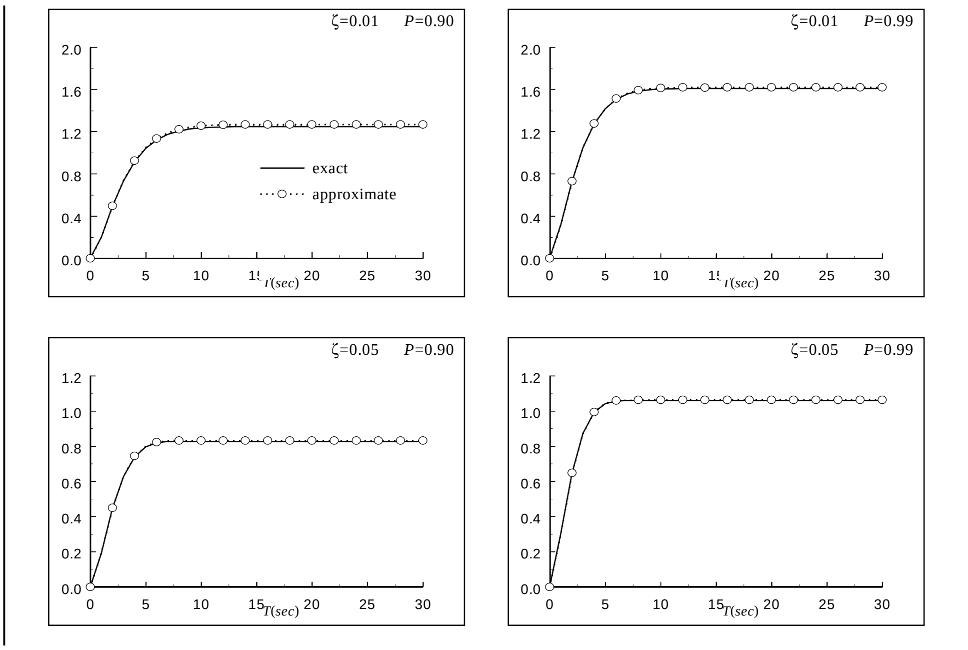

Figure 2 Vanmarcke fractile levels

was used. Also shown in Fig. 2 are the Poisson fractile level values obtained by the numerical solution of (14) in which the exact Poisson upcrossing rate from (12) was used along with the exact values of $\sigma_X^2(t)$, $\sigma_{\dot{X}}^2(t)$, and $\rho_{X\dot{X}}(t)$ from (3). In all cases considered, the approximate fractile levels match the numerically obtained solutions quite well.

APPROXIMATION OF THE VANMARCKE FRACTILE LEVELS

Strictly speaking, to apply the above approach to the nonstationary Vanmarcke case, one needs to find an equivalent stationary process such that its total number of qualified envelope upcrossings in some interval $[0,T_{eq}]$ equals the total number of qualified envelope upcrossings of the nonstationary process, the rate of which is given by (14). This represents quite a difficult task in general. An easier approach is to consider an equivalent stationary process whose number of total independent upcrossings is equal to $N_X^P(x,T)$ and whose number of independent envelope upcrossings is equal to the total number of envelope upcrossings of the nonstationary process. If these two numbers are matched for the two process and their envelopes, then it seems reasonable to expect that the total numbers of their qualified envelope upcrossings will be approximately equal. It is easy to see that the equivalent stationary process developed in the previous section satisfies the first of these two conditions. It can be shown (see Michaelov et al. 1999b) that by using $\rho_{X\dot{X}}(t)=0$ the total number of the independent upcrossings of the nonstationary envelope, $N_X^V(x,T)$, can be expressed as

$$N_V^P(x,T) \approx 2\nu_0\sqrt{\frac{\pi}{2}}\int_0^T \frac{q_X(t)x}{\sigma_X(t)}\exp\left(-\frac{x^2}{2\sigma_X^2(t)}\right)dt \tag{24}$$

Evidently, the total number of envelope upcrossings of the equivalent stationary process in the interval $[0,T_{eq}]$ is equal to

$$N_V^P(x,T) = 2\nu_0 T_{eq}\sqrt{\frac{\pi}{2}}\frac{q_{eq}x}{\sigma_{eq}}\exp\left(-\frac{x^2}{2\sigma_{eq}^2}\right) \tag{25}$$

in which q_{eq} is the bandwidth factor of the equivalent process. By equating (24) and (25) the equivalent bandwidth factor is found as

$$q_{eq} = \frac{\sigma_{eq}}{T_{eq}}\exp\left(\frac{x^2}{2\sigma_{eq}^2(t)}\right)\int_0^T \frac{q_X(t)}{\sigma_X(t)}\exp\left(-\frac{x^2}{2\sigma_X^2(t)}\right)dt \tag{26}$$

Note that this q_{eq} value depends on the x at which matching is sought. For estimating the Vanmarcke fractile value $\hat{x}^V(T,P)$, this can be approximated by using the Poisson value, $\hat{x}^P(T,P)$. Once the three equivalent process parameters σ_{eq}, T_{eq}, and q_{eq} are known, the Vanmarcke fractile level can be found from the approximate equation developed by Michaelov et al. (1996) as

$$\hat{x}^V(T,P) = \sigma_{eq}\sqrt{2\ln\left[-\frac{2\nu_0 T_{eq}}{\ln(P)}\left(1-\exp\left(-\sqrt{\frac{\pi}{2}}\,q_{eq}^{1.2}\,\frac{\hat{x}^P(P,T)}{\sigma_{eq}}\right)\right)+1\right]} \tag{27}$$

Thus, computation of the Vanmarcke fractile levels is a two-step procedure. In the first step, the Poisson fractile levels are computed according to the method outlined in the previous section. In the second step, the Vanmarcke fractile levels are computed from (27) as improvements of the Poisson fractile levels. The results obtained with the above formula are shown in Fig. 3 for the same instances of oscillator damping and probability that were considered in Fig. 2. The "exact" Vanmarcke fractile levels shown were again obtained by numerical solution of (14), in which the exact Vanmarcke upcrossing rate (13) was used along with the exact values of $\sigma_X^2(t)$, $\sigma_{\dot{X}}^2(t)$, $\rho_{X\dot{X}}(t)$, and $q_X(t)$. In all cases considered, the approximate Vanmarcke fractile levels somewhat overpredict the numerically obtained solutions.

CONCLUSIONS

In this study, simple approximations for the fractile levels of the response of linear structures to nonstationary excitation have been developed. The developed approximations are based on the extreme value CDFs resulting from application of the Poisson and Vanmarcke approaches to the extreme value problem of evolutionary Gaussian processes. The approximations of the Poisson extreme value parameters are based on considering an equivalent stationary process such that the two processes have the same number of upcrossing. The two parameters that must be determined for this equivalent stationary process are the equivalent variance and the equivalent duration. The approximation of the Vanmarcke extreme value parameters is based on the same equivalent stationary process concept but also requires the computation of an equivalent bandwidth factor. The formulas obtained are applicable to general evolutionary processes, and the results have been demonstrated for the response of a simple oscillator to a modulated white noise. The only limiting assumption is that the correlation coefficient between the nonstationary response process and its derivative can be assumed to be zero for times of interest. In all cases considered, the approximations show quite good agreement with numerically computed values of the extreme value parameters.

REFERENCES

1. Corotis, R.B., Vanmarcke, E.H., and Cornell, C.A. (1972). "First Passage of Non-stationary Random Processes," *J. Eng. Mech. Div.*, 98 (EM2), Proc. Paper 8816, pp. 401-414.

2. Michaelov, G., S. Sarkani, and Lutes, L.D. (1996). "Fractile Levels for Non-stationary Extreme Response of Simple Oscillators," *Structural Safety*, 18(1), pp. 11-31.

3. Michaelov, G., Sarkani, S., and Lutes, L.D. (1999a). "Spectral Characteristics of Nonstationary Random Processes - Response of a Simple Oscillator," *Structural Safety*, 21(3), pp. 245-267.

4. Michaelov, G., Sarkani, S., and Lutes, L.D. (1999b). "Extreme Value of the Response to Nonstatonary excitation," Submitted for publication to *Journal of Engineering Mechanics*.

5. Priestley, M.B. (1965). "Evolutionary Spectra and Nonstationary Processes," *J. Roy. Stat. Soc.*, pp. 204-307.

6. Vanmarcke, E. H. (1969). "On Measures of Reliability in Nonstationary Random Vibration," Research Report No R69-20. *Dept. of Civ. Eng.*, M.I.T.

7. Vanmarcke, E. H. (1975). "On the Distribution of the First-passage time for normal stationary processes," *J. Appl. Mech.*, 42, pp. 215-220.

8. Wilson, E.L., Der Kiureghian, A., and Bayo, E.P. (1981). "A Replacement for the SRSS Method in Seismic Analysis," *Earthquake Engineering and Structural Dynamics*, 9, pp.187-192.