PERFORMANCE BASED SEISMIC DESIGN OF BUILDING STRUCTURES WITH VISCOELASTIC DAMPERS

Satsuya SODA[1] And Yuji TAKAHASHI[2]

SUMMARY

The first part of this paper outlines mechanical properties of a viscoelastic damper (VED) which are temperature- and frequency-dependent. The second part deals with an analytical method for quantifying damping capacity of a building with the VEDs whose mechanical properties are represented by generalized Maxwell models. It is shown that the damping effect depends on the stiffness of members to support the damper, and that there is an optimum amount of the damper to provide the structure with the maximum damping factor. It is also proved that VEDs are available to assure the ultimate stability of multi-story buildings for even quite intense ground motions.

INTRODUCTION

The VED has been applied in both seismic design of new buildings and retrofit of existing ones [Soong and Dargush, 1997] because it can reduce earthquake response of structures [Soda, 1996]. In general, mechanical properties of VED, such as stiffness and damping capacity, vary depending on ambient temperature. Therefore, VEDs of appropriate capacity must be installed in buildings considering their temperature-dependent properties.

Chapter two outlines mechanical properties and an analytical model of VED. Then, in chapter three, the damping factor and the natural frequency of an SDOF system with VED are formulated. The optimum capacity of the VED, which maximizes the damping factor within a range of ambient temperature to be considered, is quantified. In chapter four, earthquake response spectra of SDOF models with optimum amount of VEDs are shown. In chapter five, earthquake response analyses of MDOF inelastic models are performed to show how effective the installation of VEDs of appropriate capacity in multi-story buildings is to assure their seismic performance.

OUTLINE OF VED

Figure 1 shows the basic mechanism of a linear VED. The symbol y and p indicate shear deformation of viscoelastic material and load, respectively. When the VED is subjected to harmonic deformation, the relationship between shear deformation and load becomes an elliptic hysteresis loop as shown in Fig. 2. The area of the loop equals energy absorbed by the VED. Load as well as energy is proportional to the shear area S and inversely proportional to the thickness d. Therefore, S/d called shape coefficient represents the capacity of the VED. Deformation and amplitude of the VED are sometimes expressed as the ratio of y to d.

Mechanical properties of the VED are represented by a complex modulus γ(if) represented by Eq. (1) [Soong and Dargush, 1997]. The real part γ'(f) of the complex modulus is called storage modulus and equivalent to stiffness of the VED. The imaginary part γ"(f), loss modulus, corresponds to damping capacity. The loss tangent tanδ(f) is defined as Eq. (2) and indicates the damping ratio of the VED. Fig. 3 is to explain these moduli and factors in relation to the hysteresis loop given by a harmonic loading.

[1] *Dept. of Arch, Faculty of Science & Engineering, Waseda University, Tokyo, Japan:Email: soda@slws1.arch.waseda.ac.jp*
[2] *Graduate School of Science & Engineering, Waseda University, Tokyo, Japan:Email: taka@slws1.arch.waseda.ac.jp*

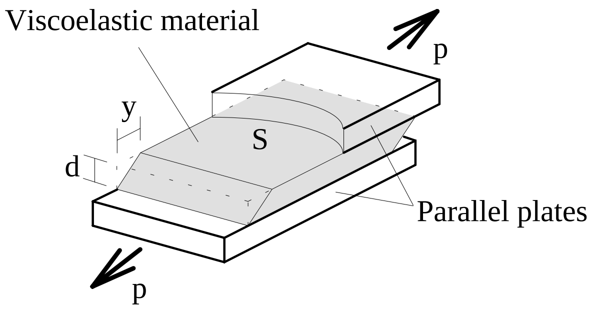


Figure 1: Basic mechanism of VED

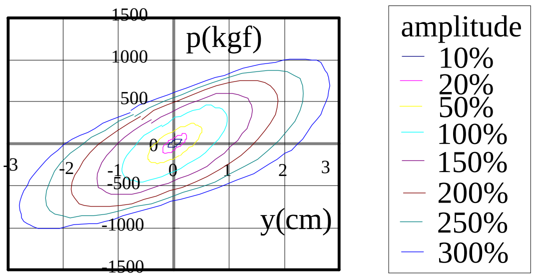


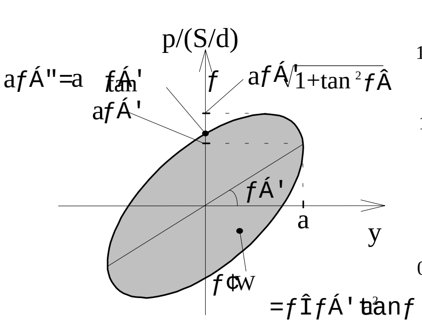
Figure 2: Hysteresis loops of VED

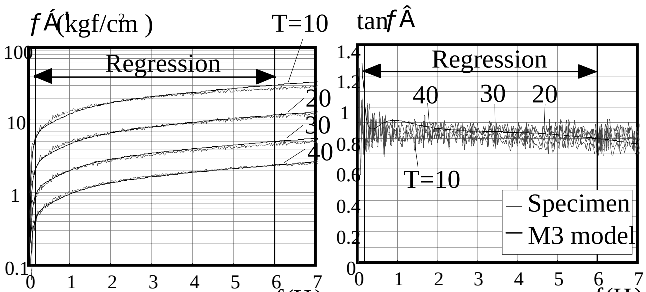
Figure 4 shows temperature- and frequency-dependent properties of complex modulus of a VED made of diene elastomer. Dotted lines are obtained by a random loading method [Soda and Takahashi, 1998]. In this figure, γ (if) is normalized by S/d, namely, expressed as the specific value for S/d = 1.0 (cm). It is shown that the tan δ of this VED is almost independent to temperature T and frequency f while the γ' is sensitive to both. Mechanical properties of this VED can be easily simulated with an M3 model shown in Fig. 5 [Soda and Takahashi, 1998]. Symbol λ_T is a temperature correction factor of the γ' shown as Eq. (3). Coefficients in Fig.5 are obtained from experimental data in Fig. 4 within the range of temperature from 10 ℃ to 40 ℃ and frequency from 0.04 Hz to 6.0 Hz. Solid lines in Fig. 4 indicate complex modulus of the M3 model. They are very close to the experimental results within the regression range regardless of T and f. It is also confirmed that the M3 model simulates hysteresis of the VED subjected to even non-stationary random excitation [Soda and Takahashi, 1998].

$$\gamma(if) = \gamma'(f) + i\gamma''(f) = \gamma'(f)\{1 + i\tan\delta(f)\} \quad (1)$$

$$\tan\delta(f) = \frac{\gamma''(f)}{\gamma'(f)} \quad (2)$$

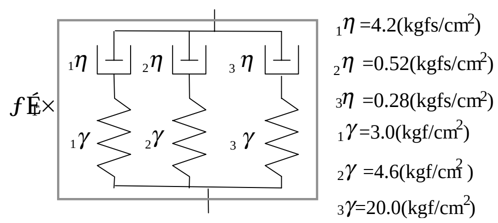
$$\lambda_T = \left(\frac{80}{T+60}\right)^{7.13} \quad (3)$$

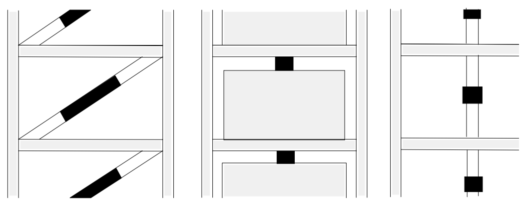




(a) Storage modulus **(b) Loss tangent**

Figure 3: Hysteresis loops of VED **Figure 4: Complex modulus of VED of diene elastomer**





(a) **(b)** (▬ :VED) **(c)**

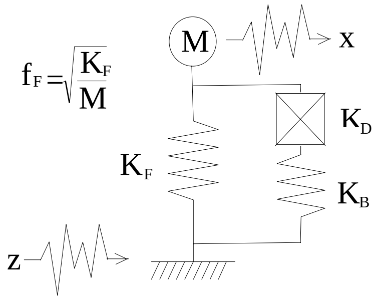
Figure 5: Analytical model of VED (M3 model) **Figure 6: Installation of VEDs in buildings**

VEDs are usually installed in each story of buildings with supporting members, for example, braces, walls and columns, as shown in Fig. 6. If the stiffness of the supporting members is small, the damping effect of the VED is small. Therefore, we must take account of the stiffness of the supporting member in predicting seismic behavior of buildings with VEDs.

QUANTIFICATION OF DAMPING EFFECTIVE OF VED

Optimum damping for linear system

A single-story building with a VED is idealized as an SDOF model shown in Fig. 7 [Fu and Kasai, 1998]. In the figure, KF, KB and KD represent the horizontal stiffness of a frame without a VED, of a supporting member and of the VED, respectively. Inherent damping of the main frame is neglected here because it is much smaller than that provided by the VED. Symbols fV and fF are the natural frequency of the frame with and without the VED. The input ground motion and the displacement of the mass are denoted by z and x, respectively.



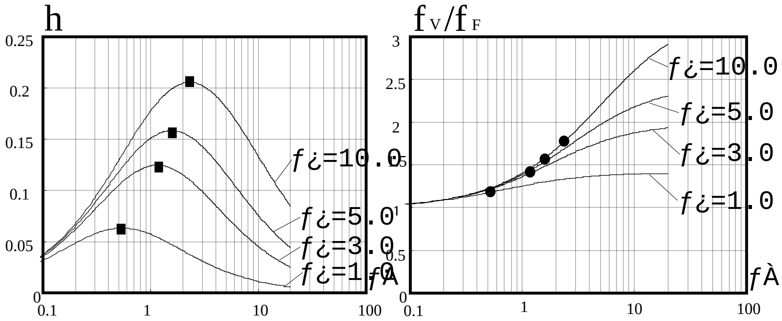


Figure 7: SDOF model with VED **(a) Damping factor** **(b) Natural frequency ratio**

Figure 8: Damping properties of SDOF model with VED [tan δ =0.85]

The equation of motion of the SDOF system is represented by Eq. (4), in which k' and k" are expressed by Eq. (5) and (6), and symbol i is an imaginary number. Parameters α and β are defined as the ratio of KB and KD to KF as defined in Eq. (7) and (8), respectively. From Eq. (4), the damping factor h and the natural frequency ratio fV/fF of the SDOF system are obtained as Eq. (9) and (10), respectively.

$$M\ddot{x} + (1 + k' + k''i)K_F x = -M\ddot{z} \quad (4)$$

$$k' = \frac{\alpha\beta(\alpha + \beta + \beta\tan^2\delta)}{(\alpha+\beta)^2 + (\beta\tan\delta)^2} \quad (5)$$

$$k'' = \frac{\alpha^2\beta\tan\delta}{(\alpha+\beta)^2 + (\beta\tan\delta)^2} \quad (6)$$

$$\alpha = \frac{K_B}{K_F} \quad (7)$$

$$\beta = \frac{K_D}{K_F} = \frac{\left(\frac{S}{d}\right)\times\gamma'}{K_F} \quad (8)$$

$$h = \frac{k''}{2(1+k')} \quad (9)$$

$$\left(\frac{f_V}{f_F}\right) = \sqrt{1+k'} \quad (10)$$

Figure 8 shows h and fV/fF with respect to α and β. the loss tangent tanδ in this case is set to be 0.85 based on the results in Fig. 4(b). It is observed that the curve of h has a peak indicated by symbols ■. The value of β (=β opt) which maximizes h is obtained as Eq. (11), and the corresponding damping factor hmax is expressed as Eq. (12) by substituting Eq. (11) into Eq. (9). Fig. 9 shows β opt and hmax in the case of tanδ = 0.85. Both of them increase monotonically with respect to α.

$$\beta_{opt} = \frac{1}{\sqrt{(1+\alpha)(1+\tan^2\delta)}}\alpha \quad (11)$$

$$h_{max} = \frac{\alpha\tan\delta}{2\{\alpha + 2 + 2\sqrt{(1+\alpha)(1+\tan^2\delta)}\}} \quad (12)$$

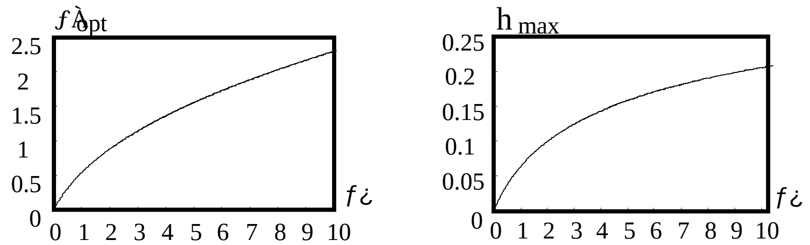
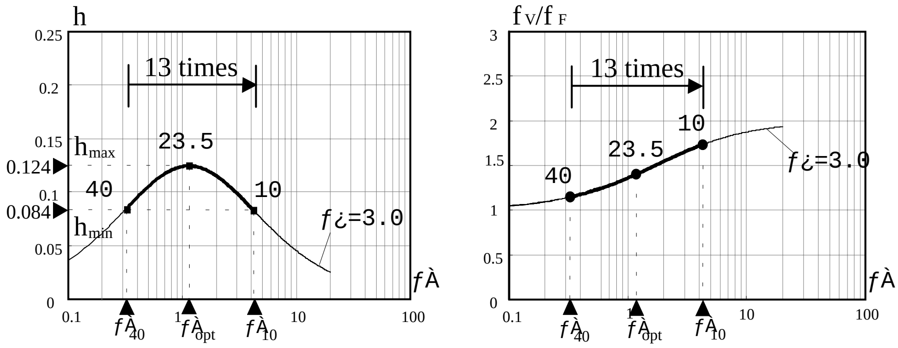


Figure 9: Optimum stiffness ratio of VED (β opt) and maximum damping factor (hmax) [tanδ =0.85]

Since the stiffness of the VED γ' or KD varies depending on ambient temperature as shown in Fig. 4, we must be very careful about determining appropriate capacity of the VED. Based on Fig. 4, let the tanδ of VED of diene elastomer be constant, about 0.85, regardless of the variation of ambient temperature and frequency. Then, h and fV/fF of the system just vary along the curve in Fig. 8(a) and (b), respectively. This means that in Eqs. (9) and (10), the stiffness ratio of the VED β only varies corresponding to the variation of ambient temperature.

Thus, we can find the capacity of the VED that maximizes the minimum damping factor hmin within the temperature range from T1 to T2 as in the following. First, X is defined as the ratio of the stiffness of the VED at T1 to that at T2 as Eq. (13). Substituting $\beta=\beta$T1 and $\beta=\beta$T2=XβT1 into Eq. (9), respectively, and solving the equation hT1=hT2, βT1 which maximizes hmin is obtained as Eq. (14). Considering Eq. (8), the capacity of the VED to be installed is determined as Eq. (15). The value of the γ'T is determined from the experimental data shown in Fig. 4(a). Eq. (12) and (16) express hmax and hmin, respectively.

Figure 10 shows h and fV/fF in the case of α=3.0, tanδ=0.85, T1=40℃, T2=10℃ and X = 13.0. The value of X is determined by substituting T=40 and T=10 into Eq. (3), respectively, and λ40 and λ10 into Eq. (13). It is shown that h varies within the range from 8.4 % to 12.4 % as the temperature goes down from 40℃ to 10℃.



(a) Damping factor **(b) Natural frequency ratio**

Figure 10: Variation of damping effects of SDOF model with VED [tanδ=0.85, X=13.0]

$$X=\frac{\beta_{T2}}{\beta_{T1}}=\frac{\gamma'_{T2}}{\gamma'_{T1}} \tag{13}$$

$$\beta_{T1}=\frac{1}{\sqrt{(1+\alpha)(1+\tan^2\delta)X}}\alpha=\frac{\beta_{opt}}{\sqrt{X}} \tag{14}$$

$$\left(\frac{S}{d}\right)=\frac{\beta_{T1}}{\gamma'_{T1}}K_F \tag{15}$$

$$h_{\min}=\frac{\alpha\tan\delta}{2\{\alpha+2+(\sqrt{X}+\frac{1}{\sqrt{X}})\sqrt{(1+\alpha)(1+\tan^2\delta)}\}} \tag{16}$$

Optimum damping for non-linear system

In the preceding section, the main frame was assumed to be elastic (KF = constant). However, when buildings are exposed to strong ground motion, they exhibit inelastic behaviors with accompanying stiffness degradation and an increase of hysteresis energy. Therefore, we should find the appropriate capacity of the VED for non-linear systems. Based on an equivalent linearization technique, behavior of inelastic models of frames is represented by the reduction ratio of stiffness keq and the equivalent damping factor heq corresponding to the maximum ductility factor μ. Using keq and heq, the equation of motion and the damping factor of the SDOF inelastic system are expressed by Eqs. (17) and (18), respectively.

$$M\ddot{x}+\{(k_{eq}+k')+(2k_{eq}h_{eq}+k'')i\}K_F x=-M\ddot{z} \tag{17}$$

$$h=\frac{2k_{Feq}h_{Feq}+k''}{2(k_{Feq}+k')} \tag{18}$$

Following the same manner as in the case of elastic models, Eq. (19) gives the stiffness ratio of the VED βT1 which maximizes hmin. Symbols a, b and c in Eq. (19) are given as Eqs. (20), (21) and (22), respectively. The optimum capacity of the VED for inelastic models can be determined by Eq.(15).

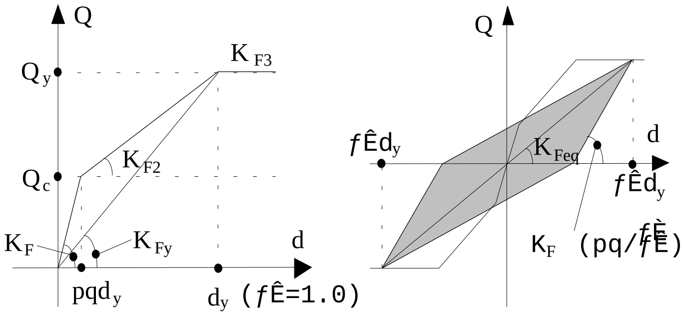
$$\beta_{T1}=\frac{-b+\sqrt{b^2-ac}}{a}\alpha \tag{19}$$

$$a=(1+\tan^2\delta)(2k_{Feq}h_{Feq}+k_{Feq}\tan\delta+\alpha\tan\delta)X \tag{20}$$

$$b=k_{Feq}h_{Feq}(1+\tan^2\delta)(X+1) \tag{21}$$

$$c=k_{Feq}(2h_{Feq}-\tan\delta) \tag{22}$$

Figure 11 shows restoring force characteristics of a modified Takeda model which is frequently used to simulate dynamic behaviors of RC frames. Symbols p and q in Fig. 11 are constants defined by Eq. (23) and (24), respectively. Fig. 12 shows βT1 for the modified Takeda model in the case of p = q = 1/3 and κ = 0.4. Each curve corresponds to βT1 for $\mu \leqq 1/9$, μ = 1.0, 2.0 and 3.0. When $\mu \leqq 1/9$, Eq. (19) coincides with Eq. (11) since keq =1.0 and heq = 0.0. Fig. 12 shows that the capacity of the VED should be reduced if inelastic behavior of the main frame is expected.



(a) Skeleton curve (b) Hysteresis loop

Figure 11: Restoring characteristics of modified Takeda model

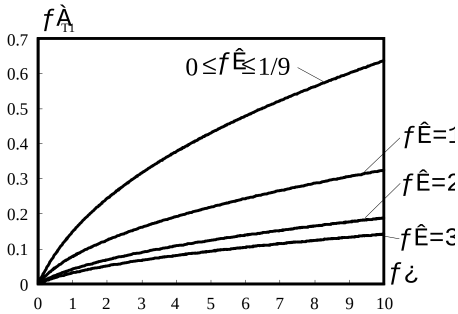


Figure 12: Stiffness ratio of VED for modified Takeda model

$$p = \frac{K_{Fy}}{K_F} \qquad (23)$$

$$q = \frac{Q_c}{Q_y} \qquad (24)$$

EARTHQUAKE RESPONSE SPECTRUM OF SDOF LNEAR SYSTEM WITH VED

Here, we discuss earthquake response of SDOF linear models with the VED. Time history of a simulated ground motion acceleration is shown in Fig. 13. The ground motion is the one that has been frequently used in Japan to assure the seismic stability of tall or base isolated buildings. It is regarded as the strongest ground motion while the buildings exist. We call this ground motion Level 2. Supposing a stronger ground motion which extremely rarely occurs, Level 2 is scaled up to 1.5 times. This ground motion is called Level 3 here. The damping factor of the frame without the VED is assumed to be 2.0 %.

In the case of α = 3.0, tanδ = 0.85, T1 = 40 ℃, T2 = 10 ℃ and X = 13.0, appropriate β40 is calculated to be 0.317 from Eq. (14), and it indicates that the stiffness of the VED at 40 ℃ is about 30 % of that of the frame alone. The Damping factor and natural frequency ratio of the SDOF model vary as shown in Fig. 10.

Figure 14 shows linear response spectra of SDOF models with and without the VEDs. In the case of a spectrum with the VED, each line indicates a spectrum at 40 ℃, 10 ℃ (hmin) and 23.5 ℃ (hmax). From Fig. 14(a), it is evident that the VED of the proper capacity can reduce the displacement response greatly, regardless of the variation of temperature. Fig. 14(b) shows that the acceleration response of short-period systems is also greatly reduced. In addition, it is shown that the VED makes spectra smoother than the supectrum without VED. This suggests that the VED is expected to make the response of buildings stable enough to get more exact predictions.

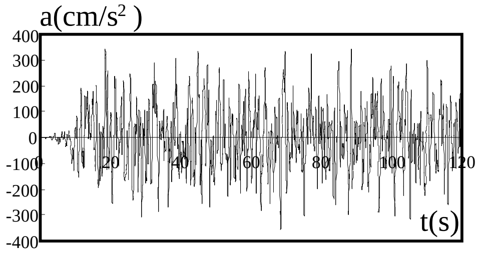
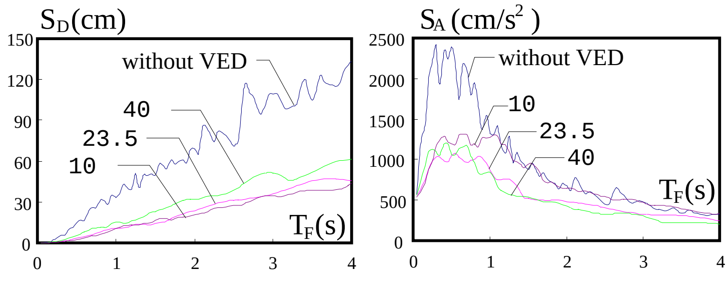


Figure 13: Time history of simulated ground acceleration



(a) Displacement (b) Acceleration

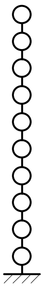
Figure 14: Response spectrum of linear system with VED

EARTHQUAKE RESPONSE ANALYSIS OF MDOF INELASTIC SYSTEM WITH VEDS

Standard building model

Next, we concentrate on the role of VEDs in multi-story RC buildings. A ten-DOF inelastic model (Fig. 15) is used. Restoring force characteristics of each story is the modified Takeda model shown in Fig. 11. Base shear coefficient is 0.3. Yielding shear force Qy and elastic stiffness KF are distributed along the height in proportion to the design shear force of the story. Cracking shear force Qc is 1/3 of Qy, and yielding deformation dy is 1.6 (cm): i.e., 1/200 of story height. Mechanical properties of the building model are shown in Table 1. The elastic period and the damping factor of the first mode are 0.64 (s) and 2.0 %, respectively, as listed in Table 2. This ten-DOF Building model is named SFM (Standard Frame Model).

Table 1: Mechanical properties of SFM



Story	M (tfs^2/cm)	K_F (tf/cm)	K_{F2} (tf/cm)	K_{F3} (tf/cm)	Q_c (tf)	Q_y (tf)	d_c (cm)	d_y (cm)	K_{Fy}/K_F
10	1.0	1258.8	K_F/3.89	0.00	Q_y/3.00	688.0	0.169	1.60	1/2.93
9		2034.2				1111.8			
8		2690.6				1470.6			
7		3262.7				1783.3			
6		3763.9				2057.3			
5		4201.2				2296.3			
4		4578.6				2502.5			
3		4898.7				2677.5			
2		5163.5				2822.2			
1		5374.2				2937.4			

Table 2: Natural period and damping factor of SFM

Mode	T_F(s)	h(%)
1	0.64	2.0
2	0.25	5.1
3	0.16	8.2

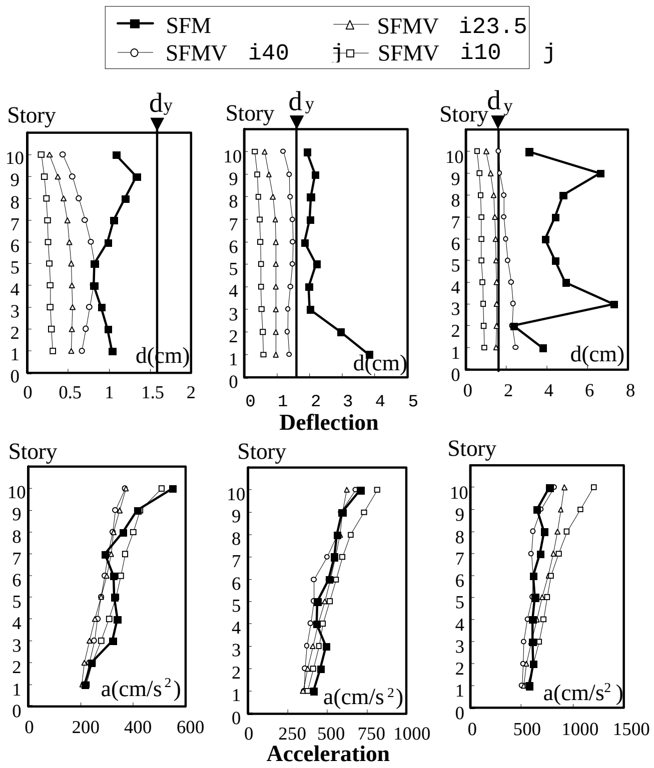
Figure 15: Ten-DOF model

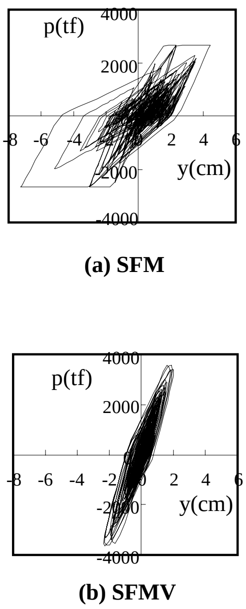
The VED is installed in every story of SFM parallel to the frame by means of supporting members. Assuming that α = 3.0, tanδ = 0.85, T1 = 40, T2 = 10 and X = 13.0, appropriate β40 for each story becomes 0.092 from Eq. 19. This indicates that the stiffness of the VED in each story is less than 10 % of the elastic stiffness KF of the same story. The capacity of the VED S/d for each story calculated from Eq. (15) is shown in Table 3. SFM with VEDs is named SFMV (Standard Frame Model with VEDs).

Table 3: Capacity of VED installed in each story of SFMV

Story	ƒ¿	ƒÁ'$_{40}$ (~10^{-3} tf^2/cm)	ƒÀ$_{40}$	S/d (cm)
10	3.0	1.20	0.092	96456
9				155864
8				206160
7				249999
6				288407
5				321912
4				350828
3				375358
2				395644
1				411789

Here, three ground motions are considered, i.e., Level1, Level2 and Level3. The first one is a simulated ground motion that is used as well as Level2. Level1 is about 0.5 times as strong as Level2, and considered as the one that occurs a few times in the lifetime of the buildings.





(a) To Level 1 (b) To Level 2 (c) To Level 3
Figure 16: Maximum response of SFM and SFMV

Figure 17: Hysteresis loops to Level 3 (at 40 ℃)

Figures 16(a), (b) and (c) show the maximum response of SFM and SFMV to Level 1, 2 and 3 ground motions, respectively. In the case of SMFV, lines with ○, and □ correspond to the response at 40 , 23.5 and 10 , respectively. Upper figures show the maximum deflection of the columns d and the lower the maximum acceleration of the masses a. A bold straight line in each upper figure represents yielding deflection dy = 1.6 (cm), i.e., μ = 1.0.

These figures show that VEDs reduce the maximum deflection without causing excessive acceleration. It should be noted that even a well-designed model is not free from such structural failure as that caused by damage concentration when it is exposed to Level 2 and Level 3 ground motions. However, the maximum deflection of the SFMV is only 2.5 (cm), which corresponds to μ = 1.5. Fig. 17 shows the hysteresis loops of the 3rd story of SFM and SFMV to Level 3 ground motion. In the case of the latter, a result at 40 ℃ is shown. From Fig. 17(b), it is found that both frame and VED absorb energy steadily without excessive deformation. It suggests that VEDs can be expected to prevent terrible damage and collapse.

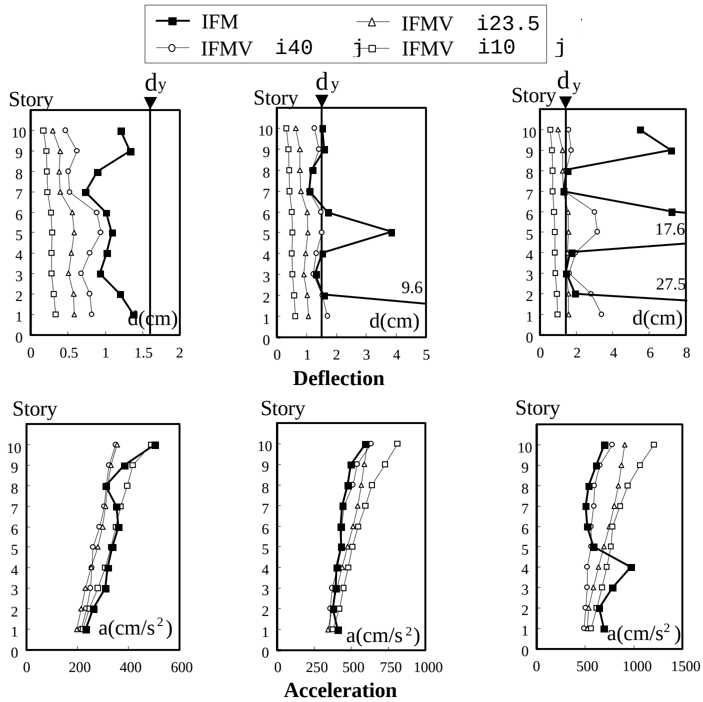
Irregular building model

Effects of the VED on a well-designed building are studied in the previous section. However, mechanical properties, such as stiffness and strength, of actual buildings do not always coincide with those that designers predict.

To take uncertain properties into consideration, an irregular model is generated by multiplying the stiffness and the strength of the SFM by random numbers whose mean value is 1.0 and standard derivation is 1/15. This model is named IFM (Irregular Frame Model). Table 4 lists the random numbers by which the stiffness and the strength of the SFM is to be multiplied. VEDs of the same capacity listed in table 3 are installed in this IFM. IFM with VEDs is named IFMV (Irregular Frame Model with VEDs). Fig. 18 shows the maximum response of IMF and IFMV in the same manner as Fig. 16. Figures 18(b) and (c) show that the maximum deflection of IFM are much greater than dy due to Level 2 and Level 3. It implies that buildings with uncertain variance may collapse even if they are subject to a considerable ground motion as well as an extreme one.

Table 4: Random number

Story	Random Number
10	0.92
9	0.92
8	1.10
7	1.16
6	0.86
5	0.84
4	0.95
3	1.04
2	0.89
1	0.83



(a) To Level 1 **(b) To Level 2** **(c) To Level 3**

Figure 18: Maximum response of IFM and IFMV

The largest maximum deformation of the IFMV is suppressed to 3.4 (cm), which corresponds to μ = 2.1. As a result, we can expect VEDs to prevent the collapse of multi-story buildings, whose mechanical properties are varied uncertainly, even when exposed to extremely strong ground motion. Thus, VED is regarded as so effective not only in the design of new buildings but also in the retrofit of existing ones.

CONCLUSION

First, mechanical properties and an analytical model of a viscoelastic damper (VED) are outlined. Then, the damping factor and the natural frequency of an SDOF system with the VED are formulated. The optimum capacity of the VED, which maximizes the damping factor within a range of ambient temperature, is quantified. Then earthquake response spectra of SDOF models with the VED of the optimum capacity are shown. They suggest that the VED can make earthquake response of buildings stable and facilitate us to make more exact predictions as well as reduce the maximum response. As a result of earthquake response analyses of MDOF inelastic models, installation of VEDs of appropriate capacity is proved to be very effective in preventing collapse of actual multi-story buildings, even if their mechanical properties have uncertain variance, to extremely strong ground motions. As a consequence, it is concluded that the VED can be expected to assure the seismic safety of existing buildings as well as new ones.

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