ANALYSIS OF SEISMIC SITE EFFECTS : BEM AND MODAL APPROACH VS EXPERIMENTS

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SUMMARY

The main purpose of the paper is the analysis of seismic site effects in alluvial basins. Both experimental investigations and numerical computations are carried out. Two different numerical methods are considered : the Boundary Element Method and a modal approach.

A Boundary Element model with a homogeneous alluvial deposit is considered in the central sedimentary basin in the city of Nice. The amplification of seismic motion is analyzed in terms of level, occuring frequency and location. In this specific site of Nice, the amplification factor is found to reach a maximum value between 10 and 30. Site effects occur in the thickest part of the basin for low frequencies (1.6 Hz) and in the thinnest part above 2.0 Hz. Shear waves (SH, SV) lead to much higher amplification but at slightly different frequencies. Comparison with experimental results are proposed.

The second numerical method is a modal approach considering the eigenmodes of the basin. They are numerous and it is generally difficult to detect those corresponding to strong site amplification. The analysis of modal excitation factor and effective modal mass values for a vertical displacement gives interesting results about main amplification frequencies of the basin. These results are also compared with experimental ones.

The boundary element method appears as a reliable and efficient approach for the analysis of seismic site effects. The use of effective modal mass in the modal approach allows the estimation of some amplification features. In the specific site considered, amplification characteristics are found to be very close in both numerical and experimental approaches.

INTRODUCTION

The local amplification of seismic motion in alluvial basins can be very important [Bard, 1985, 1994, Duval, 1996, Pitilakis, 1999, Semblat, 1999a,c]. Many different methods are used to investigate seismic wave propagation from experimental ones [Duval, 1996b, Pitilakis, 1999, Semblat, 1998a] to theoretical and numerical ones (finite element, spectral methods, boundary elements).

In the city of Nice (France), many different types of measurements (microtremors, real earthquakes) have been performed to analyze seismic wave amplification in the central alluvial basin [Duval, 1996a,b]. The efficiency of the different experimental methods is always good for main frequencies determination but somehow variable for the estimation of amplification.

The numerical analysis of wave propagation can be carried out through different approaches : finite elements [Semblat, 1997, 1998b, 1999b], spectral methods [Faccioli, 1996, Komatitsch, 1999], boundary elements [Bonnet, 1999, Semblat, 1999a,c]. The main advantage of the boundary element method is to avoid artificial truncation of the domain in the case of infinite medium. For dynamic problems, this truncation leads to artificial wave reflections giving a numerical error in the solution. Furthermore, finite element or finite difference methods involve some other drawbacks as numerical wave dispersion [Ihlenburg, 1995, Semblat, 1999b].

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As the local amplification of seismic waves is related to the vibratory resonance of the site, it is also interesting to analyze the vibration modes of the basin and the surrounding soil [Dobry, 1976, Paolucci, 1999]. It is nevertheless difficult to find out the main eigenfrequencies involved in the general amplification process. To improve the efficiency of modal approaches, one must discriminate between the numerous eigenfrequencies of the numerical model of the basin including many degrees of freedom.

EXPERIMENTAL INVESTIGATIONS

The specific site considered in located in the center of Nice (french Riviera). The alluvial basin is two thousand meters wide and its depth is around sixty meters in the deepest part (West) down to thirty meters in the thinnest part (East). Many experimental investigations have been performed using different techniques : microtremor measurements, real earthquakes (weak motion). As shown in previous publications [Duval, 1996a,b, Semblat, 1999a,c], the main frequencies of the basin are precisely estimated by both experimental methods. Two types of amplification are determined : in the deepest part of the deposit for low frequencies (between 1.0 and 1.5 Hz) and in the thinnest part for higher frequencies (above 2.0 Hz). Considering the H/V spectral ratio from microtremor recordings, it is nevertheless difficult to estimate the absolute amplification factor. In the following sections, different experimental results from real earthquakes measurements give an accurate estimation of amplification and are compared with numerical results.

BOUNDARY ELEMENT MODEL

Boundary element method

To analyze the seismic response of the site, a numerical model based on the boundary element method is firstly considered. The boundary element method can be divided into two main stages [Dangla, 1989, Bonnet, 1999] :

1 Solution of the boundary integral equation giving displacements and stresses along the border of the domain,

2 A posteriori computation for all points inside the domain using an integral representation formula.

One considers an elastic, homogeneous and isotropic solid of volume Ω and external surface ∂Ω. The problem is supposed to have an harmonic dependence on time (circular frequency ω). The displacement field is then written $u(x,t)=u(x).e^{-i\omega t}$. The displacement magnitude u(x) is solution of the following equation :

$$(\lambda + 2\mu) grad(div\, u(x)) - \mu rot(rot\, u(x)) + \rho f(x) + \rho\omega^2 u(x) = 0 \quad (1)$$

where *u* is the displacement field and *f* the volumic density of force.

The integral formulation arises from the application of the reciprocity theorem between the unknown displacement field and the fundamental solutions of a reference problem called Green's functions [Bonnet, 1999]. The reference problem generally corresponds to the case of an infinite space (or semi-infinite for SH-waves) in which a volumic force concentrated at point *y* acts along direction *e* : $\rho f^{'}(x) = \delta(x-y)e$. Numerical solution of the integral equation can be estimated by collocation method, or thanks to a variational formulation [Bonnet, 1999]. In this article, the solution of this equation is obtained by finite boundary elements discretization and then by collocation, that is application of the integral equation at each node of the mesh. The dynamic problem is analyzed in two dimensions (plane strain). Two-dimensional Green's functions of infinite medium are expressed thanks to Hankel's functions [Dangla, 1989].

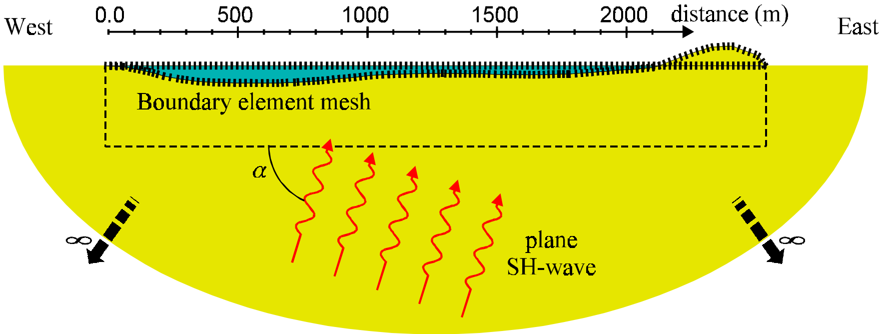

Figure 1. Boundary element mesh for a plane SH wave.

SH-wave : model and general results

In the case of SH-wave, the only displacement component is perpendicular to the propagation plane. The boundary element model is depicted in figure 1 and involves two homogeneous elastic isotropic media : the surface alluvial layer and the bedrock. Their mechanical characteristics are the following (density, shear modulus, velocity) : alluvial layer ρ_1=2000 kg/m3, μ_1=180 MPa (that is C_1=300 m/s), elastic bedrock ρ_2=2300 kg/m3, μ_2=4500 MPa (that is C_2=1400 m/s). The computations are performed thanks to the FEM/BEM code CESAR-LCPC [Humbert, 1989]. Figure 2 gives the amplification factor within the geological profile for two different frequency values. The first one (1.6 Hz) leads to a strong site amplification in the deepest part of the basin (West). The second one corresponds to important site effects in the thinnest part of the deposit (East). These results are in good agreement with experimental ones [Duval, 1996a,b].

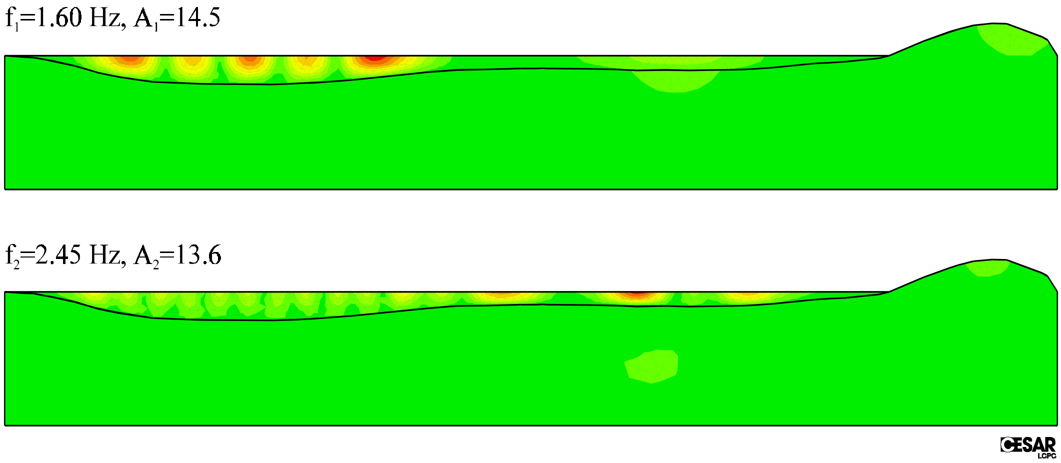

Figure 2. Estimation of amplification factor for a vertical SH-wave at different frequencies.

Comparison with experimental results

For sake of simplicity, the surface alluvial layer is assumed homogeneous. It is then necessary to estimate the influence of the mechanical characteristics of the deposit on the seismic motion amplification. Figure 3 gives the spectral ratios corresponding to three different shear moduli for the alluvial layer : μ_1=180 MPa as in the previous case, μ_2=2μ_1/3 and μ_3=μ_1/2. The absolute maximum amplification factor is determined for each frequency in every point of the surface. It is compared with experimental results (site/reference ratio) from real earthquake measurements (weak motion), taking into account standard deviation of experimental values.

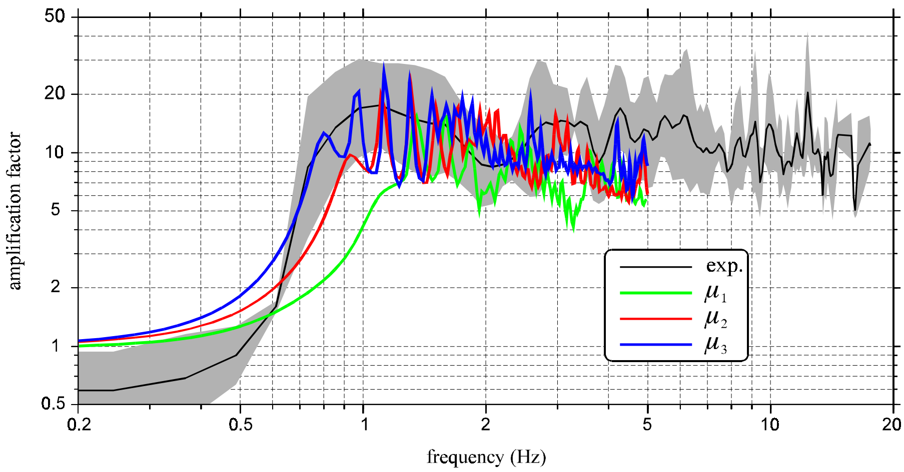

Figure 3. Maximum amplification factor vs frequency for three different shear moduli of the alluvial layer (comparison with experimental results).

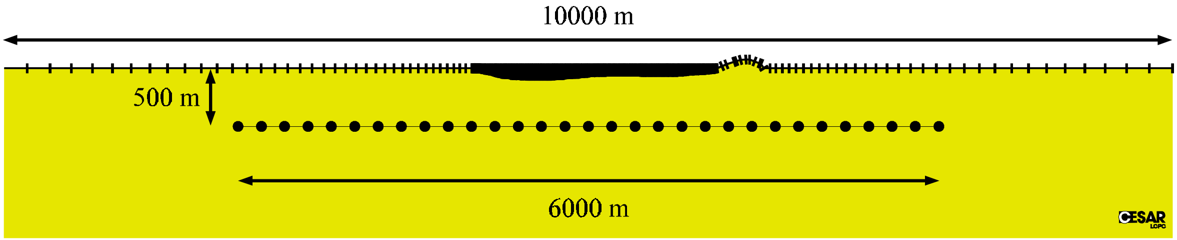

Figure 4. Boundary element mesh for P and SV waves.

The maximum values are different for the three numerical computations : it is 16.0 in the first case (μ_1=180 MPa), 25.0 in the second one ($\mu_2=3\mu_1/2$) and 30.0 in the last one ($\mu_3=\mu_1/2$). Furthermore, the amplification factor reaches its maximum value at different frequencies. In the third case ($\mu_3=\mu_1/2$), the amplification factor increases fastly with frequency. This case is much closer to the experimental results on that point and the values of amplification factor are always inside the interval of standard deviation of the experimental results. These results show the strong influence of the in situ measurements to determine the geometrical and mechanical characteristics of the different layers of the basin. The numerical approach allows for the sentivity analysis of the different parameters (mechanical, geometrical...).

P and SV-waves : numerical versus experimental results

For pressure and shear waves (P and SV), the seismic motion is within the propagation plane. It is no longer possible to use Green's functions of the infinite half-space (as for SH-waves). It is then necessary to model the free surface on both sides of the deposit. The boundary element mesh used for P and SV-waves is depicted in Figure 4 To avoid mesh truncation effects, it has to be wider than the previous mesh to take into account the infinite free surface. Its total width is chosen equal to 10000 m. The P and SV-wave seismic loadings are generated considering several ponctual sources under the alluvial deposit at a depth of 500 m. The polarization of these sources is radial for P-wave and orthoradial for SV-waves. For the analysis, the amplitude of the ponctual sources is arbitrary since the computed parameter is the amplification factor of the site. It is estimated relatively to a preliminary numerical result for which the geometry is a simple homogeneous half-space. Numerous numerical and experimental results for the different motion components are proposed in [Duval, 1996a, Semblat, 1999a,c]. In the following section, an experimental spectral ratio for vertical motion (P-wave) is compared with numerical results from a modal approach.

MODAL APPROACHES

Eigenmodes of the geological structures

Modal approaches for the analysis of geological structures

Modal approaches for the vibration analysis of geological structures is increasingly used [Paolucci, 1999, Zhao, 1996]. These approaches generally provide the fundamental vibration frequencies of geological structures taking into account their geometry [Dobry, 1976, Paolucci, 1999] or the inhomogeneity of their mechanical features [Zhao, 1996]. Considering different types of assumptions, it is then possible to estimate the fundamental frequency of a specific alluvial deposit. It is nevertheless not possible to directly compare various frequencies towards seismic motion amplification.

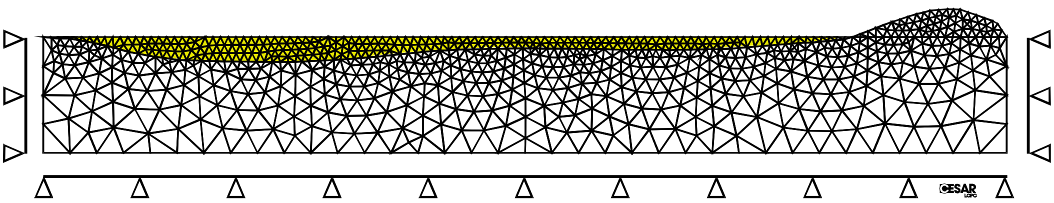

Figure 5. Finite element mesh for the determination of eigen modes.

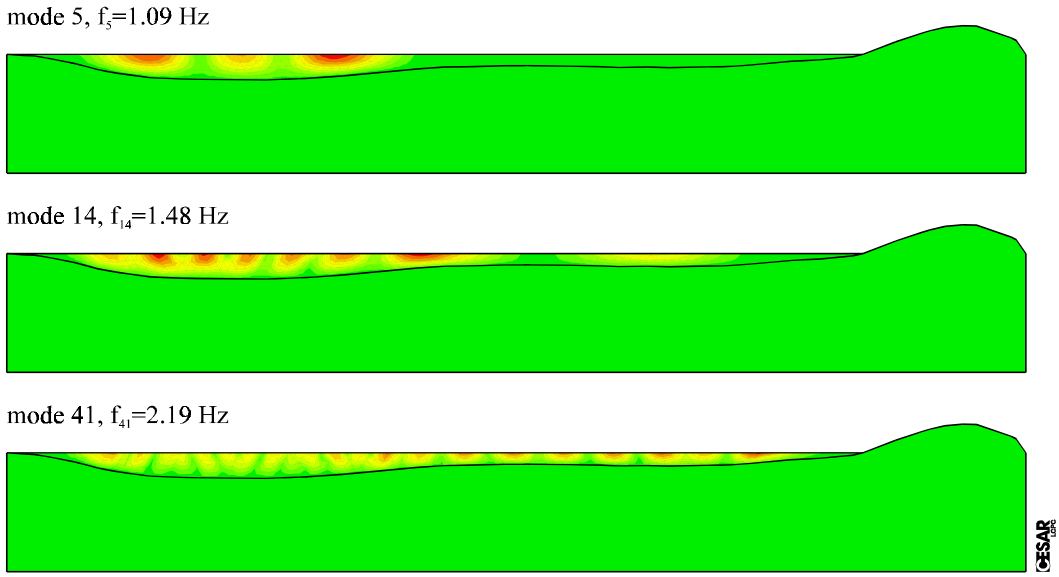

Figure 6. Eigenmodes and eigenfrequencies (5th, 14th and 41st) of the basin.

Eigenfrequencies and eigenmodes of the site

The eigenfrequencies and eigenmodes of the surface deposit and part of the surrounding bedrock are estimated by the finite element method. The computations are performed considering the 2D model depicted in figure 5 and its in-plane modes of vibration, that is vertical and horizontal displacements (plane strain). As the number of degrees of freedom of the model is large, many different eigenmodes are found in the frequency range of the problem : 49 different eigenfrequencies are determined below 2.4 Hz. Figure 6 gives the 5th, 14th and 41st eigenmodes of the site of respective eigenfrequencies 1.09 Hz, 1.48 Hz and 2.19 Hz. These modes of vibration are local modes mainly located in the surface alluvial layer. As shown in Figure 6 local modes appear in the deepest part of the deposit first (low eigenfrequencies) and local (relative) vibratory motion is stronger in the thinnest part of the basin for higher modes. These qualitative results are in agreement with experimental and numerical (BEM) ones. At this stage, it is however not possible to know which of these eigenmodes are going to lead to the highest local seismic motion. The main point is now to discriminate between the numerous frequency values those corresponding to the strongest site amplification.

EFFECTIVE MODAL MASS FOR THE ANALYSIS OF AMPLIFICATION

Modal excitation factor and effective modal mass

Considering the modes of vibration of the site determined in the previous section, we will now try to determine which of the numerous eigenmodes have the largest contribution to the response of the site under a specific excitation. In the field of structural dynamics, the dynamic response of a structure can be estimated considering its eigenmodes and using a modal superposition method. This method allows the superposition of the separate modal displacement contributions to determine the whole dynamic response of the structure [Clough, 1993]. It is generally not necessary, in the case of structures, to include all the higher order modes in the superposition process. For civil engineering structures, it is often sufficient to keep the only ten first eigenmodes to have an accurate estimation of the response. The finite element model of the basin depicted in Figure 5 has more than 1500 degrees of freedom and the number of eigenmodes in the frequency range of the problem is large. One has to determine the contribution of each mode to the dynamic response of the site.

In the modal superposition method, there are two widely used concepts for the analysis of the contribution of the different modes of a structure to its whole dynamic response : the *modal excitation factor* and the *effective modal mass*. Considering a translational excitation at the rigid-base of the structure, the modal response Y_n of the structure in the normal coordinates could be written as follows [Clough, 1993] :

$$M_n\ddot{Y}_n + C_n\dot{Y}_n + K_nY_n = \Xi_n\ddot{v}_g(t) \qquad (2)$$

where M_n, C_n and K_n are the generalized mass, damping and stiffness properties of mode n and Ξ_n is the modal excitation factor such as : $\Xi_n = {}^T\Phi_n m\{1\}$ with {1} the unit translation applied at the base of the structure.

The relative displacement vector produced in each mode n is then given by :

$$Y_n(t) = \Phi_n \frac{\Xi_n}{M_n \omega_n} V_n(t) \tag{3}$$

where $V_n(t)$ is the modal earthquake response Duhamel integral [Clough, 1993].

The modal excitation factor then expresses the contribution of the mode to the whole response of the structure. Using this parameter, the base shear force during the earthquake $F_S(t)$ can be written under this form :

$$F_S(t) = \sum_n \frac{\Xi_n^2}{M_n} \omega_n V_n(t) \tag{4}$$

The term Ξ_n^2 / M_n corresponds to the part of the total mass responding to the earthquake in mode n. It is generally called the effective modal mass of the structure. The sum of the effective modal masses of all the modes is equal to the total mass of the structure.

Modal excitation factors of the basin

Considering the finite element model depicted in Figure 5 , one assumes the seismic loading as a rigid base excitation. The dynamic excitation is then applied as a volumic translational density of force (in the vertical direction) within the whole model. The modal excitation factors and the effective modal masses for this specific excitation are computed for the fifty first eigenmodes of the geological profile, that is between 0.4 and 2.4 Hz.

The values computed for a vertical translational excitation are given in Figure 7 . Values of modal excitation factor and effective modal mass are very small for frequencies below 1.2 Hz. They fastly increase between 1.5 and 1.8 Hz. The maximum values are reached for 27th (f_{27}=1.81 Hz) and 29th (f_{29}=1.84 Hz) modes. For these modes, the percentages of effective modal mass are respectively m_{27}^{eff} =12.0% and m_{29}^{eff} =14.7%. The largest part of the mass of the model responding to the vertical earthquake excitation is in a frequency band around 1.8 Hz. These results are compared with experimental measurements in the next section. A good way to characterize the accuracy of the modal superposition method is to compute the cumulated modal mass that is the sum of the effective modal mass for all modes under a given frequency. For the finite element model considered herein, 60% of the total mass is recovered at 2.0 Hz and up to 80% at 2.5 Hz. These values are generally considered as a sufficient part of the total mass to give an accurate estimation of the dynamic response of the structure. For the geological site studied here, the graph of Figure 7 is then corresponding to the main modal contributions in which modes 27th and 29th have the largest one around a frequency value of 1.8 Hz.

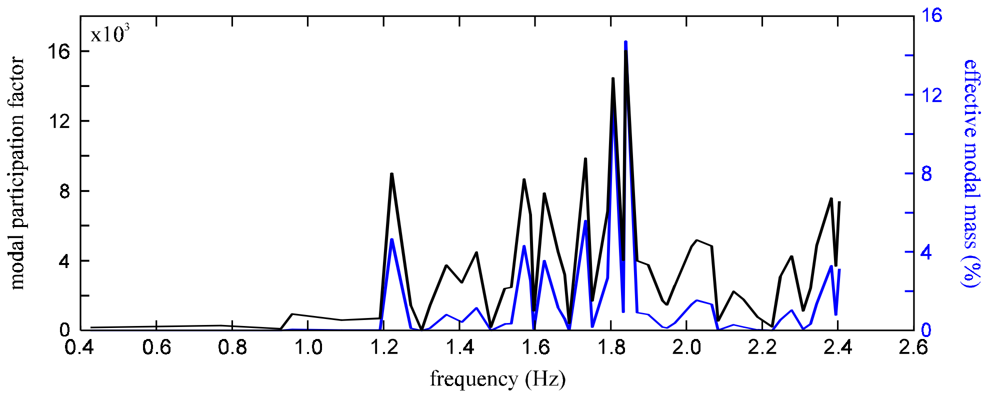

Figure 7. Modal participation factor (left) and percentage of effective modal mass (right) vs frequency.

Comparison between effective modal mass and experimental results

Since the estimation of the eigenmodes is carried out with plane-strain assumption and with a vertical translational excitation, the numerical results of the modal approach are compared with experimental spectral ratios of the vertical component of displacement. Figure 8 gives the experimental vertical spectral ratio for realearthquakes (weak motion) and the corresponding standard deviation (left scale). The percentages of effective modal mass are plotted along the right scale between 0.001 and 100%. The spectral ratio and the effective modal mass are very small for frequencies below 1 Hz. There is a fast increase of the values of these quantities between 1.5 and 2.0 Hz. Both methods (experimental and modal computation) lead to the strongest site response in the same frequency range, that is around 1.8 Hz. Above this frequency, both vertical spectral ratio and effective modal mass show a significant decrease. The agreement between seismic measurements on site and modal finite element computations seems to be very good. Through modal excitation factor and effective modal mass, modal analysis then appears as an interesting method to determine the frequency range corresponding to the strongest seismic site effects for two-dimensional geological profiles. For vertical amplification, considering the model presented in Figure 5 , P-wave BEM computations also give accurate numerical results [Semblat, 1999c].

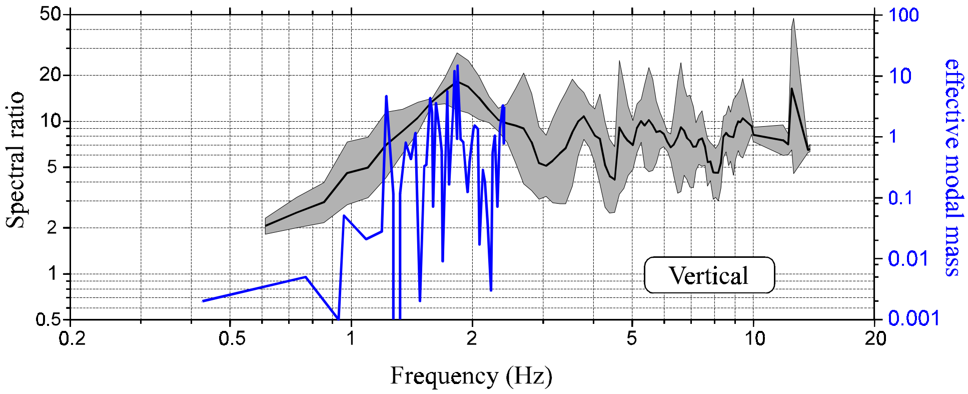

Figure 8 Experimental vertical spectral ratio (left) and percentage of effective modal mass (right) vs frequency

CONCLUSION

The analysis of seismic site effects is carried out through two different numerical methods. The specific site considered is located in the city of Nice (France) where experimental investigations give amplification factors between 10 and 30 for frequencies above 1.0 Hz.

The boundary element method is very efficient since it allows an accurate estimation of the amplification levels, the occuring frequencies as well as their location along the surface of the basin. Furthermore, the analysis of each component of displacement (North-South, East-West and vertical) is possible considering P, SV and SH-wave excitations. The influence of incidence can also be determined [Semblat, 1999a,c].

Another numerical method (modal analysis) is considered giving various eigenfrequencies of the alluvial deposit. To discriminate between these frequencies towards amplification of seismic motion, the analysis of the corresponding modal excitation factors and effective modal masses for a vertical excitation lead to higher values around highest amplification frequencies. The quantitative comparisons between the numerous eigenmodes of the finite element model lead to very interesting results. Comparisons with vertical spectral ratios determined experimentally are very satisfactory. However, for the application of the modal approach, several issues should be adressed : wave velocity distribution, effect of the model size and refinement, influence of damping, computational cost.

Finally, the Boundary Element Method is a good approach for the modelling of wave propagation and excitations sources (wave type, incidence...), the modal approach is rather a method for vibratory response analysis through the computation of the eigenfrequencies of a specific geological profile. The results presented here nevertheless show that for simple excitations, it is possible to discriminate between these eigenmodes to determine the frequency ranges giving the strongest seismic site effects.

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