

## ADV RESPONSE SPECTRUM FOR PERFORMANCE EVALUATION OF RC BUILDING WITH/WITHOUT DAMPING DEVICES

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### SUMMARY

After the 1994 Northridge Earthquake or the 1995 Hyogoken-Nanbu (Kobe) Earthquake, methods for seismic building design have been extensively reviewed and the movement to performance-oriented ones is prompted. The purpose of this study is to present an energy-based simple but accurate method to predict the maximum displacement of RC buildings exposed to strong earthquake ground motions. The method is based on taking the balance of demand and capacity energy during half-cycle of building vibration. The demand is input seismic energy and the capacity is hysteretic energy absorbed in the building. Graphically, the demand is represented by a Velocity-Displacement Response Spectrum (DVRS) curve and the capacity by a curve relating the deflection and the equivalent velocity converted from the hysteretic energy. A point of intersection of the two curves corresponds to the maximum displacement. Through some examples, availability of the method is confirmed by comparing predicted results with those by response analysis. Though the method is only applied to maximum deflection prediction in the study, it can readily be used to evaluate the seismic performance level of new and to-be-retrofitted old buildings.

### INTRODUCTION

Seismic design methods for building structures have been revised based on many lessons from severe earthquake damage. In the case of reinforced concrete buildings, the method to save most buildings from catastrophic failure by increasing their strength and ductility was almost complete. However, in view of devastating structural damage during the 1994 Northridge Earthquake or the 1995 Hyogoken-Nanbu (Kobe) Earthquake, we have come to realize that urban seismic disaster prevention requires not only the safety of buildings and habitants but also the integrity of the community. This means that it is quite important to develop a new method to provide individual building with appropriate degree of seismic safety depending on its own social mission.

Conventional seismic design method is fundamentally norm-oriented and it only specifies requirements to assure minimum resistance and ductility to prevent buildings from total collapse. On the other hand, the new design method is performance-oriented. To put this new method to practical use, we have to find a solution to two issues. One is to categorize structural seismic performance level. The other is to provide a generalized design method to assure the performance level agreed between the designer and the owner. The former issue has been discussed a lot and some reasonable proposals are obtained. On the other hand, the latter issue still remains to be considered a little more.

A method based on ADRS format[ATC,1996] to predict the maximum displacement of the buildings exposed to design earthquake ground motions has been presented. In the method, both displacement and acceleration

spectra are drawn by ADRS format on the same plane, on which load-deflection relationship of the building is superposed. Then the maximum displacement is obtained as an intersection of the spectrum (the demand) and the load-deflection curve (the capacity). This method is indeed useful because it can take into consideration the difference in hysteretic properties of the building and it allows us to find the maximum displacement graphically.

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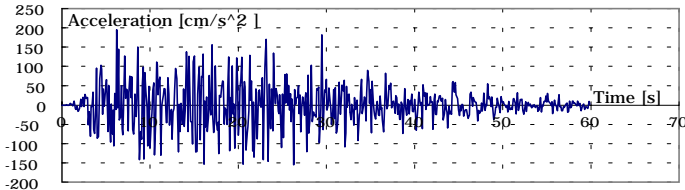
However, the method is still based on the load-displacement relationship and does not include the case that a building installs viscous or visco-elastic dampers.

Authors [Soda et. al., 1998] have already confirmed that these viscous type of damping devices are very effective to prevent buildings from such catastrophic failure as that by the damage concentration, because the resistance of these devices are velocity-dependent being capable of both absorbing input energy and distributing structural damage uniformly to every part of the building. Therefore, in this paper, an energy-based method to predict the maximum displacement of RC buildings with and without those types of damping devices will be presented. The method could be applied to evaluate seismic performance level of new and existing old buildings.

**METHOD FOR MAXIMUM RESPONSE PREDICTION**

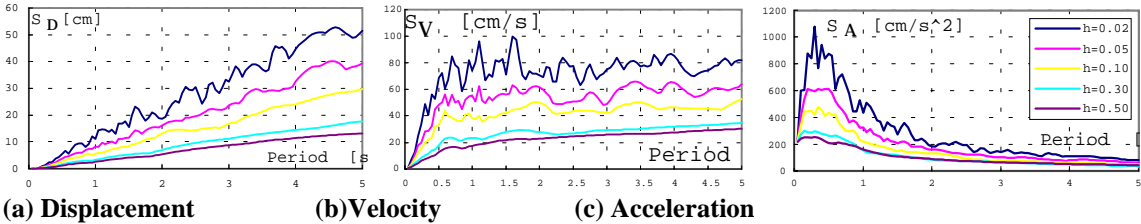
**Ground Motion and Response Spectra**

Figure one shows a time history of an artificial earthquake ground motion, which is often used to assure the seismic safety of tall buildings or base-isolated buildings in Japan. The ground motion is the one that is supposed to take place on hard soil.



**Figure 1: Time History of an Artificial Earthquake Ground Motion**

In Fig. 2, earthquake response spectra of this ground motion are shown. They are given for different values of damping factor  $h$ .



**Figure 2: Response Spectra**

**ADRS Format**

Making use of Figs. 2(a) and 2(c),  $S_D$ - $S_A$  relationship is drawn in Fig. 3. This type of presenting response spectra was first introduced [Mahaney,1993] as an ADRS (Acceleration Displacement Response Spectra) and extensively applied to seismic evaluation of concrete buildings [ATC,1996]. By just multiplying the vertical axis by structural mass, a force-displacement plane will be obtained.

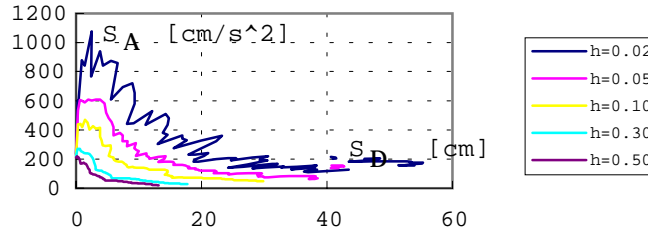
When the ADRS spectrum is transformed to a force-displacement spectrum and a load-deflection relationship of a building model is superposed in the same format, maximum displacement can be estimated as an intersection of a straight line corresponding to the equivalent stiffness of the model and a spectrum for the equivalent damping factor. More concrete procedures for the prediction is as follows:

- 1) to formulate the equivalent stiffness and the equivalent damping factor of a building model in relation to the displacement
- 2) to assume the maximum displacement
- 3) to calculate the equivalent stiffness and the equivalent damping factor
- 4) to get another approximate maximum displacement as an intersection of an ADRS curve corresponding to the equivalent damping factor and an straight line from the origin with the equivalent stiffness.

5) to repeat the procedures until the maximum displacement converges.

When rough estimate is enough, some intersections of a load-deflection curve and ADRSs for different damping factors be checked to see if the damping factor for those intersection displacement is close to the damping factor of that demand spectrum. In the case that the additional damping device is only displacement dependent,

additional strength and damping effects can easily be taken into consideration. But, in the case that the device is velocity dependent, there will be an error due to the fact that the maximum resistance does not necessary takes place when the displacement takes its maximum.

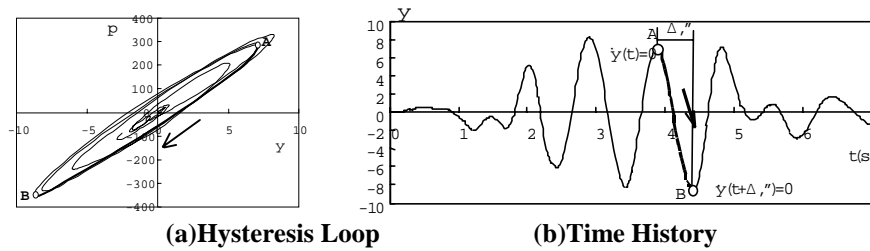


**Figure 3 Response Spectra representation by ADRS Format**

### VDRS Format

### Half Cycle Seismic Input Energy and Equivalent Velocity

Seismic input energy into a building can be defined in two ways. One is the total input energy while the ground motion continues and the other the short-period energy like that corresponding to half cycle or one cycle of building vibration. The latter has been found to be more related to maximum building displacement than the former which affects much stiffness degradation due to repeated deflection [Iwasaki, T. et. al., 1998, Soda, S., et. al., 1994, Nagahashi, S., 1992]. In this study, half cycle hysteretic energy consumed in the building during a peak-to-peak displacement like from point A at  $t$  to point B at  $t+\Delta t$  (see Fig. 4) is defined as an instantaneous seismic input energy. The reason why a half-cycle is taken out is that the center of the hysteresis loop tends to shift, especially in nonlinear vibration.



**Figure 4: Definition of Peak-to Peak Half Cycle Displacement**

The motion of a SDOF building model is expressed by Eq. (1), in which  $y$  denotes the building displacement (=deflection) and the  $z$  the ground motion displacement. Let both sides of the equation be multiplied by  $dy = ydt$  and integration from  $t$  to  $t+\Delta t$  be performed as Eq. (2), then the instantaneous kinetic energy  $\Delta E_k$ , damping energy  $\Delta E_v$ , potential energy  $\Delta E_p$  and seismic input energy  $\Delta E_i$  are obtained as in Eq. (3). These four kinds of energy are the functions of  $t$  and the maximum value of  $\Delta E_v$  will be defined by Eq. (4) as half cycle seismic input energy. Based on the relationship  $E=(m/2)v^2$ , equivalent velocity is defined as Eq. (5).

$$M\ddot{y} + C\dot{y} + Ky = -M\ddot{z} \quad (1)$$

$$\int_t^{t+\Delta t} M\ddot{y}\dot{y}dt + \int_t^{t+\Delta t} C\dot{y}\dot{y}dt + \int_t^{t+\Delta t} Ky\dot{y}dt = -\int_t^{t+\Delta t} M\ddot{z}\dot{y}dt \quad (2)$$

$$\Delta E_K + \Delta E_V + \Delta E_P = \Delta E_I \quad (3)$$

$$\Delta E_{\max} = \max[\Delta E_V] \quad (4) \quad V_{eq} = \sqrt{\frac{2\Delta E_{\max}}{M}} \quad (5)$$

### Equivalent Velocity Response Spectrum

Figure five shows the spectra for the equivalent velocity  $V_{eq}$ . Input ground motion is the same as shown in Fig. 1. It should be noted that there is little difference between the three spectra for different values of damping factor.

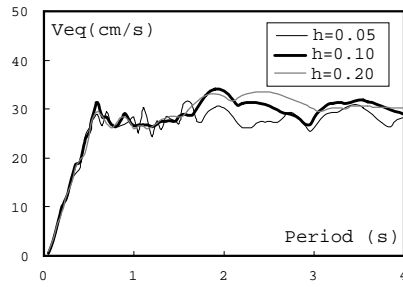


Figure 5:  $V_{eq}$  Spectra

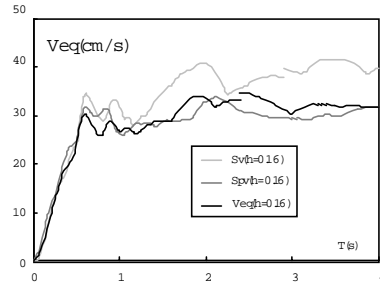


Figure 6:  $V_{eq}$  and  $S_{pv}$  Spectra for  $h=0.16$

Maximum half-cycle seismic input energy, in most cases, takes place just before or just after the maximum displacement is registered. Therefore,  $\Delta E_{\max}$  can be approximated by hysteretic energy consumed during half-cycle sinusoidal displacement with amplitude of  $S_d$ . Letting natural frequency, mass, and damping factor be  $\omega$ ,  $M$  and  $h$ , then the  $\Delta E_{\max}$  is expressed by  $S_d(h)$  as Eq. (6). And, by taking into consideration the pseudo velocity response  $S_{pv}$  is expressed as  $S_{pv}(h)=\omega S_d(h)$ ,  $V_{eq}$  will be represented as Eq.(7).

$$\Delta E_{\max} = M\pi\omega^2 S_{D(h)}^2 \quad (6)$$

$$V_{eq} = \omega S_D(h)\sqrt{2\pi h} = S_{pv}(h)\sqrt{2\pi h} \quad (7)$$

As can be seen, if  $h=1/(2\pi)$  is substituted into Eq. (7),  $V_{eq}$  will be the same as  $S_{pv}$ . Figure 6 shows the  $V_{eq}$  and  $S_{pv}$  for  $h=0.16$ . It is confirmed that the both are almost the same. A thin line in the same figure is the velocity response spectrum  $S_v$  that is, as has been indicated in many textbooks on structural dynamics, a little different from the  $S_{pv}$  in the range of relatively long natural period is observed. In the following section,  $S_{pv}$  is used to represent the maximum velocity.

### VDRS Format

Taking  $S_D$  and corresponding equivalent velocity  $V_{eq}$  calculated by Eq. (7) on horizontal and vertical axis respectively, Velocity-Displacement Response Spectrum (VDRS) is obtained as Fig.7. It is observed that the spectra except for the one for  $h=0.02$ , are very close to one another.

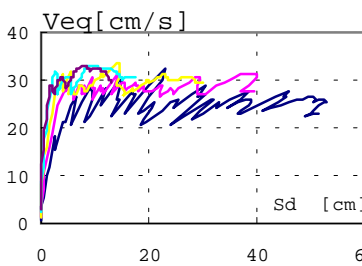


Figure 7: VDRS for Different Damping Factors

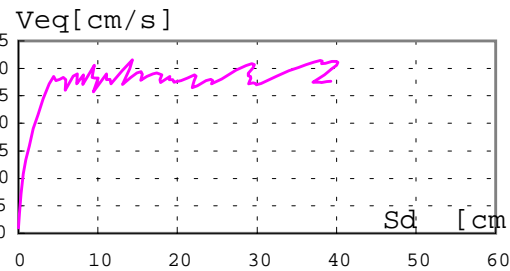


Figure 8: VDRS for  $h=0.05$

Therefore, in the practical use, a spectrum of, for example,  $h=5\%$  in Fig. 8 can be assumed to represent the VDRS curve for the ground motion used in this study.

## Maximum Displacement Prediction Method

When the load-deflection relationship of a building model is assumed, its half-cycle hysteretic energy and hence the corresponding equivalent velocity can be easily formulated in relation to the displacement or ductility factor as in the following example. Superposing this  $V_{eq}$ -deflection relationship in the same VDRS format, maximum displacement can be obtained as a point of intersection of these two curves. More concrete procedures of the method is as follows:

- 1) to formulate the equivalent velocity in relation to the displacement
- 2) to put the  $V_{eq}$ -deflection relationship on the VDRS diagram
- 3) to determine the maximum displacement as an intersection of a VDRS curve and a  $V_{eq}$ -deflection curve.

The advantage of this method is that it does not require any iterative calculation, because VDRS is, in practice, not dependent on the magnitude of damping factor. In the case that the additional damping device is only displacement dependent, additional damping capacity can simply be reflected to  $V_{eq}$ -deflection relationship of a building model itself. However, in the case that the device is velocity dependent, the capacity spectrum should include the effects of the frequency, requiring some iterative operations.

## EXAMPLES

### Analytical Model

A single-degree-of-freedom RC building model is used in the analysis. Load-deflection relationship is represented by the modified Takeda model, which is shown in Figure 9. In the figure (b),  $\bullet$  denotes the factor to determine the reversal stiffness ratio and is set to 0.4 in the analysis. Assuming that structural mass  $M=1.0$  tf·s<sup>2</sup>/cm and natural period be 1.0 second, elastic stiffness  $K_F$  is 39.5 tf/cm.  $Q_y$  is the yield strength and  $Q_c=(1/3)Q_y$  is the cracking strength. The second and the third stiffness,  $K_{F2}$  and  $K_{F3}$  are set to 1/4 and 1/1000 of  $K_F$ . Three models with different yield strength are used in the study. The Model-1, Model-2 and Model-3 each has the strength corresponding to 20 %, 30 % and 50 % of the building's weight, i.e., 980 tf.

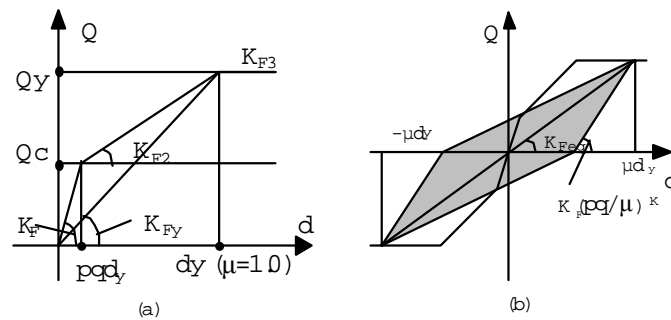


Figure 9: Modified Takeda Model

Stiffness degradation rate  $k_{Feq}$  and equivalent damping factor  $h_{Feq}$  are formulated by Eqs. (8) and (9) as functions of ductility factor  $\mu=d/d_y$ . Parameters  $p$ ,  $q$ , and  $r$  are defined by Eqs. from (10) to (12).

$$k_{Feq} = \begin{cases} 1 & (0 \leq \mu \leq pq) \\ \frac{pr}{\mu} & (pq \leq \mu \leq 1) \\ \frac{p}{\mu} & (1 \leq \mu) \end{cases} \quad (8)$$

$$h_{Feq} = \begin{cases} 0 & (0 \leq \mu \leq pq) \\ \frac{1}{\pi} \left\{ 1 - \frac{r}{q} \left( \frac{pq}{\mu} \right)^{(1-\kappa)} \right\} & (pq \leq \mu \leq 1) \\ \frac{1}{\pi} \left\{ 1 - \frac{1}{q} \left( \frac{pq}{\mu} \right)^{(1-\kappa)} \right\} & (1 \leq \mu) \end{cases} \quad (9)$$

$$p = \frac{K_{Fy}}{K_F} \quad (10)$$

$$q = \frac{Q_c}{Q_y} \quad (11)$$

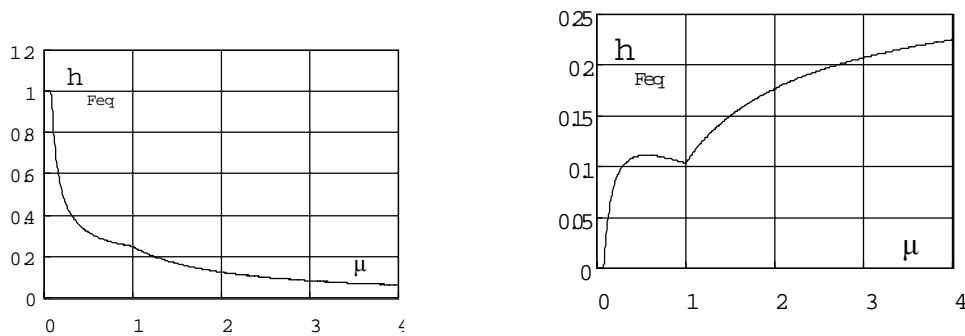
$$r = \frac{\{\mu(1-q) + q\} - pq}{1 - pq} \quad (12)$$

### Input Ground Motion

Input ground motion is an artificial one shown in Fig. 1. Its original maximum acceleration is 207.3 cm/s<sup>2</sup>. In the analysis, it is also normalized to 100, 300 and 400 cm/s<sup>2</sup> to see the effect of the intensity of the ground motion on the accuracy of prediction.

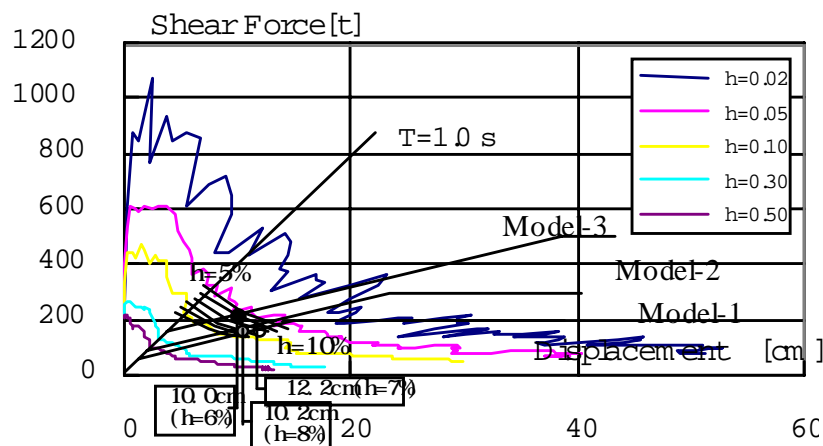
### Prediction Based on ADRS Format

Figure 10 shows  $k_{Feq}$  and  $h_{Feq}$  for the case that  $p=1/4$ ,  $q=1/3$  and  $\kappa=0.4$ .



**Figure 10: Stiffness Degradation Rate( $k_{Feq}$ ) and Equivalent Damping Factor( $h_{Feq}$ ) ( $p=1/4$ ,  $q=1/3$ ,  $\kappa=0.4$ )**

Figure eleven shows how the maximum displacement of Model-1, Model-2 and Model-3 are obtained. Since the demand, i.e., the relation between shear force and displacement is provided only for the discrete damping factors, the one corresponding to a specific damping factor is linearly interpolated.

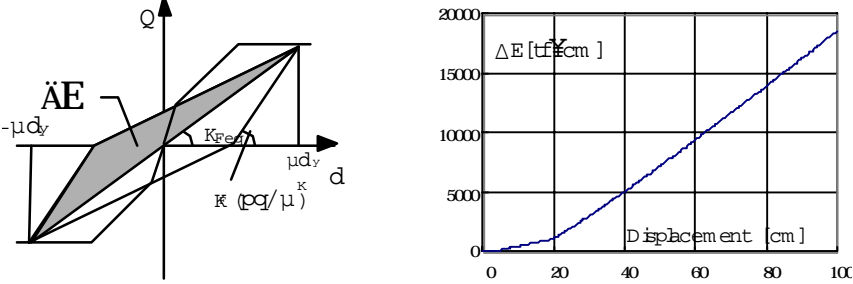


**Figure 11 Maximum Displacement of Models Predicted by ADRS Format**

In this case, points of intersection of load-deflection curve, i.e., the capacity spectra are checked to see if the damping factor for their displacement is close to the damping factor of the demand spectrum. This relatively rough prediction is quite simply performed with the aid of Fig. 10 and appropriate engineering judgment. More accurate prediction can be performed by following the procedures listed in section 2.3.

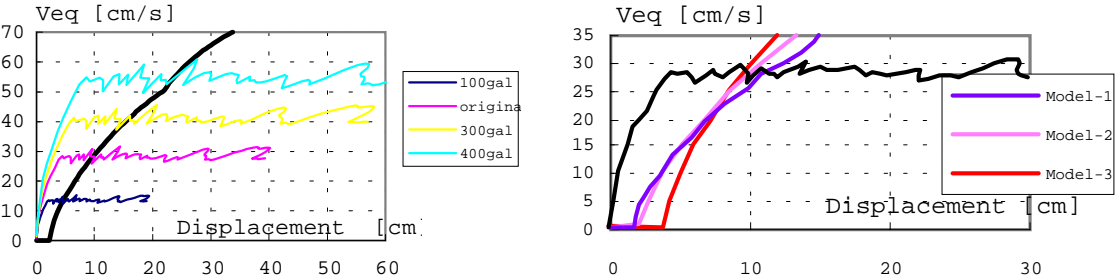
**Prediction Based on VDRS Format**

Let the half-cycle hysteretic energy  $\Delta E$ , schematically shown in Fig. 11, be formulated for the modified Takeda model with specific mechanical properties of the Model-2.



**Figure 11: Half-Cycle Hysteretic Energy**

Then a  $V_{eq}$ -deflection spectrum (capacity) of the Model-2 is obtained as shown by a solid thick line in Fig. 12(a), in which VDRS for the ground motion with four different intensities are also drawn. Those VDRSs are based on  $h=5\%$  pseudo velocity spectrum. By finding the points where the demand and the capacity spectra intersect, maximum displacement will be predicted. In fig. 12(b), input ground motion is the original one and the strength of the model is different from each other, in the same manner as the ADRS format prediction in section 3.3.



**(a)for Different Intensities of Ground Motion (b)for Different Models**  
**Figure: 12 Maximum Displacement Prediction by VDRS Format**

Table one and two compares maximum displacement by response analysis and by proposed VDRS format. It is seen that the VDRS method is quite useful to predict maximum displacement. It is also seen that predicted maximum displacement both by VDRS format and by ADRS format coincide fairly well with that by response analysis.

**Table: 1 Comparison of the Maximum Displacement**

	100 cm/s <sup>2</sup>	original	300 cm/s <sup>2</sup>	400 cm/s <sup>2</sup>
Predicted by VDRS	4.61cm	10.2cm	16.4cm	23.1cm
Response Analysis	4.88cm	10.6cm	15.4cm	24.4

**Table 2: Comparison of Maximum Displacement**

	Model-1	Model-2	Model-3
Predicted by VDRS	10.6cm	10.0cm	9.3cm
Predicted by ADRS	12.2cm	10.2cm	10.0cm
Response Analysis	10.7cm	10.6cm	10.7cm

**CONCLUSIONS**

As an important tool to put into practical use the new concept of performance-oriented seismic design of

building structures with and without damping devices, an energy-based method is studied. Important findings are as follows.

1)Equivalent velocity spectrum  $V_{eq}$  is defined based on half-cycle seismic input energy. It can simply be obtained from normal pseudo velocity response spectrum  $S_{pV}$  by the relation  $V_{eq} = S_{pV} \sqrt{2\pi h}$ .  $V_{eq}$  is, in practical use, not dependent on damping capacity of a building.

2)VDRS representation of a ground motion, i.e., a demand spectrum consists of  $V_{eq}$  on the vertical axis and displacement on the horizontal axis. Capacity of a building can also be represented by equivalent velocity, which is defined as  $V_{eq} = \sqrt{2\Delta E / M\Delta E}$  is hysteretic energy consumed in a half-cycle vibration of the building.

3)When the demand and the capacity are superposed in the same VDRS format, intersection of these spectra correspond to the maximum displacement of the building.

4)ADRS format is basically based on load-deflection relationship and so velocity-dependent resistance is not properly taken into consideration to predict maximum displacement. On the other hand, VDRS format is based on energy-deflection relationship, and the damping effects by velocity dependent damping devices can be readily considered.

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